1. (a) Given that since the four-current of a point particle is a four-vector, it must be possible to write it in covariant form. Show that it can be written as \( j^\mu(x) = e^2 \int dx \frac{dy^\mu(x)}{dy} S^{(4)}(x) \), where \( y^\mu(\tau) \) is the worldline of the particle and \( S^{(4)}(x) = S(x^0) \cdots S(x^3) \).

i.e., Show that \( j^\mu \) transforms as \( j'^\mu(x) = \Lambda^\mu_\nu j^\nu(\Lambda^{-1}(x)) \) under Lorentz transform and that in the rest frame of the particle \( j^\mu = (eS^0(x-y), 0) \).

(b) Using the expression from part (a), show that the current is conserved, i.e. \( \partial_\mu j^\mu = 0 \).
2. A sphere of dielectric permittivity \( \varepsilon \) and radius \( a \) moves with velocity \( \vec{v} = v \hat{z} \) in a uniform magnetic field \( \vec{B} = -B_0 \hat{y} \).

(a) Transforming to the rest frame of the sphere, find its induced polarization to first order in \( \frac{v}{c} \).

(b) Transforming back to the lab frame, find the electric field to first order in \( \frac{v}{c} \).

3. Consider an infinite sheet of charge density \( \sigma \) in the \( z = 0 \) plane.

Initially stationary, at time \( t = 0 \) it starts to move in the \( y \)-direction with speed \( v \).

(a) Find its current density \( \vec{J}(z, t) \).

(b) Use the Green's function to calculate \( A_y(z, t) \). Calculate \( B_x(z, t) \) and \( E_y(z, t) \).