\[
\lambda = \frac{d \Delta y}{D} = \frac{(8 \times 10^{-6} \text{ m})(9 \times 10^{-6} \text{ m})}{0.64 \text{ m}} = 0.11 \text{ nm}
\]

9. For \( m = 10^{-9} \text{ g} \) and taking the density to be \( \rho = 2 \text{ g/cm}^3 \), the volume of the particle is \( V = m/\rho = (10^{-9} \text{ g})/(2 \text{ g/cm}^3) = 5 \times 10^{-10} \text{ cm}^3 \), which corresponds to a diameter of about \( 0.001 \text{ cm} = 10^{-5} \text{ m} \). The spacing between the fringes is then

\[
\Delta y = \frac{\lambda D}{d} = \frac{(6.6 \times 10^{-20} \text{ m})(5 \times 10^6 \text{ m})}{10^{-5} \text{ m}} = 3.3 \times 10^{-8} \text{ m} = 33 \text{ nm}
\]

which is about the size of an atom!

10. \( \lambda = \frac{d \sin \phi}{2} = \frac{(0.215 \text{ nm})(\sin 55^\circ)}{2} = 0.0881 \text{ nm} \)

\[
p c = \frac{h c}{\lambda} = \frac{1240 \text{ eV} \cdot \text{ nm}}{0.0881 \text{ nm}} = 1.408 \times 10^4 \text{ eV}
\]

\[
K = \frac{p^2}{2m} = \frac{(pc)^2}{2mc^2} = \frac{(1.408 \times 10^4 \text{ eV})^2}{2(0.511 \times 10^6 \text{ eV})} = 194 \text{ eV}
\]

To achieve this kinetic energy, the electrons must be accelerated through a potential difference of \( \Delta V = +194 \text{ V} \).

11. \( p = \sqrt{2mK} = \frac{1}{c} \sqrt{2mc^2K} = \frac{1}{c} \sqrt{2(0.511 \times 10^6 \text{ eV})(175 \text{ eV})} = 1.337 \times 10^4 \text{ eV/c} \)

\[
\lambda = \frac{h}{p} = \frac{1240 \text{ eV} \cdot \text{ nm}}{1.337 \times 10^4 \text{ eV}} = 0.0927 \text{ nm}
\]

For \( n = 1 \):

\[\phi = \sin^{-1} \frac{\lambda}{d} = \sin^{-1} \frac{0.0927 \text{ nm}}{0.352 \text{ nm}} = 15.2^\circ\]

For \( n = 2 \):

\[\phi = \sin^{-1} \frac{2\lambda}{d} = \sin^{-1} \frac{2(0.0927 \text{ nm})}{0.352 \text{ nm}} = 31.8^\circ\]

For \( n = 3 \):

\[\phi = \sin^{-1} \frac{3\lambda}{d} = \sin^{-1} \frac{3(0.0927 \text{ nm})}{0.352 \text{ nm}} = 52.2^\circ\]

There is no diffracted beam for \( n = 4 \).

12. The locations of the interference maxima on the screen are given by \( d \sin \theta = n\lambda \), and for small angles we have \( \sin \theta \approx \tan \theta = y_n/D \), where \( y_n \) is the location of the \( n^{\text{th}} \) interference
\[ \Delta \lambda \sim \frac{\epsilon \lambda^2}{\Delta x} = \frac{(0.1)(12.6 \text{ cm})^2}{1.5 \times 10^2 \text{ cm}} = 0.11 \text{ cm} \]

16. The central frequency is \( f = c/\lambda = (2.997 \times 10^8 \text{ m/s})/(0.275 \text{ m}) = 1.09 \times 10^9 \text{ Hz} \). The frequency range is
\[ \Delta f \sim \frac{\epsilon}{\Delta t} = \frac{0.1}{1.27 \times 10^{-6} \text{ s}} = 7.9 \times 10^4 \text{ Hz} \]

The receiver should accept signals in a range of \( 7.9 \times 10^4 \text{ Hz} \) about a frequency of \( 1.09 \times 10^9 \text{ Hz} \).

17. For \( \Delta f = 10^4 \text{ Hz} \),
\[ \Delta t \sim \frac{\epsilon}{\Delta f} = \frac{0.1}{10^4 \text{ Hz}} = 10^{-5} \text{ s} \]

The signal processing time must be at least \( 10^{-5} \text{ s} \).

18.
\[ \lambda = \frac{\Delta x}{N} = \frac{148 \text{ m}}{36} = 4.11 \text{ m} \]
\[ \Delta \lambda \sim \frac{\epsilon \lambda^2}{\Delta x} = \frac{(0.1)(4.11 \text{ m})^2}{148 \text{ m}} = 0.011 \text{ m} \]

19. With \( \Delta v = 2.8 \times 10^4 \text{ m/s} \),
\[ \Delta x \sim \frac{\hbar}{\Delta p} = \frac{\hbar}{m \Delta v} = \frac{1.05 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.11 \times 10^{-31} \text{ kg})(2.8 \times 10^4 \text{ m/s})} = 4.1 \times 10^{-9} \text{ m} = 5.8 \text{ nm} \]

20. (a) \[ \Delta p \sim \frac{\hbar}{2 \pi \Delta x} = \frac{\hbar}{2 \pi} = \frac{1}{2 \pi} \frac{hc}{2 \pi (0.1 \text{ nm})} = \frac{1}{2 \pi} \frac{1240 \text{ eV} \cdot \text{nm}}{2 \pi (0.1 \text{ nm})} = 2000 \text{ eV} \cdot \text{c} \]

(b) \[ K = \frac{(\Delta p)^2}{2m} = \frac{(c \Delta p)^2}{2mc^2} = \frac{(2000 \text{ eV})^2}{2(0.511 \times 10^6 \text{ eV})} = 4 \text{ eV} \]

21. \[ \Delta E \sim \frac{\hbar}{\Delta t} = \frac{6.58 \times 10^{-16} \text{ eV} \cdot \text{s}}{2.0 \times 10^{-23} \text{ s}} = 33 \text{ MeV} \]

Measurements of the \( \Sigma^+ \) rest energy are likely to fall in the range 1385 MeV ± 33 MeV, or from 1352 MeV to 1418 MeV.
22. \[ \Delta t \sim \frac{\hbar}{\Delta E} = \frac{6.58 \times 10^{-16} \text{eV} \cdot \text{s}}{120 \times 10^6 \text{eV}} = 5.5 \times 10^{-24} \text{s} \]

23. \[ \Delta E \sim \frac{\hbar}{\Delta t} = \frac{6.58 \times 10^{-16} \text{eV} \cdot \text{s}}{2.1 \times 10^{-9} \text{s}} = 3.1 \times 10^{-7} \text{eV} \]

24. With \( \Delta E / E = 10^{-15} \), we have

\[ \Delta E = 10^{-15} E = 10^{-15} (75 \times 10^3 \text{ eV}) = 7.5 \times 10^{-11} \text{ eV} \]

\[ \Delta t \sim \frac{\hbar}{\Delta E} = \frac{6.58 \times 10^{-16} \text{eV} \cdot \text{s}}{7.5 \times 10^{-11} \text{ eV}} = 0.88 \times 10^{-5} \text{ s} \]

25. As we did for electrons in Example 4.9, let’s find the kinetic energy of an alpha particle with a momentum of 19.7 MeV/c:

\[ K = \frac{p^2}{2m} = \frac{(pc)^2}{2mc^2} = \frac{(19.7 \text{ MeV})^2}{2(3727 \text{ MeV})} = 0.052 \text{ MeV} \]

This is negligible compared with the typical kinetic energies of alpha particles emitted in radioactive decays. Therefore, the uncertainty principle does not limit the existence of these alpha particles inside the nucleus.

26. With \( \Delta x = 14 \text{ fm} \), we have

\[ \Delta p_x = \frac{\hbar}{\Delta x} = \frac{\hbar c}{c \Delta x} = \frac{1197 \text{ MeV} \cdot \text{fm}}{14 \text{ fm}} = 14.1 \text{ MeV}/c \]

With this uncertainty as an estimate for \( p_x \),

\[ K = \frac{p_x^2}{2m} = \frac{c^2 p_x^2}{2mc^2} = \frac{(14.1 \text{ MeV})^2}{2(938 \text{ MeV})} = 0.11 \text{ MeV} \]

This is a very small contribution to the energy of protons or neutrons in a nucleus, which are typically 10-20 MeV.

27. \[ y(x) = \int_{k_0 - \Delta k/2}^{k_0 + \Delta k/2} A(k) \cos kx \, dk = A_0 \int_{k_0 - \Delta k/2}^{k_0 + \Delta k/2} \cos kx \, dk = A_0 \left[ \frac{\sin kx}{x} \right]_{k_0 - \Delta k/2}^{k_0 + \Delta k/2} \]
22. \[ \Delta t \sim \frac{\hbar}{\Delta E} = \frac{6.58 \times 10^{-16} \text{ eV} \cdot \text{s}}{120 \times 10^6 \text{ eV}} = 5.5 \times 10^{-24} \text{ s} \]

23. \[ \Delta E \sim \frac{\hbar}{\Delta t} = \frac{6.58 \times 10^{-16} \text{ eV} \cdot \text{s}}{2.1 \times 10^{-9} \text{ s}} = 3.1 \times 10^{-7} \text{ eV} \]

24. With \( \Delta E / E = 10^{-15} \), we have

\[ \Delta E = 10^{-15} E = 10^{-15} (75 \times 10^3 \text{ eV}) = 7.5 \times 10^{-11} \text{ eV} \]

\[ \Delta t \sim \frac{\hbar}{\Delta E} = \frac{6.58 \times 10^{-16} \text{ eV} \cdot \text{s}}{7.5 \times 10^{-11} \text{ eV}} = 0.88 \times 10^{-5} \text{ s} \]

25. As we did for electrons in Example 4.9, let’s find the kinetic energy of an alpha particle with a momentum of 19.7 MeV/c:

\[ K = \frac{p_x^2}{2m} = \frac{(pc)^2}{2mc^2} = \frac{(19.7 \text{ MeV})^2}{2(3727 \text{ MeV})} = 0.052 \text{ MeV} \]

This is negligible compared with the typical kinetic energies of alpha particles emitted in radioactive decays. Therefore, the uncertainty principle does not limit the existence of these alpha particles inside the nucleus.

26. With \( \Delta x = 14 \text{ fm} \), we have

\[ \Delta p_x = \frac{\hbar}{\Delta x} = \frac{1}{c} \frac{\hbar c}{\Delta x} = \frac{1}{c} \frac{197 \text{ MeV} \cdot \text{fm}}{14 \text{ fm}} = 14.1 \text{ MeV/c} \]

With this uncertainty as an estimate for \( p_x \),

\[ K = \frac{p_x^2}{2m} = \frac{c^2 p_x^2}{2mc^2} = \frac{(14.1 \text{ MeV})^2}{2(938 \text{ MeV})} = 0.11 \text{ MeV} \]

This is a very small contribution to the energy of protons or neutrons in a nucleus, which are typically 10-20 MeV.

27. \[ y(x) = \int A(k) \cos kx \, dk = A_0 \int_{k_0 - \Delta k/2}^{k_0 + \Delta k/2} \cos kx \, dk = A_0 \sin kx \bigg|_{k_0 - \Delta k/2}^{k_0 + \Delta k/2} \]
\[
\frac{A_0}{x} \left[ \sin x (k_0 + \Delta k / 2) - \sin x (k_0 - \Delta k / 2) \right]
\]

\[
= \frac{A_0}{x} \left[ \sin k_0 x \cos \frac{x \Delta k}{2} + \cos k_0 x \sin \frac{x \Delta k}{2} - \left( \sin k_0 x \cos \frac{x \Delta k}{2} - \cos k_0 x \sin \frac{x \Delta k}{2} \right) \right]
\]

\[
= \frac{2A_0}{x} \sin \left( \frac{\Delta k}{2} x \right) \cos k_0 x
\]

28. \[y(x) = \int_{-\infty}^{+\infty} A(k) \cos kx \, dk = A_0 \int_{-\infty}^{+\infty} e^{-\left(k-k_0\right)^2/2\Delta k^2} \cos kx \, dk\]

Let \(k' = k - k_0\).

\[y(x) = A_0 \int_{-\infty}^{+\infty} e^{-k'^2/2\Delta k^2} \left[ \cos k'x \cos k_0 x - \sin k'x \sin k_0 x \right] \, dk'\]

The integral over the second term (involving the sines) vanishes because \(\sin k'x\) is an odd function of \(k'\) (the contribution of the integral from \(-\infty\) to 0 cancels the part from 0 to \(+\infty\)). The remaining integral is

\[y(x) = 2A_0 \cos k_0 x \int_0^\infty e^{-k'^2/2\Delta k^2} \cos k'x \, dk'\]

The integral is a standard form that can be found in integral tables:

\[y(x) = 2A_0 \cos k_0 x \sqrt{\pi} \frac{e^{-x^2/2\Delta k^2}}{\sqrt{2 / \Delta k}} = A_0 \Delta k \sqrt{2\pi} \frac{e^{-x^2/2\Delta k^2}}{\Delta k} \cos k_0 x\]

29. \[y(x) = A \cos(2\pi x / \lambda_1) + A \cos(2\pi x / \lambda_2) = A[\cos(2\pi x / \lambda_1) + \cos(2\pi x / \lambda_2)]\]

Using the identity \(\cos x + \cos y = 2 \cos \frac{x}{2} (x + y) \cos \frac{y}{2} (x - y)\), we get directly

\[y(x) = 2A \cos \left( \frac{\pi x}{\lambda_1} + \frac{\pi x}{\lambda_2} \right) \cos \left( \frac{\pi x}{\lambda_1} - \frac{\pi x}{\lambda_2} \right)\]

30. \[\omega_1 = 2\pi f_1 = 2\pi \frac{\nu_1}{\lambda_1} = 2\pi \frac{6}{9} = \frac{4\pi}{3} \quad \text{and} \quad k_1 = \frac{2\pi}{\lambda_1} = \frac{2\pi}{9}\]

\[\omega_2 = 2\pi f_2 = 2\pi \frac{\nu_2}{\lambda_2} = 2\pi \frac{4}{11} = \frac{8\pi}{11} \quad \text{and} \quad k_2 = \frac{2\pi}{\lambda_2} = \frac{2\pi}{11}\]
\[
\begin{align*}
&= A_0 x \left[ \sin x(k_0 + \Delta k / 2) - \sin x(k_0 - \Delta k / 2) \right] \\
&= A_0 x \left[ \sin k_0 x \cos \frac{x \Delta k}{2} + \cos k_0 x \sin \frac{x \Delta k}{2} \right] - \left( \sin k_0 x \cos \frac{x \Delta k}{2} - \cos k_0 x \sin \frac{x \Delta k}{2} \right) \\
&= \frac{2A_0}{x} \sin \left( \frac{\Delta k}{2} x \right) \cos k_0 x
\end{align*}
\]

28. \[ y(x) = \int_{-\infty}^{+\infty} A(k) \cos kx \, dk = A_0 \int_{-\infty}^{+\infty} e^{-\frac{(k-k_0)^2}{2\Delta k^2}} \cos kx \, dk \]

Let \( k' = k - k_0 \).

\[ y(x) = A_0 \int_{-\infty}^{+\infty} e^{-\frac{k'^2}{2\Delta k^2}} \left[ \cos k'x \cos k_0 x - \sin k'x \sin k_0 x \right] dk' \]

The integral over the second term (involving the sines) vanishes because \( \sin k'x \) is an odd function of \( k' \) (the contribution of the integral from \( -\infty \) to 0 cancels the part from 0 to \( +\infty \)). The remaining integral is

\[ y(x) = 2A_0 \cos k_0 x \int_{0}^{+\infty} e^{-\frac{k'^2}{2\Delta k^2}} \cos k'x \, dk' \]

The integral is a standard form that can be found in integral tables:

\[ y(x) = 2A_0 \cos k_0 x \sqrt{\pi} \frac{e^{-\frac{x^2}{2\Delta k^2}}}{\sqrt{2 / \Delta k}} = A_0 \Delta k \sqrt{2\pi} \frac{e^{-\frac{x^2}{2\Delta k^2}}}{\Delta k} \cos k_0 x \]

29. \[ y(x) = A \cos(2\pi x / \lambda_1) + A \cos(2\pi x / \lambda_2) = A \left[ \cos(2\pi x / \lambda_1) + \cos(2\pi x / \lambda_2) \right] \]

Using the identity \( \cos x + \cos y = 2 \cos \frac{1}{2}(x + y) \cos \frac{1}{2}(x - y) \), we get directly

\[ y(x) = 2A \cos \left( \frac{\pi x}{\lambda_1} + \frac{\pi x}{\lambda_2} \right) \cos \left( \frac{\pi x}{\lambda_1} - \frac{\pi x}{\lambda_2} \right) \]

30. \[ \omega_1 = 2\pi f_1 = 2\pi \frac{v_1}{\lambda_1} = 2\pi \frac{6}{9} = \frac{4\pi}{3} \quad \text{and} \quad k_1 = \frac{2\pi}{\lambda_1} = \frac{2\pi}{9} \]

\[ \omega_2 = 2\pi f_2 = 2\pi \frac{v_2}{\lambda_2} = 2\pi \frac{4}{11} = \frac{8\pi}{11} \quad \text{and} \quad k_2 = \frac{2\pi}{\lambda_2} = \frac{2\pi}{11} \]
31. (a) \[ v_{\text{phase}} = \omega / k \]

\[
\frac{dv_{\text{phase}}}{dk} = \frac{d}{dk} \left( k v_{\text{phase}} \right) = v_{\text{phase}} + k \frac{dv_{\text{phase}}}{dk}
\]

\[
\frac{dv_{\text{phase}}}{dk} = \frac{d}{dk} \left( 2 \pi \sqrt{\frac{2}{k}} \right) = -2 \pi \frac{k}{2} = \frac{dv_{\text{phase}}}{dk} \left( \frac{\lambda}{k} \right)
\]

\[ v_{\text{group}} = v_{\text{phase}} - \lambda \frac{dv_{\text{phase}}}{d\lambda} \]

(b) The index of refraction \( n \) for light in glass decreases as \( \lambda \) increases (shorter wavelengths are refracted more than longer wavelengths); that is \( dn/d\lambda < 0 \). Because \( n = c/v_{\text{phase}} \), \( dn/d\lambda \) and \( dv_{\text{phase}}/d\lambda \) have opposite signs and so \( dv_{\text{phase}}/d\lambda > 0 \). Thus \( v_{\text{group}} > v_{\text{phase}} \).

32. \[ v_{\text{phase}} = \sqrt{\frac{b}{\lambda}} = \sqrt{\frac{bk}{2\pi}} = \frac{\omega}{k} \quad \text{or} \quad \omega = \sqrt{\frac{b}{2\pi}} k^{3/2} \]

\[ v_{\text{group}} = \frac{d\omega}{dk} = \sqrt{\frac{3}{2} \pi k^{3/2}} = \frac{3}{2} \sqrt{\frac{bk}{2\pi}} = \frac{3}{2} v_{\text{phase}} \]

33. \[ K = E - mc^2 = \sqrt{p^2 c^2 + m^2 c^4} - mc^2 \]

\[ \frac{dK}{dp} = \frac{1}{2} \left( p^2 c^2 + m^2 c^4 \right)^{-1/2} (2pc) = \frac{pc^2}{\sqrt{p^2 c^2 + m^2 c^4}} = \frac{pc^2}{E} = \frac{mv}{mc^2} \frac{\sqrt{1 - v^2 / c^2}}{\sqrt{1 - v^2 / c^2}} = v \]

34. (a) With a node at each end (say, at \( x = 0 \) and \( x = L \)) and no other nodes, we must have one half-wave between the two nodes. Thus \( L = \lambda_{1}/2 \) or \( \lambda_{1} = 2L \). If there is an additional node at the midpoint (\( x = L/2 \)), then there is a full wave between the two ends, and \( L = \lambda_{2} \) or \( \lambda_{2} = 2L/2 \). The next shorter wavelength has (in addition to the nodes at either end) nodes at \( x = L/3 \) and \( x = 2L/3 \), so there are three half-waves between the ends: \( L = 3\lambda_{1}/2 \) or \( \lambda_{n} = 2L/3 \). Continuing in this way, we see that in the \( n^{\text{th}} \) case there are \( n \) half-waves in the length \( L \), so \( L = n(\lambda_{n}/2) \) or \( \lambda_{n} = 2L/n \).

(b) With \( p_{n} = h/\lambda_{n} = nh/2L \), we see that \( cp_{n} \) is of order keV, so nonrelativistic equations can safely be used:
Thus \( K_1 = 1.50 \text{ eV}, K_2 = 6.00 \text{ eV}, K_3 = 13.5 \text{ eV} \).

35. \[
\lambda = \frac{h}{p} = \frac{hc}{\sqrt{2mc^2K}} = \frac{1240 \text{ eV} \cdot \text{nm}}{\sqrt{2(940 \times 10^6 \text{ eV})(0.0105 \text{ eV})}} = 0.279 \text{ nm}
\]

From the Bragg scattering formula (Eq. 3.16), we have

\[
\sin \theta = \frac{n\lambda}{2d} = \frac{(1)(0.279 \text{ nm})}{2(0.247 \text{ nm})} = 0.565 \quad \text{or} \quad \theta = 34.4^\circ
\]

For second-order \((n = 2)\) scattering at that angle, \( \lambda = (2d \sin \theta) / 2 = 0.140 \text{ nm} \). The wavelength is reduced by half, so the momentum is doubled and the kinetic energy increases by a factor of 4 to 0.0420 eV. For third-order scattering \((n = 3)\), the kinetic energy is 9 times as great, or 0.0945 eV. The scattered beam at that angle will consist of all energies that are \( n^2 \) times the original energy \((n = 1, 2, 3, \ldots)\).

36. (a) The mass of a nitrogen molecule is 14 u. The average molecular kinetic energy is \( \frac{1}{2}kT \), so the de Broglie wavelength is

\[
\lambda = \frac{h}{p} = \frac{hc}{\sqrt{2mc^2K}} = \frac{1240 \text{ eV} \cdot \text{nm}}{\sqrt{2(14 \text{ u})(931.5 \times 10^6 \text{ eV/u})(1.5)(8.617 \times 10^{-5} \text{ eV/K})(293 \text{ K})}} = 0.0279 \text{ nm}
\]

(b) The number of nitrogen molecules per unit volume is

\[
n = \frac{pN_A}{M} = \frac{(1.292 \text{ kg/m}^3)(6.02 \times 10^{23} \text{ molecules/mole})}{(0.028 \text{ kg/mole})} = 2.78 \times 10^{25} \text{ molecules/m}^3
\]

and the average spacing between molecules is \( n^{-1/3} = 3.30 \times 10^{-9} \text{ m} = 3.3 \text{ nm} \). The de Broglie wavelength is 2 orders of magnitude smaller than the molecular spacing, so that quantum effects are unimportant in gases at room temperature.

(c) Let’s estimate that quantum effects would be significant if the de Broglie wavelength were about 1/10 of the molecular separation (0.33 nm):

\[
p = h \frac{1 \ h c}{\lambda = c} = \frac{11240 \text{ eV} \cdot \text{nm}}{0.33 \text{ nm}} = 3760 \text{ eV/c}
\]

\[
K = \frac{p^2}{2m} = \frac{p^2c^2}{2mc^2} = \frac{(3760 \text{ eV})^2}{2(28 \text{ u})(931.5 \times 10^6 \text{ eV/u})} = 2.71 \times 10^{-4} \text{ eV}
\]

The molecules have this tiny amount of average kinetic energy at a temperature
\[ K_n = \frac{p_n^2}{2m} = \frac{c^2 p_n^2}{8m c^2} = \frac{n^2 h^2 c^2}{8mc^2 \lambda^2} = n^2 \frac{(1240 \text{ eV} \cdot \text{nm})^2}{8(511,000 \text{ eV})(0.50 \text{ nm})^2} = n^2 (1.50 \text{ eV}) \]

Thus \( K_1 = 1.50 \text{ eV}, K_2 = 6.00 \text{ eV}, K_3 = 13.5 \text{ eV} \).

35. \( \lambda = \frac{\hbar}{p} = \frac{hc}{\sqrt{2m c^2 K}} = \frac{1240 \text{ eV} \cdot \text{nm}}{\sqrt{2(940 \times 10^6 \text{ eV})(0.0105 \text{ eV})}} = 0.279 \text{ nm} \)

From the Bragg scattering formula (Eq. 3.16), we have

\[ \sin \theta = \frac{n \lambda}{2d} = \frac{(1)(0.279 \text{ nm})}{2(0.247 \text{ nm})} = 0.565 \quad \text{or} \quad \theta = 34.4^\circ \]

For second-order \( (n = 2) \) scattering at that angle, \( \lambda = (2d \sin \theta) / 2 = 0.140 \text{ nm} \). The wavelength is reduced by half, so the momentum is doubled and the kinetic energy increases by a factor of 4 to 0.0420 eV. For third-order scattering \( (n = 3) \), the kinetic energy is 9 times as great, or 0.0945 eV. The scattered beam at that angle will consist of all energies that are \( n^2 \) times the original energy \((n = 1, 2, 3, \ldots)\).

36. (a) The mass of a nitrogen molecule is 14 u. The average molecular kinetic energy is \( \frac{1}{2} kT \), so the de Broglie wavelength is

\[ \lambda = \frac{\hbar}{p} = \frac{hc}{\sqrt{2m c^2 K}} = \frac{1240 \text{ eV} \cdot \text{nm}}{\sqrt{2(14 \text{ u})(931.5 \times 10^6 \text{ eV/u})(1.5)(8.617 \times 10^{-5} \text{ eV/K})(293 \text{ K})}} = 0.0279 \text{ nm} \]

(b) The number of nitrogen molecules per unit volume is

\[ n = \frac{\rho N_A}{M} = \frac{(1.292 \text{ kg/m}^3)(6.02 \times 10^{23} \text{ molecules/mole})}{(0.028 \text{ kg/mole})} = 2.78 \times 10^{25} \text{ molecules/m}^3 \]

and the average spacing between molecules is \( n^{-1/3} = 3.30 \times 10^{-9} \text{ m} = 3.3 \text{ nm} \). The de Broglie wavelength is 2 orders of magnitude smaller than the molecular spacing, so that quantum effects are unimportant in gases at room temperature.

(c) Let’s estimate that quantum effects would be significant if the de Broglie wavelength were about 1/10 of the molecular separation (0.33 nm):

\[ p = \frac{\hbar}{\lambda} = \frac{hc}{c \lambda} = \frac{11240 \text{ eV} \cdot \text{nm}}{0.33 \text{ nm}} = 3760 \text{ eV/c} \]

\[ K = \frac{p^2}{2m} = \frac{p^2 c^2}{2mc^2} = \frac{(3760 \text{ eV})^2}{2(28 \text{ u})(931.5 \times 10^6 \text{ eV/u})} = 2.71 \times 10^{-4} \text{ eV} \]

The molecules have this tiny amount of average kinetic energy at a temperature
\[
T = \frac{2K}{3k} = \frac{2(2.71 \times 10^{-4} \text{ eV})}{3(8.617 \times 10^{-5} \text{ eV/K})} = 2.1 \text{ K}
\]

Nitrogen is no longer a gas at this temperature, so our calculation using the formula for the mean molecular energy of gases is not correct. However, it does suggest that if quantum effects are to become important in gases, they will occur only at low temperatures. (Recall the discussion in Chapter 1 about how the equipartition of energy fails for the rotational and vibrational motions of some gases at even moderate temperatures, so other effects of quantum behavior may be observable at these temperatures.)

37. For both the photon and the electron,

\[
p = \frac{\hbar}{\lambda} = \frac{1}{c} \frac{hc}{\lambda} = \frac{1}{c} \frac{1240 \text{ eV} \cdot \text{nm}}{0.281 \text{ nm}} = 4.41 \times 10^3 \text{ eV/c}
\]

For the photon,

\[E = pc = 4.41 \times 10^3 \text{ eV}\]

For the electron,

\[K = \frac{p^2}{2m} = \frac{p^2c^2}{2mc^2} = \frac{(4.41 \times 10^3 \text{ eV})^2}{2(511,000 \text{ eV})} = 19.1 \text{ eV}\]

38. (a) The initial nucleus is at rest, the final momenta of the helium and the neutron must sum to zero: \(p_n + p_{\text{He}} = 0\), and so \(p_n = -p_{\text{He}}\). The energy released in the decay appears as the kinetic energy of the final products: \(K_n + K_{\text{He}} = 0.89 \text{ MeV}\). Using nonrelativisitve kinetic energies, we have

\[
K_n + K_{\text{He}} = \frac{p_n^2}{2m_n} + \frac{p_{\text{He}}^2}{2m_{\text{He}}} = \frac{p_n^2}{2m_n} = \frac{p_n^2}{2m_n} \left(1 + \frac{m_n}{m_{\text{He}}}\right) = 0.89 \text{ MeV}
\]

\[
K_n = \frac{0.89 \text{ MeV}}{1 + m_n / m_{\text{He}}} = \frac{0.89 \text{ MeV}}{1 + 1/4} = 0.71 \text{ MeV}
\]

(b) \[\Delta E \sim \frac{\hbar}{\Delta t} = \frac{6.58 \times 10^{-16} \text{ eV} \cdot \text{s}}{1.0 \times 10^{-21} \text{ s}} = 0.66 \text{ MeV}\]
(b) \((p^2)_{av} = (p_x^2 + p_y^2 + p_z^2)_{av} = (\Delta p_x)^2 + (\Delta p_y)^2 + (\Delta p_z)^2 = 3(\Delta p_x)^2\)

\[ K_{av} = \frac{(p^2)_{av}}{2m} = \frac{e^2(p^2)_{av}}{2mc^2} = \frac{3(990 \text{ eV})^2}{2(65 \text{ u})(931.5 \times 10^6 \text{ eV/u})} = 2.4 \times 10^{-5} \text{ eV} \]

(c) In a cube of copper 1.0 cm on edge (0.141 mole), the energy is

\[(0.141 \text{ mole})(6.02 \times 10^{23} \text{ atoms/mole})(2.4 \times 10^{-5} \text{ eV/atom}) = 2.04 \times 10^{18} \text{ eV} = 0.33 \text{ J} \]

This energy is small compared with the internal energy (roughly 1000 J), but it is not quite as negligibly small as the energy of the electronic motion (see Problem 36). This energy of 0.33 J is independent of temperature, so it becomes relatively more important as the temperature of the copper is reduced (thereby decreasing the internal energy). This is one example of the phenomenon of “zero-point motion,” a certain minimum energy that a confined quantum system must have. There is no counterpart to this zero-point motion in classical physics.

42. When the beam passes through a hole of width \(\Delta x = d\), there is a resulting uncertainty in the transverse momentum of order \(\Delta p_x \sim \hbar/d\) and thus in the transverse velocity of \(\Delta v_x \sim \hbar/md\). From Eq. 4.16, we have \((p^2)_{av} = (\Delta p_x)^2\) or \((v^2)_{av} = (\Delta v_x)^2\). The diameter of the beam grows larger than its original diameter by an amount \(\Delta d = t(\Delta v_x)\), where \(t\) is the time the beam has been traveling. The speed of the atoms as they leave the oven at a temperature \(T\) is found from

\[ K = \frac{1}{2}mv^2 = \frac{3}{2}kT \quad \text{so} \quad v = \sqrt{\frac{3kT}{m}} \]

The beam travels the distance \(L\) at a speed \(v\) in a time \(t = L/v\), and thus

\[ \Delta d = t(\Delta v_x) = \frac{L}{d} = \frac{Lh}{md\sqrt{3kT/m}} = \frac{Lh}{d\sqrt{3mkT}} \]

\[ = \frac{(2 \text{ m})(1.05 \times 10^{-34} \text{ J} \cdot \text{s})}{(0.003 \text{ m})\sqrt{3(7 \text{ u})(1.66 \times 10^{-27} \text{ kg/u})(1.38 \times 10^{-23} \text{ J/K})(1500 \text{ K})}} = 3 \times 10^{-9} \text{ m} \]

The spreading of the beam due to the uncertainty principle is thus a negligible effect.

43. From Eq. 4.15 with \(p_{x,av} = 0\), we have \((\Delta p_x)^2 = (p_{x,av}^2)\) and similarly in the \(y\) direction we have \((\Delta p_y)^2 = (p_{y,av}^2)\). The kinetic energy is \(K = p^2/2m = (p_x^2 + p_y^2)/2m\), and using the minimum estimates for the uncertainties we have
\[
K = \frac{(\Delta p_x)^2 + (\Delta p_y)^2}{2m} = \frac{(\hbar / 2\Delta x)^2 + (\hbar / 2\Delta y)^2}{2m} = \frac{\hbar^2 c^2}{8mc^2} \left[ \frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2} \right] \\
= \frac{(197 \text{ eV} \cdot \text{nm})^2}{8(511,000 \text{ eV})} \left[ \frac{1}{(1.25 \text{ nm})^2} + \frac{1}{(2.76 \text{ eV})^2} \right] = 7.3 \text{ meV}
\]

44. (a)

(b) The wave packet has amplitude mostly in the region from about \(x = -3\) to \(x = +3\), so \(\Delta x \sim 6\).

(c) There are 3 complete oscillations in the region from about \(x = -2\) to \(x = +2\), so \(\lambda \sim 4/3 = 1.3\).

(d)

\[
\Delta \lambda \sim \frac{\epsilon \lambda^2}{\Delta x} = \frac{(0.1)(1.3)^2}{6} = 0.03
\]