Problem 1

\[ \mu = 0.8 \frac{c}{c} = \frac{4}{5} \frac{c}{c} \quad ; \quad \gamma = \frac{1}{\sqrt{1 - \frac{\mu^2 c^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{16}{25}}} = \frac{3}{5} \]

Time on earth is dilated. \( \tau = 240 \text{ ns} \) = proper time

\[ \Delta t = \gamma \tau = \frac{3}{5} \times 240 \text{ ns} = 400 \text{ ns} \]

(a) Distance traveled in earth’s reference frame:

\[ D = \mu \times \Delta t = \frac{4}{5} \times 300,000 \text{ km} \times 400 \times 10^{-9} \text{ s} = 0.096 \text{ km} \]

\[ D = 96 \text{ m} \]

(b) According to moving observer, time is \( \tau \), not \( \Delta t \)

\[ D' = D \cdot \frac{3}{5} = 57.6 \text{ m} \]
Problem 2

(a) \( x_1 = \text{LA}, \ x_2 = \text{NY} \)
\[ t_1 = \text{candle in LA}, \ t_2 = \text{candle in NY} \]
\[ t_1 = t_2 = t \text{ since candles are lighted simultaneously in earth's frame} \]
\[ t_1' = \gamma (t_1 - \frac{ux_1}{c^2}) \Rightarrow \text{since } t_1 = t_2, \text{ subtract } \]
\[ t_2' = \gamma (t_2 - \frac{ux_2}{c^2}) \]
\[ t_2' - t_1' = -\frac{ux_2}{c^2} (x_2 - x_1) < 0 \text{ since } x_2 > x_1, \]
so \( t_2' < t_1' \Rightarrow \text{candle was first lighted in NY,} \]
\[ \Rightarrow (\text{ii}) \text{ the NY twin is older according to you} \]

(b) Assume candles are lit when plane is exactly midway between LA and NY.
Light from NY will get to the plane first, since it is moving towards NY.
You assume light travels at speed \( c \), according to Einstein.
Since you were midway when candles were lit, and got the light from NY first, you conclude that candle in NY was lit first \( \Rightarrow \text{NY twin is older.} \)
Problem 3

\[ \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{1}{4}}} = \frac{2}{\sqrt{3}} = 1.155 \]

\[ \Delta t_0 = \Delta t / \gamma \]

According to an observer in the car,

\[ \Delta t = \frac{600 \text{m}}{0.5 \text{c}} = \frac{600 \text{m} \cdot 5}{0.5 \times 3 \times 10^8 \text{m}} = 400 \times 10^{-8} \text{s} = 4 \mu \text{s} \]

So according to your stopwatch,

\[ \Delta t_0 = \frac{4 \mu \text{s}}{1.155} = 3.46 \mu \text{s} \] (a)

(b) According to observer on car/ground, \[ \Delta t = 4 \mu \text{s} \]
Problem 7

\[ c = \lambda f \]

When you are driving towards the light:

\[ f_1 = \frac{\lambda}{\sqrt{1 - \frac{v}{c}}} \]

When you are driving away from the light:

\[ f_2 = \frac{\lambda}{\sqrt{1 + \frac{v}{c}}} \]

\[ \frac{f_1}{f_2} = \frac{\lambda_2}{\lambda_1} = \frac{\text{red}}{\text{green}} = \frac{700}{550} \Rightarrow \gamma = \frac{1 + \frac{v}{c}}{1 - \frac{v}{c}} = 1.2727 \]

\[ \gamma (1 - \frac{v}{c}) = 1 + \frac{v}{c} \Rightarrow \gamma - \frac{v}{c} \gamma = 1 + \frac{v}{c} \Rightarrow \frac{\mu}{c} (\gamma + 1) = \gamma - 1 \Rightarrow \frac{\mu}{c} = \frac{\gamma - 1}{\gamma + 1} = 0.12 \]

\[ \Rightarrow \mu = 0.12c = 300,000 \frac{\text{km}}{s} \times 0.12 \]

\[ \Rightarrow \mu = 36,000 \text{ km/s} \] (a)

(b) On the police car's side:

\[ \lambda = \lambda_2 \sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}} = \lambda_1 \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}} \]

We can use either relation with the found value of \( \frac{v}{c} \)

\[ \text{or} \]

\[ \lambda^2 = \lambda_1 \cdot \lambda_2 \Rightarrow \lambda = \sqrt{\lambda_1 \lambda_2} \]

\[ \lambda = \sqrt{700 \times 550} \text{ nm} = 620 \text{ nm} \]

\[ \Rightarrow \text{police sees orange light} \]
Problem 5

\[ v' = \text{your speed with respect to wellway} \]

\[ \mu = 0.45c = \text{speed of wellway} \]

\[ v' = \frac{v - \mu}{1 - \frac{\mu v}{c^2}} \]

You want your speed \( v \) with respect to the ground to be \( 2\mu = 0.9c \)

\[ v' = \frac{2\mu - \mu}{1 - 2\mu^2} = \frac{0.45c}{1 - 2(0.45)^2} = 0.76c \]

\[ v' = 0.76c = \text{your speed with respect to wellway} \]

(b) According to Galileo, your speed with respect to the ground would be

\[ v_{\text{Gal}} = \mu + v' = 1.21c \]
Bonus problem

If you travel at speed \( u \), you properly is

\[
\Delta t = \sqrt{1 - \frac{u^2}{c^2}} \Delta t'
\]

You need to age at half the rate that you would on earth, so

\[
\sqrt{1 - \frac{u^2}{c^2}} = \frac{1}{2} \Rightarrow 1 - \frac{u^2}{c^2} = \frac{1}{4} \Rightarrow \frac{u^2}{c^2} = \frac{3}{4} = 0.25
\]

\[
\frac{u}{c} = \frac{\sqrt{3}}{2} \Rightarrow u = 0.866c = 259,808 \text{ km/s}
\]

Time on earth = .2 years = \( 2 \times 365 \times 24 \times 3600 \text{ s} = 6.3 \times 10^7 \text{ s} = \Delta t'

Distance for one trip to moon and back:

\( D = 2 \times 384,000 \text{ km} = 768,000 \text{ km} \)

So the number of times, \( N \) is

\[
N = \frac{u \cdot \Delta t'}{D} = \frac{259,808 \times 6.3 \times 10^7}{768,000}
\]

\[
N = 21,312,375 \text{ times}
\]

Alternative answer: you figure out a way to travel at the speed of light. Then, you don't age at all. Doing that in 1 year, you travel a distance \( D' = C \times 3.15 \times 10^7 \text{ s} = 9.46 \times 10^{12} \text{ km} \)

that is

\[
N = \frac{D'}{D} = 12,318,750 \text{ times}
\]