$\boxed{\text{Week 5}}$

1. The action for a point particle is given by $S = \int d\lambda \left[-mc\sqrt{U_\mu U^\mu} - \frac{q}{c} U^\mu A_\mu \right]$

(a) Vary the action to find the E.O.M. for a point particle, $\frac{dU^\mu}{d\tau} = \frac{q}{mc} F^{\mu\nu} U_\nu$. Show that this reduces to the standard Lorentz force eq. at low velocities.

(b) Show that the action is invariant under gauge transformations $A_\mu \to A_\mu + \partial_\mu \varphi$, for any ~~sca~~ scalar function φ.

2. Consider an infinite straight wire with current I, and an ~~charge q~~ electron initially moving parallel to the wire with velocity v_0 and at a distance a from it.

(a) Find the relativistic Lagrangian for this system in cylindrical coordinates.

(b) Use the Lagrangian to find three constants of the motion.

(c) Describe the trajectory of the ~~particle~~ electron. What is the max. distance r_{max} ~~of the particle from the~~ it ~~can~~ will reach from the ~~axis~~ axis of the wire.

3. An ~~plan~~ EM wave is incident on a perfectly conducting ~~medium~~ plane, at rest. The reflection coefficient $R = \left|\frac{E_I}{E_R}\right|^2 = 1$. Now suppose the plane is moving at a velocity v in the direction of propagation of the incident wave. What is the reflection coefficient in the ~~laboratory frame~~ lab frame. (Note: For an EM wave in vacuum $\vec{B} = \hat{n} \times \vec{E}$, where $\hat{n}$ is the direction of propagation.)

4. (a) ~~The~~ Green's function G_{2d} for the 2d wave eq. satisfies $(\partial_{x_0}^2 - \partial_{x_1}^2 - \partial_{x_2}^2)\, G_{2d} = \delta(x_0)\delta(x_1)\delta(x_2)$. Use the method of fourier transforms to show that $G_{2d}(x_0, \vec{x}) = \frac{1}{2\pi} \frac{\Theta(x_0 - |\vec{x}|)}{\sqrt{x_0^2 - |\vec{x}|^2}}$,

where Θ is the Heaviside step function.

(b) Repeat part (a) for the 1d ~~case~~ wave eq. to show that $G_{1d}(x_0, x) = \frac{1}{2}\Theta(x_0 - |x|)$

(c) Show that one can obtain the Green's function in n-1 ~~spatial~~ dim. G_{n-1} from the Green's function in n dim. G_n by integrating over the additional coordinate, i.e.

$$G_{n-1}(x_0, \ldots, x_{n-1}) = \int dx_n \, G_n(x_0, \ldots, x_n).$$

Starting from ~~G_{3d}~~ $G_{3d}(x_0, \vec{x}) = \frac{1}{4\pi} \frac{\delta(x_0 - |\vec{x}|)}{|\vec{x}|}$, find G_{2d} and G_{1d} and compare with the results above.

(d) Comment on the different nature of solutions to the wave eq. in 1d, 2d, and 3d. What is the ~~[illegible]~~ spacetime domain of influence of a point source?