Physics 216/116 Winter 2020

Problem Set IV: Due at Last Class – Hard Deadline

1) Short answer

Each of these questions does not require more than a few lines of calculation. *Don't* make them longer than they need be. Do state your reasoning clearly. Try to do these closed book.

a) What is the width of the laminar boundary layer at the bottom of a viscous fluid rotating at Ω ?

b) What is the width of a laminar boundary layer of a stagnation flow?

c) What shaped eddy is most effective at Rayleigh–Bernard convection when $\Omega^2 >> g\alpha\beta$? Estimate the anisotropy.

d) How will the strength of a vortex tube evolve when the fluid density within rises linearly in time?

e) A sphere of radius R moves thru an inviscid fluid line (i.e. obeys Euler, not Navïer–Stokes) with a free surface. The sphere moves at *V*, at various depths *d*. What happens? Estimate the drag on the sphere.

2) Now consider a rotating fluid which is also compressible and *self-gravitating*. For the latter, include a body force $\underline{f} = -\nabla\phi$ where:

$$\nabla^2\phi = 4\pi G\rho$$

Take $\underline{\Omega} = \Omega\hat{z}$, as usual.

a) For $\underline{k} = k\hat{z}$, show

$$\omega^2 = k^2 c_s^2 - 4\pi G\rho_0 .$$

Welcome to the Jeans instability! What might be the significance of the marginally stable length scale?

b) For $\underline{k} = k\hat{x}$, show:

$$\omega^2 = k^2 c_s^2 - 4\pi G \rho_0 + 4\Omega^2 .$$

When are all modes stable?

c) Why might this result be of interest in the context of galaxy structure?

d) How might Jeans instabilities evolve nonlinearly? Why might cooling effects be important here?

3) Calculate the boundary layer thickness and flow field near a semi-infinite plate, assuming laminar flow. This is the Blasius problem. Follow the self-similarity approach discussed in Landau and Lifshitz. Get as far as you can with the numerical factors.

4) **Required for Grads**

Traveling shock in Burgers' equation

During our class, we found the steady shock for the 1D Burgers' equation

$$\partial_t u(x,t) + \frac{1}{2}\partial_x u^2 = v\partial_x^2 u,$$

when we maintain the velocities $u \rightarrow \pm U$ as $x \rightarrow \mp\infty$.

a) Generalize the solution to the case when we maintain the velocities at infinity $u(-\infty) = U_- \ and \ u(+\infty) = U_+$ with $U_- > U_+$.

b) Verify that the dissipation remains finite in the limit $v \rightarrow 0$.

c) The velocity of the shock that you have found above is a special case of the Rankine-Hugoniot condition. By integrating the conservation law

$$\partial_t w(x,t) + \partial_x f(w) = 0$$

around a shock moving with velocity V_s, derive the Ranke-Hugonoit relation $V_s = \frac{f_+ - f_-}{w_+ - w_-}$ where the subscripts $\pm$ refer to the value on the proximal right and left of the shock, respectively.

5) Short Answer: Keep your answers here brief and focused.

a) Estimate the number of modes needed to simulate a high Re flow, assuming you resolve the dissipation scale (many simulations don't!).

b) Estimate the Nusselt number for turbulent boundary layer flow. What is the ratio of Nu to that for the corresponding laminar case?

c) Briefly describe the layer structure of turbulent flow over a rough surface with roughness scale y_0.

d) Does the continuum model of a fluid improve or decline in validity as Re increases? Take the flow subsonic.

e) Calculate the spectrum of Burgers turbulence, assuming the flow may be written as a superposition of uncorrelated shocks. Hint: Consider a convenient way to write the flow v.

6) More Short Answers

a) Estimate the largest water droplet size that will survive suspended in a turbulent air flow of Reynolds number Re and integral scale L. Don't worry about gravity — i.e. assume the experiment is done in the space shuttle.

b) Estimate the temperature of a body of size R in a flow v_0 with temperature T_0. Hint: Consider viscous heating effects.

c) What are the limiting forms of the dependence of Nusselt number on Prandtl number in a laminar boundary layer when Re>>1 and Pr>>1?

7) **Required for Grads**

This problem asks you to synthesize your recently acquired knowledge of fluids and turbulence with your well-developed understanding of waves and radiation so as to understand the acoustic emission of turbulence. [Ref: Landau and Lifshitz.]

a) Calculate the pressure field radiated by a patch of turbulence by solving formally the inhomogeneous wave equation with source $\rho\partial^2 T_{ik}/\partial x_1 \partial x_k$ where $T_{ik} = v_i v_k$. Show where this set up comes from.

b) Specialize your result to the far field, and use the retarded time dependence to relate space to time derivatives.

c) Decompose the pressure field into parts, related to:

$\ddot{T}_{ik} = \left(\ddot{T}_{ik} - \frac{1}{3}\ddot{T}_{ll}\delta_{ik}\right) + \frac{1}{3}\ddot{T}_{ll}\delta_{ik} = Q_{ik} + Q\delta_{ik}$, and calculate the emitted intensity. What are the multipole moments of the acoustic radiation field?

d) Use the now-standard simplifying tricks to show that the acoustic energy emitted per mass and per time scales as $\epsilon_s \sim v^8/lc_s^5$. From this, estimate ϵ_s/ϵ. What can one deduce concerning the efficiency of acoustic emission from subsonic turbulence? Here ϵ is the turbulent dissipation rate, as usual.

e) Briefly discuss how the story might change if one were considering convection-driven turbulence. To do this, consider how buoyancy effects enter.

f) Extra Credit: Do the calculation implied in (e).