(1) Consider a monatomic ideal gas in the presence of a temperature gradient $\nabla T$. Answer the following questions within the framework of the relaxation time approximation to the Boltzmann equation.

(a) Compute the particle current $j$ and show that it vanishes.

(b) Compute the ‘energy squared’ current,

$$j_{\varepsilon^2} = \int d^3p \varepsilon^2 v f(r, p, t) .$$

(c) Suppose the gas is diatomic, so $c_p = \frac{7}{2}k_B$. Show explicitly that the particle current $j$ is zero. Hint: To do this, you will have to understand the derivation of eqn. 8.85 in the Lecture Notes and how this changes when the gas is diatomic. You may assume $Q_{\alpha\beta} = F = 0$.

(2) Consider a classical gas of charged particles in the presence of a magnetic field $B$. The Boltzmann equation is then given by

$$\frac{\varepsilon - \hbar k_B T^2}{m c} f_0 \cdot \nabla T - \frac{e}{mc} v \times B \cdot \frac{\partial \delta f}{\partial v} = \left( \frac{\partial f}{\partial t} \right)_{\text{coll}} .$$

Consider the case where $T = T(x)$ and $B = B\hat{z}$. Making the relaxation time approximation, show that a solution to the above equation exists in the form $\delta f = v A(\varepsilon)$, where $A(\varepsilon)$ is a vector-valued function of $\varepsilon(v) = \frac{1}{2}mv^2$ which lies in the $(x, y)$ plane. Find the energy current $j_\varepsilon$. Interpret your result physically.

(3) A photon gas in equilibrium is described by the distribution function

$$f^0(p) = \frac{2}{e^{p/k_B T} - 1} ,$$

where the factor of 2 comes from summing over the two independent polarization states.

(a) Consider a photon gas (in three dimensions) slightly out of equilibrium, but in steady state under the influence of a temperature gradient $\nabla T$. Write $f = f^0 + \delta f$ and write the Boltzmann equation in the relaxation time approximation. Remember that $\varepsilon(p) = cp$ and $v = \frac{\partial f}{\partial p} = c\hat{p}$, so the speed is always $c$.

(b) What is the formal expression for the energy current, expressed as an integral of something times the distribution $f$?

(c) Compute the thermal conductivity $\kappa$. It is OK for your expression to involve dimensionless integrals.
(4) Suppose the relaxation time is energy-dependent, with \( \tau(\varepsilon) = \tau_0 e^{-\varepsilon/\varepsilon_0} \). Compute the particle current \( j \) and energy current \( j_\varepsilon \) flowing in response to a temperature gradient \( \nabla T \).

(5) Use the linearized Boltzmann equation to compute the bulk viscosity \( \zeta \) of an ideal gas.

(a) Consider first the case of a monatomic ideal gas. Show that \( \zeta = 0 \) within this approximation. Will your result change if the scattering time is energy-dependent?

(b) Compute \( \zeta \) for a diatomic ideal gas.

(6) Consider a two-dimensional gas of particles with dispersion \( \varepsilon(k) = Jk^2 \), where \( k \) is the wavevector. The particles obey photon statistics, so \( \mu = 0 \) and the equilibrium distribution is given by

\[
f^0(k) = \frac{1}{e^{\varepsilon(k)/k_B T} - 1}.
\]

(a) Writing \( f = f^0 + \delta f \), solve for \( \delta f(k) \) using the steady state Boltzmann equation in the relaxation time approximation,

\[
v \cdot \frac{\partial f^0}{\partial r} = -\frac{\delta f}{\tau}.
\]

Work to lowest order in \( \nabla T \). Remember that \( v = \frac{1}{\hbar} \frac{\partial \varepsilon}{\partial k} \) is the velocity.

(b) Show that \( j = -\lambda \nabla T \), and find an expression for \( \lambda \). Represent any integrals you cannot evaluate as dimensionless expressions.

(c) Show that \( j_\varepsilon = -\kappa \nabla T \), and find an expression for \( \kappa \). Represent any integrals you cannot evaluate as dimensionless expressions.