Justify all your answers to all problems. Write clearly.

Time dilation; Length contraction: $\Delta t = \gamma \Delta t_0$ ; $L = L_0 / \gamma$ ; $c = 3 \times 10^8 \text{m/s}$

Lorentz transformation: $x' = \gamma(x - ut)$ ; $y' = y$ ; $t' = \gamma(t - ux / c^2)$

Velocity: $v'_x = \frac{v_x - u}{1 - u v_x / c^2}$ ; $v'_y = \frac{v_y}{\gamma(1 - u v_x / c^2)}$ ; $\gamma = \frac{1}{\sqrt{1 - u^2 / c^2}}$

Inverse transformations: $u \rightarrow -u, \text{primed} \leftrightarrow \text{unprimed}$; Doppler: $f' = f \sqrt{\frac{1 + u / c}{1 - u / c}}$

Momentum: $\vec{p} = \gamma m \vec{v}$ ; Energy: $E = \gamma mc^2$ ; Kinetic energy: $K = (\gamma - 1)mc^2$

$E = \sqrt{p^2 c^2 + m^2 c^4}$ ; rest energy: $E_0 = mc^2$

Electron: $m_e = 0.511 \text{MeV/c}^2$ ; Proton: $m_p = 938.26 \text{MeV/c}^2$ ; Neutron: $m_n = 939.55 \text{MeV/c}^2$

Atomic unit: $1u = 931.5 \text{MeV/c}^2$ ; electron volt: $1eV = 1.6 \times 10^{-19} J$

Photoelectric effect: $eV_s = K_{max} = hf - \phi = hc / \lambda - \phi$ ; $\phi$ = work function

Stefan law: $I = \sigma T^4$ ; $\sigma = 5.67037 \times 10^{-8} \text{W/m}^2 \cdot \text{K}^4$ ; Wien's law: $\lambda_m T = 2.8978 \times 10^{-3} \text{m} \cdot \text{K}$

$I(T) = \int_0^\infty I(\lambda, T) d\lambda$ ; $I = (c / 4)u$ ; $u(\lambda, T) = N(\lambda)E_{av}(\lambda, T)$ ; $N(\lambda) = \frac{8\pi}{\lambda^5}$

Boltzmann distribution: $N(E) = Ce^{E/kT}$ ; $N = \int_0^\infty N(E) dE$ ; $E_{av} = \frac{1}{N} \int_0^\infty EN(E) dE$

Classical: $E_{av} = kT$ ; Planck: $E_n = n\epsilon = nhf$ ; $N = \sum_n N(E_n)$ ; $E_{av} = \frac{1}{N} \sum_n E_n N(E_n)$

Planck: $E_{av} = \frac{hc / \lambda}{e^{hc / \lambda kT} - 1}$ ; $hc = 1.240 \text{eV} \cdot \text{nm}$ ; $\lambda_m T = hc / 4.96 k$ ; $\sigma = \frac{2\pi^5 k^4}{15c^2 h^3}$

Boltzmann constant: $k = (1/11,604) eV / K$

Compton scattering: $\gamma = \frac{h}{m_c c}$ (1 cos ) ; $\frac{\gamma}{m_c} = 0.0243 A$

de Broglie: $\frac{h}{p} = f = \frac{E}{h}$ ; $= 2 f$ ; $k = \frac{2}{2} + E = h$ ; $p = h k$ ; $E = \frac{p^2}{2m}$

Wave packets: $y(x,t) = \sum_j a_j \cos(k_j x - \omega_j t)$, or $y(x,t) = \int dk a(k) e^{ik(x - \omega t)}$ ; $\Delta k \Delta x \sim 1$ ; $\Delta \omega \Delta t \sim 1$
group and phase velocity: \( v_g = \frac{d\omega}{dk} \); \( v_p = \frac{\omega}{k} \); Heisenberg: \( \Delta x\Delta p \sim \hbar \); \( \Delta t\Delta E \sim \hbar \)

Schrodinger equation: \( -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + U(x) \frac{(x,t)}{t} = i\hbar \frac{\partial}{\partial t} \frac{(x,t)}{t} \); \( (x,t) = (x)e^{-\frac{E}{\hbar}t} \)

Time independent Schrodinger equation: \( -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + U(x) \frac{(x)}{E} = \frac{\hbar^2}{2m} \frac{\partial}{\partial t} \frac{(x)}{t} \); \( \frac{dx}{*} = 1 \)

\( \propto \) square well: \( \psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \); \( E_n = \frac{\pi^2 \hbar^2 n^2}{2mL^2} \); \( \frac{\hbar^2}{2m_e} = 0.0381 eV nm^2 \) (electron)

2D square well: \( \Psi(x,y) = \Psi_1(x)\Psi_2(y) \); \( E = \frac{\pi^2 \hbar^2}{2m} \left( \frac{n_1^2}{L_1^2} + \frac{n_2^2}{L_2^2} \right) \)

Harmonic oscillator: \( \psi_n(x) = H_n(x)e^{-\frac{m_0 \omega x^2}{2\hbar}} \); \( E_n = (n + \frac{1}{2})\hbar \omega \); \( E = \frac{p^2}{2m} + \frac{1}{2} m_0 \omega x^2 = \frac{1}{2} m_0 \omega A^2 \)

Step potential: reflection coeff: \( R = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2} \); \( T = 1 - R \); \( k = \sqrt{\frac{2m}{\hbar^2}(E - U)} \)

Tunneling: \( \psi(x) \sim e^{-\alpha x} \); \( T = e^{-2\alpha \Delta x} \); \( \alpha(x) = \sqrt{\frac{2m[U(x) - E]}{\hbar^2}} \)

\( \hbar c = 1,240 eV \cdot nm \); \( \hbar c = 197.3 eV \cdot nm \)

Electron: \( m_e = 0.511 MeV / c^2 \); Proton: \( m_p = 938.26 MeV / c^2 \); Neutron: \( m_n = 939.55 MeV / c^2 \)

Rutherford scattering: \( N(\theta) = \text{constant} \times \frac{1}{\sin^2(\theta / 2)} \); \( U = \frac{kZ e^2}{r} \); \( k = 1/(4\pi\epsilon_0) = 1.44 eV \cdot nm \)

Bohr atom: \( r_n = (a_0 / Z)n^2 \); \( E_n = -E_0 Z^2 / n^2 \); \( E_0 = \frac{ke^2}{2a_0} \); \( a_0 = \frac{\hbar^2}{m_e ke^2} \); \( L = m_e \nu r = n\hbar \)

Spherically symmetric potential: \( n, \ell, m, (r, \theta, \phi) = P_{\ell m} (r) Y_{\ell m} (\theta, \phi) \); \( Y_{\ell m} (\theta, \phi) = P_{\ell m} (\theta) e^{im\phi} \)

Angular momentum: \( \vec{L} = \vec{r} \times \vec{p} \); \( \langle L_\ell \rangle = \frac{\hbar}{i} \frac{\partial}{\partial \phi} \); \( \langle L^2 \rangle = (\ell + 1)\hbar^2 Y_{\ell m}^* \); \( \langle L_z \rangle Y_{\ell m} = m_\ell h Y_{\ell m} \)

quantum numbers: \( n = 1, 2, 3, \ldots \); \( 0 \leq \ell \leq n - 1 \); \( -\ell \leq m_\ell \leq \ell \)

Radial probability density: \( P(r) = r^2 |R_{nl}(r)|^2 \); Energy: \( E_n = \frac{ke^2 Z^2}{2a_0 n^2} \)
Ground state of hydrogen-like ions: \[ Y_{1,0,0} = \frac{1}{\hbar^2} \frac{Z}{a_0} e^\frac{Zr}{a_0} \] 

Orbital magnetic moment: \[ \mu = -\frac{e}{2m_e} \vec{L} \] \[ \mu_z = -\mu_b m_i \] \[ \mu_b = \frac{e\hbar}{2m_e} = 5.79 \times 10^{-5} \text{eV/T} \]

Energy in a magnetic field: \[ U = -\vec{\mu} \cdot \vec{B} \]

Spin 1/2: \[ s = \frac{1}{2}, \quad |S| = \sqrt{s(s+1)} \hbar \] \[ S_z = m_i \hbar \] \[ m_s = \pm 1/2 \] \[ -s = \frac{e}{2m_e} g \vec{S} \quad g=2 \]

Orbital + spin mag moment: \[ \vec{\gamma} = \frac{e}{2m_e} (\vec{L} + g \vec{S}) \] Energy in mag. field: \[ U = -\vec{\gamma} \times \vec{B} \]

Subshells: 1s, 2s, 2p, 3s, 3p, 4s, 3d, 4p, 5s, 4d, 5p, 6s, 4f, 5d, 6p, 7s, 5f, 6d

2(2\ell + 1) electrons per subshell. Screened electron: \[ E_n = (-13.6\text{eV})Z_{\text{eff}}^2 / n^2 \]

12 problems total

**Problem 1** (6 points)

You are running at speed 0.6c and pass a long car that is at rest with respect to the ground. When you go by the back of the car you turn on your stopwatch, and you stop it when you go by the front of the car. Your stopwatch shows that 5µs went by.

(a) How long is this car, in meters, as measured by an observer in the car?
(b) According to an observer in the car, how long did it take for you to run by the car?

**Problem 2** (6 points)

Two equal masses m are moving towards each other at speed u. They collide and stick together. Their final mass is 3m.

(a) What is the value of u/c?
(b) When you stand on one of the masses, how fast is the other mass moving towards you? Give the answer as a fraction of c.

**Problem 3** (6 points)
A 100 W light bulb has a filament with surface area 500 mm$^2$.
(a) What is the temperature of the surface of the filament?
(b) At what wavelength (in nm) does this light bulb emit maximum power?

**Problem 4** (6 points)
An electron is moving at speed 3000 m/s in the +x direction. A photon moving in the -x direction scatters off it, and the electron reverses its motion, i.e. ends up moving in the -x direction at the same speed, 3000 m/s.
(a) What was the wavelength of the incident photon, in nm?
(b) What is the wavelength of the scattered photon, in nm?

**Problem 5** (6 points)
A proton and an electron both have de Broglie wavelength 0.001 nm.
(a) Find the kinetic energy of the proton, in eV.
(b) Find the kinetic energy of the electron, in eV.
Hint: one is relativistic, one is not.

**Problem 6** (6 points)
An electron is in a stationary state of an infinite square well of length 6A. It is equally likely to be found at distance 1A and 3A from the left wall, and its energy is less than 10eV.
(a) Find its energy in eV and its quantum number n.
(b) How much more likely is it to find this electron at a distance 1A than at a distance 0.5A from the left wall?

**Problem 7** (6 points)
When an electron in the second excited state of a one-dimensional infinite well of width L makes a transition to the first excited state, it emits a photon of wavelength 100 nm.
(a) What is the wavelength of the photon emitted when this electron makes a transition from the first excited state to the ground state?
(b) What is the length L of this well, in nm?

**Problem 8** (6 points)
An electron in the ground state of a one-dimensional harmonic oscillator potential has classical amplitude of oscillation 0.2 nm.
(a) Estimate its average kinetic energy (in eV) using the uncertainty principle.
(b) Calculate its ground state energy (in eV).

**Problem 9** (6 points)
When 10,000 electrons of energy $E=5\text{eV}$ are incident on the barrier of thickness $L=0.2\text{nm}$ shown above, 200 electrons tunnel through to the other side.

(a) If this barrier was instead $0.4\text{nm}$ thick, how many electrons would have tunneled through?

(b) What is the height of this barrier, in eV?

**Problem 10 (6 points)**
An electron in a Bohr orbit of a hydrogen-like ion has angular momentum $4\hbar$ and energy $-54.4\text{eV}$.

(a) What is the atomic number $Z$ of this ion (number of protons in the nucleus)?

(b) What is the radius of this orbit in terms of $a_0$, the Bohr radius?

**Problem 11 (6 points)**
An electron is in a 3d state of hydrogen, described by the wavefunction

$$\psi(r,\theta,\phi) = Cr^2 e^{-r/\alpha_0} \sin^2 \theta \, \rho \, \phi$$

(a) What is the angle that the angular momentum vector makes with the $z$-axis? Give your answer in degrees.

(b) What is the average value of the radius for this electron, in terms of $a_0$?

**Problem 12 (6 points)**
An electron is in the 2p state of hydrogen in the presence of a magnetic field $10\text{T}$. Ignoring electron spin, find the wavelengths of all the different photons that can be emitted when this electron makes a transition to the ground state. Give your answers in nm with 5 significant digits.