Problem Set 1: Due 23 Jan

1) Complete the calculation of the induced mass of potential flow around a sphere, which was begun and discussed in class. In particular, show the energy of potential flow is

\[
E = \rho \left[ 4\pi (\mathbf{A} \cdot \mathbf{u}) - \frac{V_0 u^2}{2} \right] = m_{ik} \frac{u_i u_k}{2},
\]

where \(\mathbf{A}\) is the dipole moment of the flow and \(V_0\) is the volume of the body in motion at \(\mathbf{u}\). Compute \(m_{ik}\), the induced mass tensor. What is its value for a sphere?

2) Derive the energy relation

\[
\frac{\partial}{\partial t} \left( \frac{\rho v^2}{2} + \rho \varepsilon \right) = -\nabla \cdot \left( \rho \mathbf{v} \left( \frac{v^2}{2} + \omega \right) \right)
\]

from the continuity, Euler and energy equations. Here, \(\omega\) is the enthalpy density.

For 216:

3) Consider a small body immersed in a fluid flow which oscillates. Derive the general relation between the velocity of the body and that of the fluid. What is the result for a spherical body of density \(\rho_0\)?

4) a) Derive the dispersion relation for an azimuthally symmetric wave propagating along the \(\hat{z}\) axis and in radius in an ideal incompressible, unbounded fluid rotating at \(\Omega = \Omega_0 \hat{z}\).

b) Now assume the fluid is bounded by a cylindrical wall at \(r = R\). What is the profile of radial velocity?

For 116:

3) Acheson, 1.5

4) Acheson, 1.4