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Ambipolar Diffusion and Implications for Star Formation

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ABSTRACT

Star formation simulations and studies have struggled to describe the mechanism of angular momentum transport in protostellar systems. Ideal magnetohydrodynamics (MHD) does not form large Keplerian disks that are formed easily in hydrodynamical simulations due to effective magnetic braking and a build up of magnetic flux in the center of the collapsing system. Studying non-ideal MHD effects such as ambipolar diffusion improves the regulation of magnetic flux pile-up when forming stars and their disks. Ambipolar diffusion enables neutral particles to redistribute the magnetic field to decrease the field intensity during collapse. This improves formation of disks, prevents a violent release of magnetic flux that could displace the core and/or the disk, and allows for stronger outflows to occur with Alfven waves carrying angular momentum out of the system. In this paper I will discuss the failures of ideal MHD in prestellar core collapse. I will describe the physical framework for ambipolar diffusion in non-ideal MHD. I will also review solutions that have been suggested over time and compare them to the non-ideal MHD effect, ambipolar diffusion.

Keywords: MHD, Star Formation, Ambipolar Diffusion

1. INTRODUCTION

Molecular clouds are the structures that harbor the formation of stars (Morris et al. 1974). Giant molecular clouds (GMCs) can have masses $M_{GMC} \sim 10^5$ $M_\odot$ with clumps of masses $M_c \sim 10^3$-10^4 M$_\odot$ (Stark & Blitz 1978). The Jeans mass, M_J, for the clumps is only a few $M_\odot$. These clouds cannot be collapsing on free fall timescales or the rate of star formation in the galaxy would be too high (Morris et al. 1974). Because $M_J < M_c$, a molecular cloud clump cannot be supported by thermal pressure alone (Lizano & Shu 1987). Magnetic fields have the longevity and the strength to provide mechanical support against collapse (Lizano & Shu 1987). If it were not for the magnetic fields, clouds would collapse on the free fall time scale because the Bonnor-Ebert critical mass for a molecular cloud is only 5.8 $M_\odot$ (Mouschovias 1991). Typical magnetic fields in molecular clouds are 10-100μG, and it has been shown that even relatively weak fields can still provide effective support against gravity (Mouschovias (1991) & Mouschovias (1976)).

With the importance that magnetic fields have in supporting the molecular cloud structure, some problems arise. The longevity of magnetic fields makes them a resilient obstacle to rapid star formation (Lizano & Shu 1987). A magnetic flux, Φ, can support a cloud if its' mass does that exceed a critical mass of

$$M_{cr} = 0.15\frac{\Phi}{G^1/2} = 10^3 M_\odot \left(\frac{B}{30\mu G}\right)\left(\frac{R}{2pc}\right)^2,$$

where B is the magnetic field strength (Lizano & Shu 1987). If the mass of the cloud clump is greater than the critical mass the self-gravity of the cloud can overtake the magnetic field support even when we include flux-freezing conditions inducing a magnetic diluted collapse (Scott & Black 1980). The rapid contraction will compress the embedded magnetic fields above the starting values forming large A and B stars (Lizano & Shu 1987). If the clump is not greater than the critical mass, the cloud cannot collapse indefinitely by self-gravity (Lizano & Shu 1987). So how does the magnetic field get removed so the cloud can collapse and form a prestellar core?

Star formation also involves angular momentum conservation, and thus angular momentum transport. For decades scientists have struggled to describe the mechanism of angular momentum transport when forming a central star and a protostellar disk. Disks play a central role in star formation and typically have masses *sim* 0.1 M$_\odot$ and diameters of *sim* 100 AU. It is believed that most of the mass of Solar-type star is assembled through a disk (Lizano & Shu 1987). Numerical simulations of conservation of angular momentum and disk formation show a much higher angular momentum value in the core than what is observed Mellon & Li (2008). A large amount of angular momentum must be lost during collapse, and must come from another mechanism that is not in hydrodynamic calculations. Adding in magnetic effects with simplified ideal magnetohydrodynamics (MHD), flux-freezing and very effective magnetic braking prevents disk formation and causes a massive pile-up of magnetic flux in the center of the collapsing system (Masson et al. 2016). This results in little to no disk formation and highly magnetized protostars that do not match observations (Masson et al. 2016). A solution to these problems is to go to non-ideal MHD effects following the early efforts by Mestel & Spitzer (1956) and efforts by Mellon & Li (2008) on ambipolar diffusion and by Machida & Matsumoto (2011) on generic resistivity. Non-ideal MHD may not be able to entirely solve the problems in forming disks and low magnetized protostars, but a physical dissipative scale for the magnetic flux could help regulate pile-up of magnetic flux in the center of collapsing systems (Masson et al. 2016). In Section 2, I will describe the ideal MHD in star formation and where it fails. In Section 3, I will describe non-ideal MHD and the effects that enable star formation. In Section 4, I will summarize these ideas and state where the current literature is today on the role of magnetic fields in star formation and its connection to planet formation.

2. IDEAL MHD IN STAR FORMATION

2.1. *Physical Framework*

Ideal MHD has a set of equations to describe phenomena. The first equation is the mass conservation equation,

$$\frac{\partial \rho}{\partial t} + \nabla(\rho \boldsymbol{v}) = 0,$$

where ρ is the mass density and $\boldsymbol{v}$ is the fluid velocity vector. The next equation is the momentum conservation equation,

$$\rho \left(\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} \right) = -\nabla P + \boldsymbol{J} \times \boldsymbol{B} + \rho \boldsymbol{g},$$

where $\boldsymbol{B}$ is the magnetic field, P is total pressure and $\boldsymbol{g}$ is the gravitational acceleration vector. We also have the induction equation,

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B})$$

and the most important property of magnetic fields,

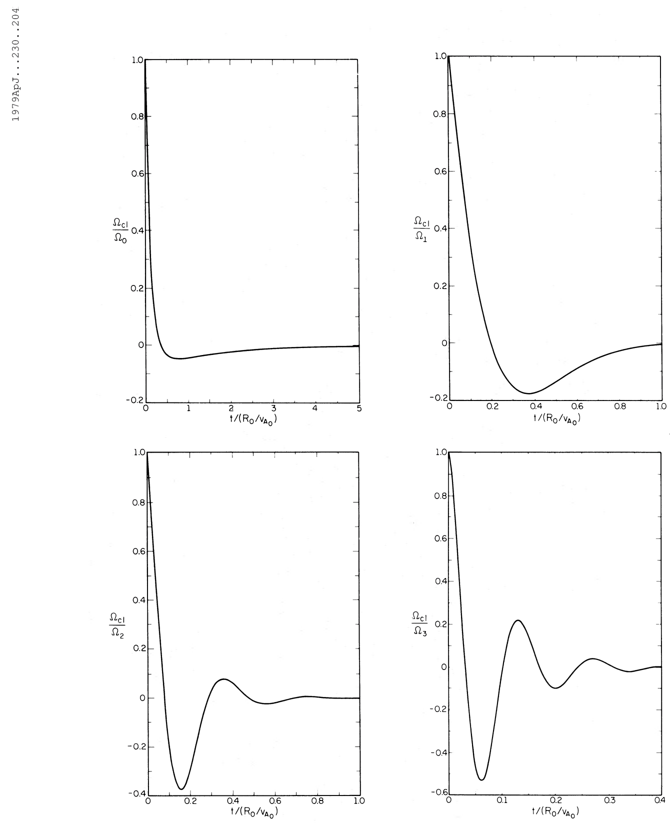
$$\nabla \cdot \boldsymbol{B} = 0.$$

One other property that comes from the ideal MHD framework is flux-freezing condition that comes from Ohm's Law,

$$\boldsymbol{E} = -\boldsymbol{V} \times \boldsymbol{B},$$

where $\boldsymbol{E}$ is the electric field. Flux freezing is where the magnetic field lines are "frozen" into the plasma (Shu 1992).

In 1979 Mouschovias and Paleologou worked on solving these equations for a rigid rotating cloud of uniform density with cylindrical symmetry, in an outside medium with its own magnetic field. They show that the magnetic field increases outside of the cloud as it contracts, allowing the Alfven speed to increase. They calculate a decay in angular momentum and show that the momentum is being carried out of the cloud and to the surrounding medium (see Figure 1 in this paper/Fig 2(a-d) from Mouschovias & Paleologou (1979)). This phenomena is called magnetic braking. Its main effect is slowing down rotationally supported structures that arise during the gravitational collapse (Masson et al. 2016). The braking comes from magnetic tensions that prevent distortion of the field lines that are coupled to the flow through the ionized particles. The angular momentum is then transported along the field lines with a speed close to the Alfven speed (Masson et al. 2016). But, the ideal MHD framework and magnetic braking start to break down when looking at disk formation and magnetic flux pile-up in the center of the system (Masson et al. 2016). Galli et al. (2006) analyzed the collapse of isothermal magnetized rotating clouds within the ideal MHD framework. As the collapse happens, the magnetic field frozen into the plasma gets trapped and amplified. They concluded that magnetic field dissipation is required to match observations of low magnetized protostars (Galli et al. 2006).



FIGS. 2*a* (*upper left*), 2*b* (*upper right*), 2*c* (*lower left*), and 2*d* (*lower right*).—The angular velocity of the cloud (Ω_{cl}) as a function of time for four different density ratios, $\rho_{cl}/\rho_{ext} = 1, 10, 10^2, 10^3$, respectively. In each case, Ω_{cl} is normalized to its initial (arbitrary) value. The unit of time is the same in all four graphs and is equal to the Alfvén crossing time across the cloud radius at the time of cloud formation, when $\rho_{cl}/\rho_{ext} = 1$. A larger density ratio represents a later stage in the contraction of the cloud.

Figure 1. From Mouschovias & Paleologou (1979), the angular velocity of the cloud Ω_{cl} as a function of four density ratios of the cloud density (ρ_{cl}) to the surrounding density (ρ_{ext}). The angular velocity is normalized to its initial value and the time unit is the Alfven crossing time across the cloud radius.

This early work was mostly focused on the prestellar core phase of star formation, but what about the protostellar phase? These two phases are separated by a stage in which the central density goes to infinity (Li et al. 1996). In the protostellar phase, magnetic braking can be so efficient as to stifle the formation of a disk for moderate levels of cloud magnetization, which is a critical step of star formation (Mellon & Li 2008). In particular, magnetic braking disrupts the rotationally supported disk by causing supersonic collapse in regions within the disk. Even with weak field cases, the magnetic field still is dynamically relevant and can be amplified so its pressure dominates over the thermal pressure in the disk and the surrounding region. Mellon & Li (2008) also found that if a core is strongly magnetized, the disk formation is completely suppresed so the bulk of the angular momentum must be ejected out of the system through slow braking driven wind that has yet to be observed. Galli et al. (2006) also found that infalling gas tended to spiral directly into the center of the system without forming a supported disk. Since disks are observed around most, if not all, young stellar objects, the ideal MHD framework has met its limit.

Another important aspect of star formation is that of outflows. The piling-up of the toroidal component of the magnetic field during collapse leads to a growing vertical structure that Masson et al. (2016) calls the magnetic tower. The magnetically driven outflow is reinforced and amplified in the ideal MHD case. This comes from the magnetic field pile up in the center of the system, the bending of the radial field lines, and the increasing toroidal field around the collapsing system (Masson et al. 2016). Masson et al. (2016) also looked into what the large scale structure of the field lines would look like in ideal MHD and found that there was a strange "split monopole topology" of the magnetic field where field lines are being pinched in the equatorial plane. While this does allow for an outflow feature, that structure isn't always stable and can lead to the velocity field falling back onto the core instead of flowing out of the system (Masson et al. 2016). Most of these strange features are relaxed or even completely removed when moving into the non-ideal MHD framework.

3. NON-IDEAL MHD AND ITS ROLE IN STAR FORMATION

3.1. *Non-Ideal MHD and Ambipolar Diffusion*

In this section we move to a two fluid model depiction of MHD that includes non-linear effects. Molecular clouds are lightly ionized, meaning that there will be a relative drift of the charged and neutral gas species (Shu 1992). The charged particles feel the electromagnetic forces directly, and the neutrals only experience them through collisions with the ionized species. If there were no collisions and friction because of the relative drift, the neutrals would not be affected by the embedded magnetic fields at all (Shu 1992).

Ambipolar diffusion comes from the fact that neutral particles will not have the same mean velocity as the ionized species (Shu 1992). Thus, there will be a resistance to the relative drift causing a frictional drag force per unit volume described by,

$$\boldsymbol{f_d} = \gamma \rho_n \rho_i (\boldsymbol{v_i} - \boldsymbol{v_n}),$$

where ρ_n and ρ_i are the mass density of the neutrals and the ions, and γ is a drag coefficient. The drag coefficient for molecular clouds was calculated by Draine et al. (1983) as

$$\gamma = 3.5 \times 10^{13} cm^3 g^{-1} s^{-1}.$$

This will change some of the MHD equations stated in Section 2. In particular it will affect the time evolution of the magnetic field,

$$\frac{\partial \boldsymbol{B}}{\partial t} + \nabla \times (\boldsymbol{B} \times \boldsymbol{v_i}) = 0,$$

where we assume the magnetic field is frozen in the plasma of the ions and the electrons (Shu 1992). For the neutrals the equation looks different,

$$\frac{\partial \boldsymbol{B}}{\partial t} + \nabla \times (\boldsymbol{B} \times \boldsymbol{v_n}) = \nabla \times \left(\frac{\boldsymbol{B}}{4\pi\gamma\rho_n\rho_i} \times [\boldsymbol{B} \times (\nabla \times \boldsymbol{B})] \right),$$

this equation constitutes a nonlinear diffusion equation for the magnetic field (Shu 1992). Ambipolar diffusion enables the neutral species to overcome magnetic field lines and redistribute the field lines (Masson et al. 2016).

Masson et al. (2016) conducted simulations involving a magnetized clump of uniform density in solid body rotation that collapsed under its own gravity. They used a barotropic equation of state to mimic radiative transfer and paid attention to magnetic flux conservation, magnetic braking, and flux release. Their results showed that the strongest magnetic fields inside the collapsing core tend to be on order of $10^{-1}G$ in non-ideal MHD, and are at least ten times stronger in ideal MHD (Masson et al. 2016). Ambipolar diffusion is able to prevent the magnetic field from amplification because of a diffusion barrier that develops when the core has collapsed isothermally to a number density of atoms of $n_H \approx 10^{-13}$ g cm^{-3}.

In Masson et al. (2016), they found that disks with Keplerian velocity profiles, as found in hydrodynamical simulations, form in all non-ideal MHD simulations of the magnetized clumps as they collapse. A more recent study compared ideal MHD simulations and non-ideal MHD simulations and found that when diffusion is present, the protostar has a large amount of rotation that leads to the formation of gravitationally stable disks (Vaytet et al. 2018). Vaytet et al. (2018) found that the rotation of the core and the formation of the disk rolls the magnetic field lines into a toroidal topology that can support an outflow. Figure 2, shows the difference in velocity vector fields and gas density between the ideal MHD simulations of core collapse and disk formation and the non-ideal MHD simulations from Vaytet et al. (2018). This figure shows how the rotation of the core is extremely important in the formation of disks.

In the non-ideal MHD framework, ambipolar diffusion corrects a lot of the obscure and unobserved phenomena from ideal MHD. In particular, when looking at outflows during collapse, the magnetic tower (outflow structure) is less

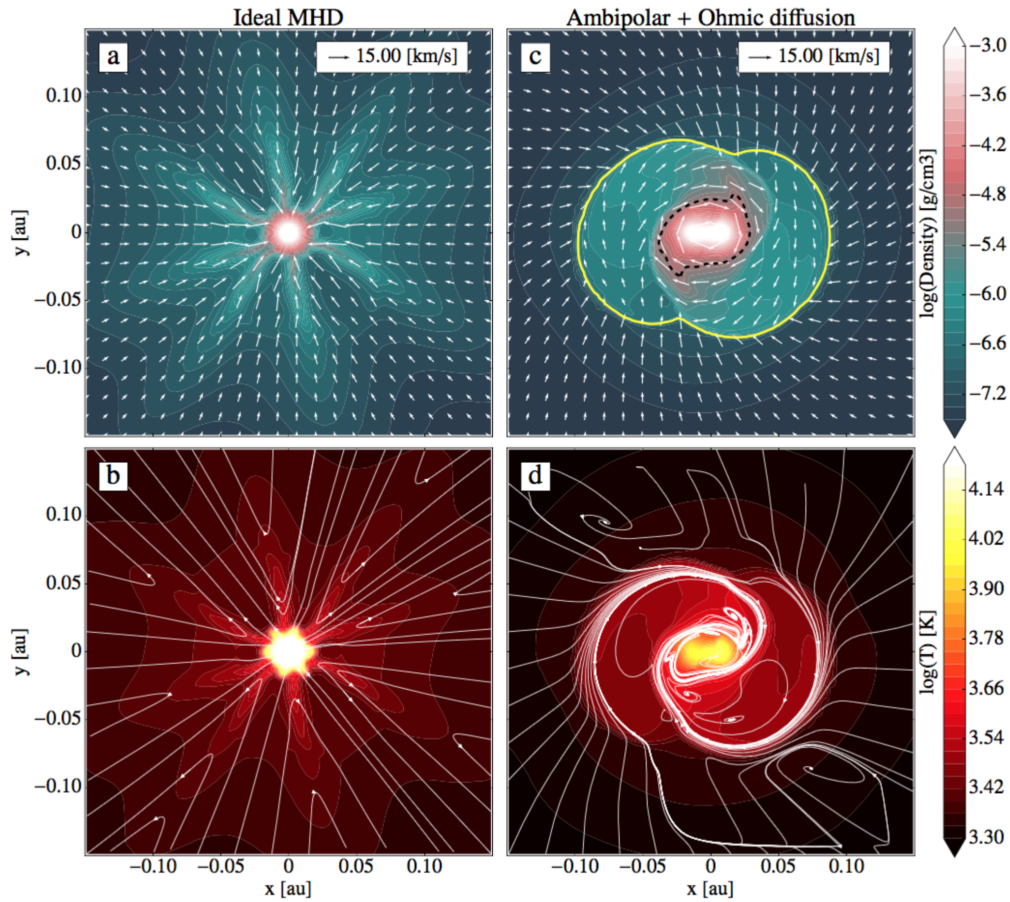


Figure 2. From Vaytet et al. (2018): Slices of the gas density with velocity vectors run with ideal MHD and non-ideal MHD. In c, there is a yellow contour that marks the disk limit at a density of $\rho > 10^{-6.7}$ g cm^{-3}.

dense, has a lower magnetic field intensity, yields a lower Alfven velocity, and explains weaker magnetic braking since the angular momentum is being carried away by slower Alfven waves (Masson et al. 2016). In the ideal case, field lines get pinched, especially in the center of the system, when ambipolar diffusion is included, there is a redistribution of magnetic field lines and pinching of field lines is not necessary (Masson et al. 2016). Thus, while magnetic flux freezing and braking play a role in formation of structures during the collapse, ideal MHD strongly amplifies fields and launch powerful magnetically supported outflows and non-ideal MHD weakens this processes and in some cases causes them to disappear.

3.2. *Other Non-Ideal Effects*

Ambipolar diffusion is not the only non-ideal MHD effect. Ohmic dissipation also has been shown to play a role in star formation. Ohmic dissipation is the process of transfering magnetic energy (or electric energy) into heat. This process can be very effective in dissipating the magnetic field by increasing the drift velocity to a faster velocity than the velocity of the gas. Figure 3 is from Susa et al. (2015) illustrates the ratio of the drift velocity of the magnetic field lines to the gas velocity for four metallicities. When the drift velocity overtakes the gas velocity the magnetic field dissipates Susa et al. (2015). Ohmic dissipation depends on metallicity. For decreasing metallicity, the charge carriers are available so the gas becomes more resistive (Susa et al. 2015). Susa et al. (2015) also found that if you assume a constant metallicity, the magnetic field dissipates for a wider range of density, especially for lower ionization rates. Since molecular clouds are mostly lightly ionized, Ohmic dissipation of the magnetic field could definitely play a major role in star formation.

Another mechanism for diffusing the magnetic field during collapse and creating large disks is turbulent reconnection. The frozen-in condition for magnetic fields still holds, but when you allow turbulent reconnection there can be changes

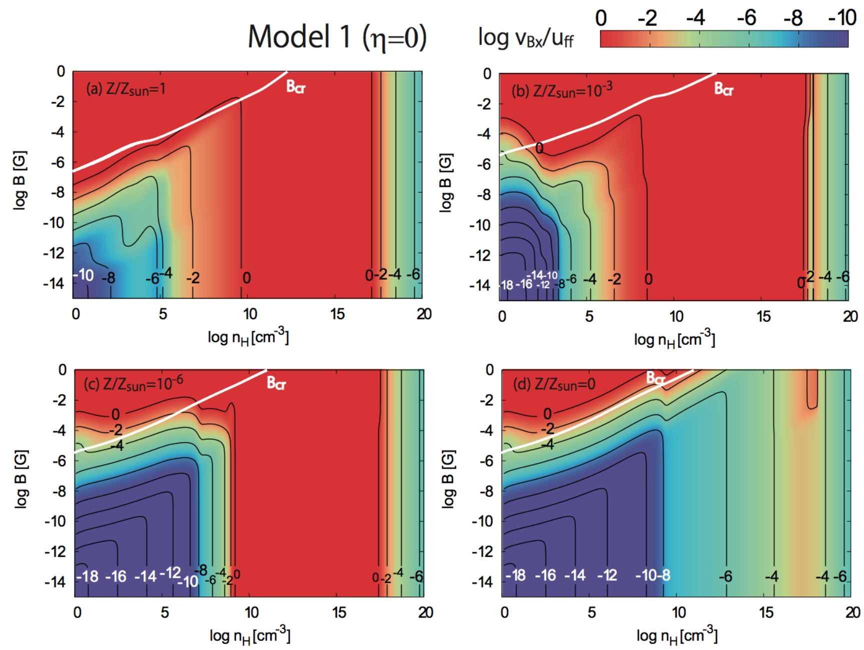


Figure 3. From Susa et al. (2015): The ratio of v_{B_x}/u_{ff}, where v_{B_x} is the magnetic field drift velocity and u_{ff} is the free fall velocity of the gas, for the n_H -B plane with respect to the ionization rate of our galaxy ($\eta = 1$).$Thismodelisfor \eta = 0$. The numbers on the contour are logarithmic values and panels a-d represent different metallicities. The metallicities are (a) $Z/Z_\odot = 1$, (b) 10^{-3}, (c) 10^{-6}, and (d) 0.

in the magnetic field topology (Santos-Lima et al. 2012). When simulating and modeling this phenomena, it was shown that there was a decoupling between the gas and the magnetic field due to reconnection when using a one-fluid approximation (i.e., without having ambipolar diffusion, or ohmic dissipation) (Santos-Lima et al. 2012). Turbulent reconnection, when allowed, can decrease the magnetic flux to mass ratio and can increase the gas density in the center of the system. Santos-Lima et al. (2012) showed that turbulent magnetic reconnection diffusivity can transport magnetic flux to the outskirts of the core and allow the formation of a rotationally supported disk with a close Keplerian profile in 3D MHD numerical simulations. In turbulent diffusivity you modify the conservation of momentum equation to include extra terms,

$$\rho\left(\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v}\right) = -c_s^2\nabla\rho + \boldsymbol{J}\times\boldsymbol{B} + \rho\boldsymbol{g} + \boldsymbol{f},$$

where c_s^2 is the sound speed, and $\boldsymbol{f}$ is the injected turbulence (Santos-Lima et al. 2012). The 3D simulation showed that a high density disk arises in the central region with a radius of 150 AU, and is surrounded by turbulent debris further out. The model was able to reduce the magnetic braking effects, resulting in Keplerian-like rotation in the formed disks. It is not clear that the disks produced are rotationally supported and they can still be highly magnetized Lam et al. (2019). Figure 4 shows a set of plots from Santos-Lima et al. (2012) depicting the ideal MHD 3D models, resistive MHD model (not talked about in this work), and the turbulent model. Clearly, none of them perfectly match the hydrodynamical simulations, but to have turbulent MHD on its own gives a very close approximation. It could be interesting to pursue a combination of turbulent effects, resistive effects, and ambipolar effects in creating a hydrodynamic Keplerian circumstellar disk.

4. SUMMARY AND CONNECTION TO PLANET FORMATION

It is still not completely understood how stars and their circumstellar disks are formed out of lightly ionized molecular clouds. Many issues arise including the conservation of angular momentum, the need for a redistribution of magnetic flux so it doesn't pile-up in the center of the collapsing system, and the need for rotationally stable circumstellar disks. As I have shown, ideal MHD fails to reproduce massive disks, causes violent outflows, and creates pinch-field topology and high magnetic flux in the center of the system. When non-ideal MHD is considered, the magnetic field topology

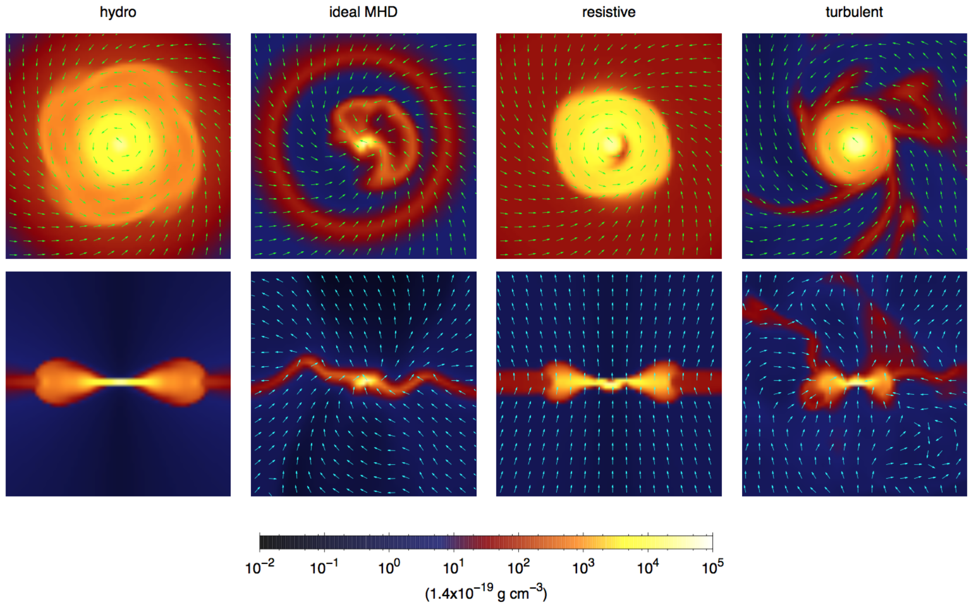


Figure 4. From Santos-Lima et al. (2012): Face-on (top), and edge on (bottom), density maps of the central slices of the formed core and disks. The arrows overplotted represent the velocity field. From right to left: the pure hydrodynamic rotating system, the ideal MHD model, the anomalous resistivity model, and the turbulent MHD model. All the models had a initial magnetic field in the vertical direction, and each image has a side of 1000 AU.

can change, so the magnetic flux can be diffused out as the cloud collapses. I focus heavily on ambipolar diffusion being one of the most important phenomena to allow diffusion, and to allow disks that are almost Keplerian to form. But, ambipolar diffusion is not the only non-ideal MHD effect to produce disks and reduce flux build up. Ohmic dissipation and turbulent reconnection can achieve almost the same results, but some combination of all three tends to match observations much better (Vaytet et al. 2018).

The magnetization and stability of circumstellar disks play an important role in our universe. Disks are the birthplace of planets, including the ones in our Solar System. We have discovered over 4,000 exoplanets to date, so the prevalence indicates that disk formation must be common around young stars. Images from ALMA have greatly improved our understanding of disks. Observations have revealed molecular lines that show Keplerian velocity fields and exoplanets that are in the process of forming in the disk like PDS 70 system (Keppler et al. 2018). Figure 5 shows ALMA images of the PDS 70 system that reveals a complex disk structure with one planet newly formed. Understanding the underlying physics of star formation and disk formation can greatly improve our knowledge of how our Solar System formed in the first place.

Planet formation occurs in the early stages of star formation after the disk has formed around the protostar. In those early stages the disk is still interacting with the core and is delivering mass and angular momentum to the forming disk. The disk needs to be able to fragment in order to form planets. But, Duffin & Pudritz (2008) found that magnetic fields suppress disk fragmentation. They later included non-ideal MHD effets, but this did not enhance the prospects for fragmentation. Spiral arm structures can compress the gas and aid in fragmentation. This feature was seen especially in turbulent reconnection diffusion discussed above. So, the problem is far from being solved. The presence of magnetic fields in the circumstellar stellar disk can prevent formation of planets. This consequence isn't solved by moving from ideal MHD to non-ideal MHD effects. These effects are creating believable disks and velocity profiles, but they are all still somewhat magnetized. We know most stars have had disks at some point during formation. We know that planets and even low mass star companions form from gravitational instabilities and/or core accretion in disks of young stars of all masses Öberg et al. (2011). Diving deeper into what non-ideal MHD and hydrodynamic phenomena create disks and protostars that create the environment for formation of planets is essential is understanding how the Earth, and the Solar System planets came to be.

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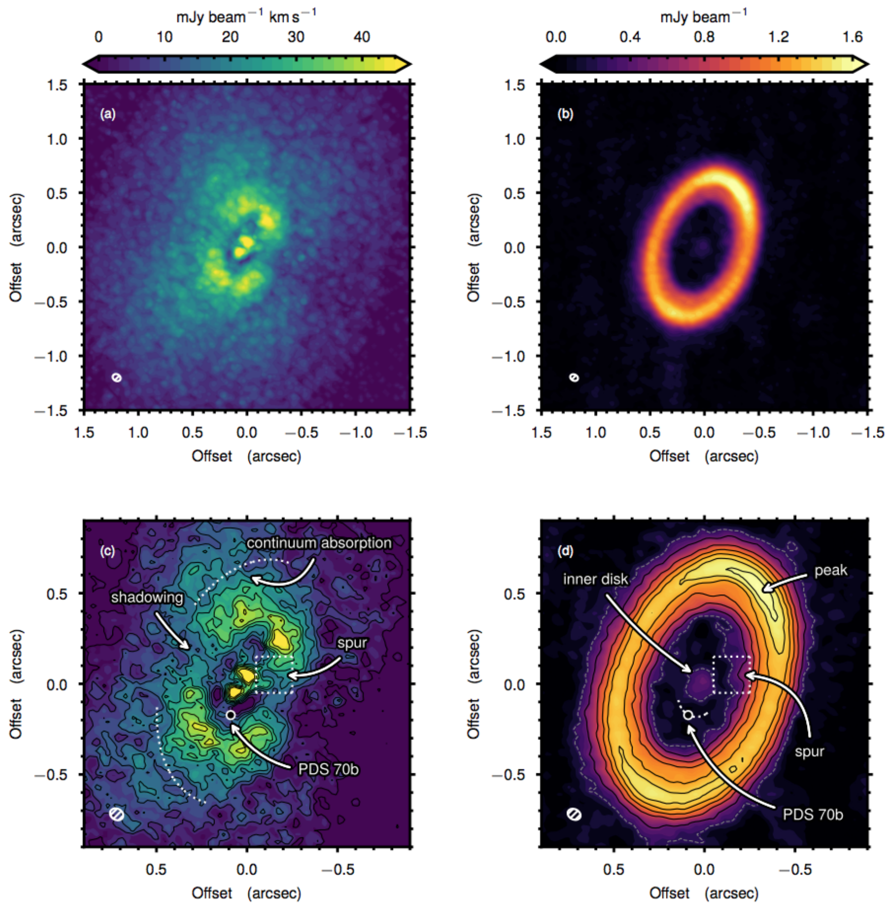

Figure 5. From Keppler et al. (2019): ALMA observations of ^{12}C (left column), and the 350.6 GHz continuum (right column). The bottom row provides a closer view of the system and annotates the stretching of color to bring out specific details.

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