\[ A_2 = A \frac{\cos \pi/3}{\cos \phi} = 0.661A \]

3. \[ E_1 = \frac{\hbar^2}{8mL^2} = 5.6 \text{ eV} \]

With \( L' = 2L \),
\[ E'_1 = \frac{\hbar^2}{8mL'^2} = \frac{\hbar^2}{8m(2L)^2} = \frac{1}{4} \frac{\hbar^2}{8mL^2} = \frac{1}{4} (5.6 \text{ eV}) = 1.4 \text{ eV} \]

4. With \( \lambda_n = 2L/n \),
\[ \lambda_1 = \frac{2(0.144 \text{ nm})}{1} = 0.288 \text{ nm} \quad \lambda_2 = \frac{2(0.144 \text{ nm})}{2} = 0.144 \text{ nm} \quad \lambda_3 = \frac{2(0.144 \text{ nm})}{3} = 0.096 \text{ nm} \]

5. The smallest energy is (using Equation 5.3)
\[ E_1 = \frac{\hbar^2}{8mL^2} = \frac{(hc)^2}{8(mc^2)L^2} = \frac{(1240 \text{ eV} \cdot \text{nm})^2}{8(511,000 \text{ eV})(0.062 \text{ nm})^2} = 98 \text{ eV} \]

Then \( E_2 = 2^2 E_1 = 391 \text{ eV} \) and \( E_3 = 3^2 E_1 = 881 \text{ eV} \).

6. With \( L = 1.2 \times 10^{-14} \text{ m} = 10 \text{ fm} \),
\[ E_1 = \frac{\hbar^2}{8mL^2} = \frac{(hc)^2}{8(mc^2)L^2} = \frac{(1240 \text{ MeV} \cdot \text{fm})^2}{8(940 \text{ MeV})(12 \text{ fm})^2} = 1.4 \text{ MeV} \]

7. (a) At \( x = a \), \( \psi_1 = \psi_2 \) and \( d\psi_1 / dx = d\psi_2 / dx \):
\[ 0 = (a - d)^2 - c \quad \text{and} \quad -2ab = 2(a - d) \]
From the second equation, \( d = a(b+1) \). Inserting this into the first equation, we find \( c = a^2b^2 \).

(b) With \( \psi_2 = \psi_3 \) at \( x = w \), we get \((w-d)^2 - c = 0\), or

\[
 w = d + \sqrt{c} = a(b+1) + \sqrt{a^2b^2} = a(2b+1)
\]

The slope is discontinuous at \( w \) suggesting an infinite discontinuity in the potential energy at that location.

8. (a) The regions with \( x < a \) and \( x > a \) do not contribute to the normalization. The normalization integral is

\[
 \int |\psi(x)|^2 \, dx = \int_a^b b^2(a^2 - x^2)^2 \, dx = b^2 \int_a^b (a^4 - 2a^2x^2 + x^4) \, dx = b^2 \left( a^4x - 2a^2x^3 + \frac{x^5}{5} \right) \biggr|_{-a/2}^{+a/2}
\]

Evaluating the integral and setting it equal to 1, we find

\[
 b^2 \left( 2a^2 - 4a^4 + \frac{2a^5}{5} \right) = 1 \quad \text{or} \quad b = \sqrt{\frac{15}{16a^5}}
\]

(b) \( P(x) \, dx = |\psi(x)|^2 \, dx = b^2(a^2 - x^2)^2 \, dx \), and with \( x = a/2 \) and \( dx = 0.010a \) we obtain

\[
 P(x) \, dx = \frac{15}{16a^5} \left( \frac{a^2}{4} - a^2 \right)^2 (0.010a) = 0.0053
\]

(c) \( P(a/2 : a) = \int_{a/2}^a |\psi(x)|^2 \, dx = \int_{a/2}^a b^2(a^2 - x^2)^2 \, dx = \frac{15}{16a^5} \left( a^4x - 2a^2x^3 + \frac{x^5}{5} \right) \biggr|_{a/2}^{+a/2} = \frac{15}{16a^5} \left[ a^4 \left( a - \frac{a}{2} \right) - 2a^2 \left( a^3 - \frac{a^3}{3} \right) + \frac{1}{5} \left( a^5 - \frac{a^5}{32} \right) \right] = 0.104
\]

9. With \( \psi(x) = Cx e^{-bx} \), we have \( d\psi/dx = Ce^{-bx} - bCxe^{-bx} \) and

\[
 \frac{d^2\psi}{dx^2} = -2bCe^{-bx} + b^2Cxe^{-bx}
\]

We now substitute \( \psi(x) \) and \( d^2\psi/dx^2 \) into the Schrödinger equation:
\[-\frac{\hbar^2}{2m} (-2bCe^{-bx} + b^2Cxe^{-bx}) + U(x)Cxe^{-bx} = ECxe^{-bx}\]

Canceling the common factor of $Ce^{-bx}$ and solving for $E$,

\[E = \frac{\hbar^2 b}{mx} - \frac{\hbar^2 b^2}{2m} + U(x)\]

The energy $E$ will be a constant only if the two terms that depend on $x$ cancel each other:

\[\frac{\hbar^2 b}{mx} + U(x) = 0 \quad \text{or} \quad U(x) = -\frac{\hbar^2 b}{mx}\]

The cancellation of the two terms depending on $x$ leaves only the remaining term for the energy:

\[E = -\frac{\hbar^2 b^2}{2m}\]

10. (a) The regions with $x < -L/2$ and $x > +L/2$ do not contribute to the normalization integral. The remaining integral is:

\[\int \left| \psi(x) \right|^2 \, dx = \int_{-L/2}^{0} C^2 (2x/L + 1)^2 \, dx + \int_{0}^{L/2} C^2 (-2x/L + 1)^2 \, dx\]

\[= C^2 \left( 4x^3/3L^2 + 2x^2/L + x \right)_{-L/2}^{0} + C^2 \left( 4x^3/3L^2 - 2x^2/L + x \right)_{0}^{L/2} = C^2 L/3\]

Setting the integral equal to 1 gives $C = \sqrt{3/L}$.

(b) $P(x) \, dx = \left| \psi(x) \right|^2 \, dx = C^2 (4x^2/L^2 - 4x/L + 1) \, dx$ and with $x = 0.250L$ and $dx = 0.010L$,

\[P(x) \, dx = \frac{3}{L} \left( \frac{4 \, L^2}{L^2} - \frac{4 \, L}{4} + 1 \right) (0.010L) = 0.0075\]

(c) $P(0: L/4) = \int_{0}^{L/4} \left| \psi(x) \right|^2 \, dx = C^2 \int_{0}^{L/4} \left( 4 \, x^2/L^2 - 4 \, L \, x + 1 \right) \, dx = \frac{3}{L} \left( \frac{4 \, x^3}{3L^2} - \frac{4 \, x^2}{L^2} + x \right)_{0}^{L/4} = \frac{1}{2}$

(d) $\langle x \rangle = \int \left| \psi(x) \right|^2 \, x \, dx = \frac{3}{L} \int_{-L/2}^{0} \left( 4 \, x^2/L^2 + 4 \, x/L + 1 \right) \, x \, dx + \int_{0}^{L/2} \left( 4 \, x^2/L^2 - 4 \, x/L + 1 \right) \, x \, dx$
It is apparent from the shape of the wave function that the equal probability densities for positive and negative $x$ cancel to give an average of zero.

$$\langle x^2 \rangle = \int |\psi(x)|^2 x^2 \, dx = \frac{3}{L} \int_{-L/2}^{0} \left( \frac{4x^2}{L} + \frac{4x}{L} + 1 \right) x^2 \, dx + \int_{0}^{L/2} \left( \frac{4x^2}{L} - \frac{4x}{L} + 1 \right) x^2 \, dx$$

$$= \frac{3}{L} \left( \frac{4x^5}{5L^2} + \frac{x^4}{L} + \frac{x^3}{3} \right)_{-L/2}^{0} + \frac{3}{L} \left( \frac{4x^5}{5L^2} - \frac{x^4}{L} + \frac{x^3}{3} \right)_{0}^{L/2} = \frac{L^2}{40}$$

The rms value is then $x_{\text{rms}} = \sqrt{\langle x^2 \rangle} = \sqrt{L^2/40} = 0.158L$.

11. (a) The normalization integral is

$$\int_{-\infty}^{\infty} |\psi(x)|^2 \, dx = \int_{-\infty}^{\infty} A^2 x^2 e^{-2bx} \, dx = 2A^2 \int_{0}^{\infty} x^2 e^{-2bx} \, dx = 2A^2 \left( \frac{2}{(2b)} \right) = 1$$

so $A = \sqrt{2b^3}$.

(b) The wave function can be written as $\psi(x) = Ae^{bx}$ for $x < 0$ and $\psi(x) = Ae^{-bx}$ for $x > 0$.

$$\int_{-\infty}^{\infty} |\psi(x)|^2 \, dx = \int_{-\infty}^{0} A^2 e^{2bx} \, dx + \int_{0}^{\infty} A^2 e^{-2bx} \, dx = \frac{2A^2}{2b}$$

and $A = \sqrt{b}$.

12. For the $x > 0$ wave function, we have

$$\frac{d\psi}{dx} = -Abe^{-bx} \quad \text{and} \quad \frac{d^2\psi}{dx^2} = Ab^2 e^{-bx}$$

Substituting into the Schrödinger equation then gives

$$-\frac{\hbar^2}{2m} Ab^2 e^{-bx} + UAe^{-bx} = EAe^{-bx} \quad \text{or} \quad -\frac{\hbar^2 b^2}{2m} + U = E$$

This is consistent with a constant value of $U$, which we can take to be 0, giving $E = -\hbar^2 b^2 / 2m$. Repeating the calculation for the $x < 0$ wave function gives an identical result.
So the potential energy is a constant (zero) for \( x < 0 \) and for \( x > 0 \). What happens at \( x = 0 \)? Note that the wave function is continuous at \( x = 0 \) (both give \( \psi = A \) at \( x = 0 \)) but that the derivative \( d\psi/dx \) is not continuous. This suggests an infinite discontinuity in \( U(x) \) at \( x = 0 \), and because the wave functions approach 0 as \( x \to \infty \) there must be a negative potential energy that produces the bound states. So the potential energy is

\[
U(x) = \begin{cases} 
0 \quad x < 0, x > 0 \\
-\infty \quad x = 0
\end{cases}
\]

This type of function is known as a delta function.

13. (a) \[
E_1 = \frac{\hbar^2}{8mL^2} = \frac{(hc)^2}{8(mc^2)L^2} = \frac{(1240 \text{ eV} \cdot \text{nm})^2}{8(511,000 \text{ eV})(0.285 \text{ nm})^2} = 4.63 \text{ eV}
\]

\( 4 \to 1: \quad \Delta E = E_4 - E_1 = 16E_1 - E_1 = 15E_1 = 15(4.63 \text{ eV}) = 69.5 \text{ eV} \)

(b) \( 4 \to 3: \quad \Delta E = E_4 - E_3 = 16E_1 - 9E_1 = 7E_1 = 7(4.63 \text{ eV}) = 32.4 \text{ eV} \)

\( 4 \to 2: \quad \Delta E = E_4 - E_2 = 16E_1 - 4E_1 = 12E_1 = 12(4.63 \text{ eV}) = 55.6 \text{ eV} \)

\( 3 \to 2: \quad \Delta E = E_3 - E_2 = 9E_1 - 4E_1 = 5E_1 = 5(4.63 \text{ eV}) = 23.2 \text{ eV} \)

\( 3 \to 1: \quad \Delta E = E_3 - E_1 = 9E_1 - E_1 = 8E_1 = 8(4.63 \text{ eV}) = 37.0 \text{ eV} \)

\( 2 \to 1: \quad \Delta E = E_2 - E_1 = 4E_1 - E_1 = 3E_1 = 3(4.63 \text{ eV}) = 13.9 \text{ eV} \)

14. With \( E_1 = 1.54 \text{ eV} \) and \( E_n = n^2E_1 \) we have

\[
\Delta E_3 = E_3 - E_1 = 9E_1 - E_1 = 8E_1 = 8(1.54 \text{ eV}) = 12.3 \text{ eV}
\]

\[
\Delta E_4 = E_4 - E_1 = 16E_1 - E_1 = 15E_1 = 15(1.54 \text{ eV}) = 23.1 \text{ eV}
\]

15. \[
\int_0^L A^2 \sin^2 \left(\frac{n\pi x}{L}\right) dx = A^2 \left[ \frac{L}{n\pi} \right] \int_0^n \sin^2 u \, du \] with \( u = n\pi x / L \). The integral is a standard form that can be found in integral tables:

\[
A^2 \left[ \frac{L}{n\pi} \right] \int_0^n \sin^2 u \, du = A^2 \left[ \frac{L}{n\pi} \left( \frac{1}{2}u - \frac{1}{4}\sin 2u \right) \right]_0^n = A^2 \frac{L}{2}
\]

Setting the integral equal to 1 for normalization gives \( A^2L/2 = 1 \) or \( A = \sqrt{2/L} \).
16. (a) \( P(0: L/3) = \int_0^{L/3} |\psi_1(x)|^2 \, dx = \int_0^{L/3} \frac{2}{L} \sin^2 \frac{\pi x}{L} \, dx = \frac{2}{\pi} \int_0^{\pi/3} \sin^2 u \, du \)

\[
= \frac{2}{\pi} \left( \frac{u}{4} - \frac{\sin 2u}{4} \right) \Bigg|_0^{\pi/3} = 0.1955
\]

(b) \( P(L/3: 2L/3) = \int_{L/3}^{2L/3} |\psi_1(x)|^2 \, dx = \int_{L/3}^{2L/3} \frac{2}{L} \sin^2 \frac{\pi x}{L} \, dx = \frac{2}{\pi} \int_{\pi/3}^{2\pi/3} \sin^2 u \, du \)

\[
= \frac{2}{\pi} \left( \frac{u}{4} - \frac{\sin 2u}{4} \right) \Bigg|_{\pi/3}^{2\pi/3} = 0.6090
\]

(c) \( P(2L/3: L) = \int_{2L/3}^{L} |\psi_1(x)|^2 \, dx = \int_{2L/3}^{L} \frac{2}{L} \sin^2 \frac{\pi x}{L} \, dx = \frac{2}{\pi} \int_{2\pi/3}^{\pi} \sin^2 u \, du \)

\[
= \frac{2}{\pi} \left( \frac{u}{4} - \frac{\sin 2u}{4} \right) \Bigg|_{2\pi/3}^{\pi} = 0.1955
\]

17. (a) \( P(x)dx = |\psi_1(x)|^2 \, dx = \frac{2}{L} \sin^2 \frac{3\pi x}{L} \, dx = \frac{2}{0.189 \text{ nm}} \left( \frac{3\pi (0.188 \text{ nm})}{0.189 \text{ nm}} \right) 0.001 \text{ nm} = 2.63 \times 10^{-5} \)

(b) \( P(x)dx = \frac{2}{L} \sin^2 \frac{3\pi x}{L} \, dx = \frac{2}{0.189 \text{ nm}} \left( \frac{3\pi (0.031 \text{ nm})}{0.189 \text{ nm}} \right) 0.001 \text{ nm} = 0.0106 \)

(c) \( P(x)dx = \frac{2}{L} \sin^2 \frac{3\pi x}{L} \, dx = \frac{2}{0.189 \text{ nm}} \left( \frac{3\pi (0.079 \text{ nm})}{0.189 \text{ nm}} \right) 0.001 \text{ nm} = 5.42 \times 10^{-3} \)

(d) A classical particle has a uniform probability to be found anywhere within the region, so \( P(x)dx = (0.001 \text{ nm})/(0.189 \text{ nm}) = 5.29 \times 10^{-3} \).

18. With \( E = E_0 (n_x^2 + n_y^2) \) the levels above \( 50E_0 \) are as follows:

<table>
<thead>
<tr>
<th>( n_x )</th>
<th>( n_y )</th>
<th>( E )</th>
<th>( n_x )</th>
<th>( n_y )</th>
<th>( E )</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>4</td>
<td>52E_0</td>
<td>6</td>
<td>5</td>
<td>61E_0</td>
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<tr>
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<td>5</td>
<td>6</td>
<td>61E_0</td>
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<td>3</td>
<td>7</td>
<td>58E_0</td>
<td>1</td>
<td>8</td>
<td>65E_0</td>
</tr>
</tbody>
</table>
The level at $E = 65E_0$ is 4-fold degenerate.

19. With $E = E_0(n_x^2 + n_y^2 / 4)$ the levels are as follows:

<table>
<thead>
<tr>
<th>$n_x$</th>
<th>$n_y$</th>
<th>$E$</th>
<th>$n_x$</th>
<th>$n_y$</th>
<th>$E$</th>
</tr>
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<td>3</td>
<td>6.25$E_0$</td>
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<td>5</td>
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<td>1</td>
<td>9.25$E_0$</td>
</tr>
<tr>
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<td>2</td>
<td>5.00$E_0$</td>
<td>1</td>
<td>6</td>
<td>10.00$E_0$</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>5.00$E_0$</td>
<td>3</td>
<td>2</td>
<td>10.00$E_0$</td>
</tr>
</tbody>
</table>

The levels at $E = 5.00E_0$ and $E = 10.00E_0$ are both 2-fold degenerate.

20. Using Equations 5.39 and 5.40, we have

$$\frac{\partial \psi}{\partial x} = g(y) \frac{df}{dx} = g(y)(k_x A \cos k_x x - k_x B \sin k_x x)$$

$$\frac{\partial^2 \psi}{\partial x^2} = g(y) \frac{d^2 f}{dx^2} = g(y)(-k_x^2 A \sin k_x x - k_x^2 B \cos k_x x) = -k_x^2 g(y) f(x)$$

$$\frac{\partial \psi}{\partial y} = f(x) \frac{dg}{dy} = f(x)(k_y C \cos k_y y - k_y D \sin k_y y)$$

$$\frac{\partial^2 \psi}{\partial y^2} = f(x) \frac{d^2 g}{dy^2} = f(x)(-k_y^2 C \sin k_y y - k_y^2 D \cos k_y y) = -k_y^2 f(x) g(y)$$

With $U(x, y) = 0$ inside the well, Equation 5.37 gives

$$-\frac{\hbar^2}{2m} \left[ -k_x^2 f(x) g(y) - k_y^2 f(x) g(y) \right] = Ef(x) g(y)$$

and so $E = \frac{\hbar^2}{2m} (k_x^2 + k_y^2)$. 

---

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21. Let $E_0 = \hbar^2 \pi^2 / 2mL^2$. The energy states are then

<table>
<thead>
<tr>
<th>$n_x$</th>
<th>$n_y$</th>
<th>$n_z$</th>
<th>$E$</th>
<th>Degeneracy</th>
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<table>
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<th>Degeneracy</th>
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<td>(2,2,3), (2,3,2), (3,2,2)</td>
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<td>$12E_0$</td>
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<tr>
<td>$3E_0$</td>
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22. With $\psi(x) = Ae^{-ax^2}$, the normalization integral is

$$\int_{-\infty}^{\infty} |\psi(x)|^2 \, dx = A^2 \int_{-\infty}^{\infty} e^{-2ax^2} \, dx = 2A^2 \int_{0}^{\infty} e^{-2ax^2} \, dx = A^2 \sqrt{\frac{\pi}{2a}}$$

The integral is a standard form that can be found in integral tables. Setting this result equal to 1 for the normalization condition and using Equation 5.49 for $a$, we obtain $A^2 \sqrt{\pi/2a} = 1$ or