Problem Set I: due TBD

1. Kulsrud; Chapter 3, #1 — simple, but important educational. Highlights duality of description in terms of field lines and fluid elements.

2. Kulsrud; Chapter 3, #3 — introduction to magnetic helicity


4. *Electron MHD* (EMHD)

This extended problem introduces you to EMHD and challenges you to apply what you've learned about MHD to understand the structures of a different system of fluid equations. In EMHD, the ions are stationary and the "fluid" is a fluid of electrons. EMHD is useful in problems involving fast Z-pinches, filamentation and magnetic field generation in laser plasmas, Fast Igniter, etc.

The basic equations of EMHD are the electron momentum balance equation

\[
\frac{\partial}{\partial t} \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} = \frac{q}{m} \mathbf{E} - \frac{\nabla P}{\rho} - \frac{q}{mc} \mathbf{v} \times \mathbf{B} - \mathbf{v},
\]

(2) \[ \mathbf{J} = nqv, \]

and continuity

\[
\nabla \cdot \mathbf{J} = 0.
\]

Note that here, Ampere's law forces incompressibility of the mass flow \( \mathbf{v} \). Here \( \mathbf{v} \) is the electron fluid velocity, \( n \) is the electron-ion collision frequency, \( q = |e| \), \( m = m_e \). Of course, Maxwell's equations apply, but the displacement current is neglected.
i.) **Freezing-in**

Determine the freezing-in law for EMHD by taking the curl of Eqn. (1) and using the identity

$$-\mathbf{v} \cdot \nabla \mathbf{v} = \mathbf{v} \times \nabla \left( \frac{v^2}{2} \right).$$

Assume the electrons have $p = p(r)$. Approach this problem by trying to derive an equation for "something" which has the structure of the induction equation in MHD. Discuss the physics - what is the "something" and what is it frozen into? In retrospect, why is the frozen-in quantity obvious? How is freezing-in broken?

ii.) **Large Scale Limit**

Show that for $\ell^2 >> \frac{c^2}{\nu}$, the dynamical equations for EMHD reduce to

$$\frac{\partial B}{\partial t} + \nabla \times \left( \frac{J}{nq} \times B \right) = \nabla \times \left( \frac{J}{nq} \right)$$

$$\nabla \cdot J = 0; \quad \nabla \cdot B = 0.$$

a) Show that density remains constant here.

b) Formulate an energy theorem for EMHD in this limit, by considering the energy content of a "blob" of EMHD fluid.

c) Discuss the frozen-in law in this limit.