4

The Particle Nature of Matter

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4-3 Thomson's device will work for positive and negative particles, so we may apply $\frac{q}{m} \approx \frac{V\theta}{B^2 ld}$.

(a) $$\frac{q}{m} \approx \frac{V\theta}{B^2 ld} = (2\,000\text{ V})\frac{0.20\text{ radians}}{\left(4.57\times10^{-2}\text{ T}\right)^2}(0.10\text{ m})(0.02\text{ m}) = 9.58\times10^{7}\text{ C/kg}$$

(b) As the particle is attracted by the negative plate, it carries a positive charge and is a proton. $\left(\frac{q}{m_p} = \frac{1.60\times10^{-19}\text{ C}}{1.67\times10^{-27}\text{ kg}} = 9.58\times10^{7}\text{ C/kg}\right)$

(c) $$v_x = \frac{E}{B} = \frac{V}{\mathrm{d}B} = \frac{2\,000\ \text{V}}{0.02\ \text{m}}\left(4.57\times10^{-2}\ \text{T}\right) = 2.19\times10^{6}\ \text{m/s}$$

(d) As $v_x \sim 0.01c$ there is no need for relativistic mechanics.

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4-8 (a) From Equation 4.16 we have $\Delta n \infty \left(\frac{\sin\phi}{2}\right)^{-4}$ or $\Delta n_2 = \Delta n_1 \frac{\left(\frac{\sin\phi_1}{2}\right)^4}{\left(\frac{\sin\phi_2}{2}\right)^4}$. Thus the number of α's scattered at 40 degrees is given by

$$\Delta n_2 = (100\ \text{cpm})\frac{\left(\sin\frac{20}{2}\right)^4}{\left(\sin\frac{40}{2}\right)^4} = (100\ \text{cpm})\left(\frac{\sin 10}{\sin 20}\right)^4 = 6.64\ \text{cpm}\ .$$

Similarly

$$\Delta n \text{ at 60 degrees} = 1.45 \text{ cpm}$$
$$\Delta n \text{ at 80 degrees} = 0.533 \text{ cpm}$$
$$\Delta n \text{ at 100 degrees} = 0.264 \text{ cpm}$$

(b) From 4.16 doubling $\left(\frac{1}{2}\right)m_\alpha v_\alpha^2$ reduces Δn by a factor of 4. Thus Δn at 20 degrees $= \left(\frac{1}{4}\right)(100 \text{ cpm}) = 25 \text{ cpm}$.

(c) From 4.16 we find $\frac{\Delta n_{\text{Cu}}}{\Delta n_{\text{Au}}} = \frac{Z_{\text{Cu}}^2 N_{\text{Cu}}}{Z_{\text{Au}}^2 N_{\text{Au}}}$, $Z_{\text{Cu}} = 29$, $Z_{\text{Au}} = 79$.

$$\begin{aligned} N_{\text{Cu}} &= \text{number of Cu nuclei per unit area} \\ &= \text{number of Cu nuclei per unit volume*foil thickness} \\ &= \left[\left(8.9 \text{ g/cm}^3\right)\left(\frac{6.02\times10^{23} \text{ nuclei}}{63.54 \text{ g}}\right)\right]t = 8.43\times10^{22}t \\ N_{\text{Au}} &= \left[\left(19.3 \text{ g/cm}^3\right)\left(\frac{6.02\times10^{23} \text{ nuclei}}{197.0 \text{ g}}\right)\right]t = 5.90\times10^{22}t \end{aligned}$$

So $\Delta n_{\text{Cu}} = \Delta n_{\text{Au}}(29)^2 \frac{8.43\times10^{22}}{(79)^2}\left(5.90\times10^2\right) = (100)\left(\frac{29}{79}\right)^2\left(\frac{8.43}{5.90}\right) = 19.3 \text{ cpm}$.

4-9 The initial energy of the system of α plus copper nucleus is 13.9 MeV and is just the kinetic energy of the α when the α is far from the nucleus. The final energy of the system may be evaluated at the point of closest approach when the kinetic energy is zero and the potential energy is $k(2e)\frac{Ze}{r}$ where r is approximately equal to the nuclear radius of copper. Invoking conservation of energy $E_i = E_f$, $K_\alpha = (k)\frac{2Ze^2}{r}$ or

$$r = (k)\frac{2Ze^2}{K_\alpha} = \frac{(2)(29)\left(1.60\times10^{-19}\right)^2\left(8.99\times10^9\right)}{\left(13.9\times10^6 \text{ eV}\right)\left(1.60\times10^{-19} \text{ J/eV}\right)} = 6.00\times10^{-15} \text{ m}.$$

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4-11 $\frac{1}{\lambda}=R\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)$. For the Balmer series, $n_f=2$; $n_i=3,\ 4,\ 5,\ \ldots$. The first three lines in the series have wavelengths given by $\frac{1}{\lambda}=R\left(\frac{1}{2^2}-\frac{1}{n^2}\right)$ where $R=1.097\,37\times10^7\ \text{m}^{-1}$.

$$\text{1st line: } \frac{1}{\lambda}=R\left(\frac{1}{4}-\frac{1}{9}\right)=\left(\frac{5}{36}\right)R;\ \lambda=\frac{36}{5R}=656.112\text{ nm}$$

$$\text{2nd line: } \frac{1}{\lambda}=R\left(\frac{1}{4}-\frac{1}{16}\right)=\left(\frac{3}{16}\right)R;\ \lambda=\frac{16}{3R}=486.009\text{ nm}$$

$$\text{3rd line: } \frac{1}{\lambda}=R\left(\frac{1}{4}-\frac{1}{25}\right)=\left(\frac{21}{100}\right)R;\ \lambda=\frac{100}{21R}=433.937\text{ nm}$$

4-12 $\frac{1}{\lambda}=R\left(\frac{1-1}{n^2}\right)$ where $n=2,\ 3,\ 4,\ \ldots$ and $R=1.097\,373\,2\times10^7\ \text{m}^{-1}$;

$$\text{For } n=2:\ \lambda=R^{-1}\left(1-\frac{1}{2^2}\right)^{-1}=1.215\,02\times10^{-7}\text{ m}=121.502\text{ nm (UV)}$$

$$\text{For } n=3:\ \lambda=R^{-1}\left(1-\frac{1}{3^2}\right)^{-1}=1.025\,17\times10^{-7}\text{ m}=102.517\text{ nm (UV)}$$

$$\text{For } n=4:\ \lambda=R^{-1}\left(1-\frac{1}{4^2}\right)^{-1}=1.972\,018\times10^{-7}\text{ m}=97.201\,8\text{ nm (UV)}$$

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4-14 (a) $r_n=\frac{n^2\hbar^2}{m_e ke^2}$; where $n=1,\ 2,\ 3,\ \ldots$

$$r_n=n^2\frac{\left(1.055\times10^{-34}\text{ Js}\right)^2}{\left(9.11\times10^{-31}\text{ kg}\right)\left(9.0\times19^9\ \text{Nm}^2/\text{C}^2\right)\left(1.6\times10^{-19}\text{ C}\right)^2}=\left(0.052\,9\text{ nm}\right)n^2$$

For $n=1$: $r_n=0.052\,9$ nm

For $n=2$: $r_n=0.212\,1$ nm

For $n=3$: $r_n=0.477\,2$ nm

(b) From Equation 4.26, $v=\left(\frac{ke^2}{m_e r}\right)^{1/2}$

$$v_1=\left[\frac{\left(8.99\times10^9\ \text{Nm}^2/\text{C}^2\right)\left(1.60\times10^{-19}\text{ C}\right)^2}{\left(9.11\times10^{-31}\text{ kg}\right)\left(0.052\,9\times10^{-9}\text{ m}\right)}\right]^{1/2}=2.19\times10^6\ \text{m/s}$$

$$v_2=1.09\times10^6\ \text{m/s}$$

$$v_3=7.28\times10^5\ \text{m/s}$$

(c) As $c = 3.0\times10^{8}$ m/s, $v \ll c$ and no relativistic correction is necessary.

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4-16 For Li^{2+}, $Z = 3$ from Equation 4.36

$$E_n = -\frac{13.6Z^2}{n^2 \text{ eV}} = -\frac{122.4}{n^2 \text{ eV}}$$

0

$E_3 = -13.6$ eV

$E_2 = -30.6$ eV

$E_1 = -122.4$ eV

So $E_1 = -122.4$ eV
$E_2 = -30.6$ eV
$E_3 = -13.6$ eV, etc.

4-17 $r = \dfrac{n^2\hbar^2}{Zm_eke^2} = \left(\dfrac{n^2}{Z}\right)\left(\dfrac{\hbar^2}{m_eke^2}\right)$; $n = 1$

$$r = \frac{1}{Z}\left[\frac{\left(1.055\times10^{-34}\text{ Js}\right)^2}{\left(9.11\times10^{-31}\text{ kg}\right)\left(9\times10^{9}\text{ Nm}^2/\text{C}^2\right)\left(1.6\times10^{-19}\text{ C}\right)^2}\right] = \frac{5.30\times10^{-11}\text{ m}}{Z}$$

(a) For He^{+}, $Z=2$, $r=\dfrac{5.30\times10^{-11}\ \text{m}}{2}=2.65\times10^{-11}\ \text{m}=0.026\,5\ \text{nm}$

(b) For Li^{2+}, $Z=3$, $r=\dfrac{5.30\times10^{-11}\ \text{m}}{3}=1.77\times10^{-11}\ \text{m}=0.017\,7\ \text{nm}$

(c) For Be^{3+}, $Z=4$, $r=\dfrac{5.30\times10^{-11}\ \text{m}}{4}=1.32\times10^{-11}\ \text{m}=0.013\,2\ \text{nm}$

4-18 (a) $\Delta E=(13.6\ \text{eV})\left(\dfrac{1}{1^2}-\dfrac{1}{3^2}\right)=12.1\ \text{eV}$

(b) Either $\Delta E=12.1\ \text{eV}$ or $\Delta E=(13.6\ \text{eV})\left(\dfrac{1}{1}-\dfrac{1}{2^2}\right)=10.2\ \text{eV}$ and

$\Delta E=(13.6\ \text{eV})\left(\dfrac{1}{2^2}-\dfrac{1}{3^2}\right)=1.89\ \text{eV}$.

4-19 (a) $\Delta E=(-13.6\ \text{eV})\left(\dfrac{1}{n_i^2}-\dfrac{1}{n_f^2}\right)=(-13.6\ \text{eV})\left(\dfrac{1}{9}-\dfrac{1}{4}\right)=1.89\ \text{eV}$

(b) $E=\dfrac{hc}{\lambda}\Rightarrow\lambda=\dfrac{hc}{E}=\left(4.14\times10^{-15}\ \text{eV s}\right)\dfrac{3\times10^{8}\ \text{m/s}}{1.89\ \text{eV}}=658\ \text{nm}$

(c) $c=\lambda f\Rightarrow f=\dfrac{c}{\lambda}=\dfrac{3\times10^{8}\ \text{m/s}}{657\times10^{-9}\ \text{m}}=4.56\times10^{14}\ \text{Hz}$

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4-22 $E = K + U = \frac{mv^2}{2} - \frac{ke^2}{r}$. But $\frac{mv^2}{2} = \left(\frac{1}{2}\right)\frac{ke^2}{r}$. Thus $E = \left(\frac{1}{2}\right)\left(\frac{-ke^2}{r}\right) = \frac{U}{2}$, so $U = 2E = 2(-13.6\text{ eV}) = -27.2\text{ eV}$ and $K = E - U = -13.6\text{ eV} - (-27.2\text{ eV}) = 13.6\text{ eV}$.

4-23 (a) $r_1 = (0.052\,9\text{ nm})n^2 = 0.052\,9\text{ nm}$ (when $n = 1$)

(b) $m_e v = m_e\left(\frac{ke^2}{m_e r}\right)^{1/2}$

$$m_e = \left[\frac{(9.1\times10^{-31}\text{ kg})(9\times10^{9}\text{ Nm}^2/\text{C}^2)}{5.29\times10^{-11}\text{ m}}\right]^{1/2}\times(1.6\times10^{-19}\text{ C})$$

$M_e v = 1.99\times10^{-24}\text{ kg m/s}$

(c) $L = m_e vr = (1.99\times10^{-24}\text{ kg m/s})(5.29\times10^{-11}\text{ m})$, $L = 1.05\times10^{-34}(\text{kg m}^2/\text{s}) = \hbar$

(d) $K = |E| = 13.6\text{ eV}$

(e) $U = -2K = -27.2\text{ eV}$

(f) $E = K + U = -13.6\text{ eV}$

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4-25 (a) $\Delta E = hf = (13.6 \text{ eV})\left(\frac{1}{n_{\mathrm{f}}^2} - \frac{1}{n_{\mathrm{i}}^2}\right)$ or $f = (13.6 \text{ eV})\left(\frac{\frac{1}{9} - \frac{1}{16}}{4.14\times10^{-15} \text{ eV s}}\right) = 1.60\times10^{14} \text{ Hz}$

(b) $T = \frac{2\pi r_n}{v}$ so $f_{\text{rev}} = \frac{1}{T} = \frac{v}{2\pi r_n}$. Using $v = \left(\frac{ke^2}{m_e r_n}\right)^{1/2}$, $f_{\text{rev}} = \left(\frac{ke^2}{mr_n}\right)^{1/2}$. For $n = 3$, $r_3 = (3)^2 a_0$ and

$$f_{\text{rev}} = \frac{\left(8.99\times10^9 \text{ Nm}^2/\text{C}^2\right)\left(1.60\times10^{-19} \text{ C}\right)^2}{\frac{\left[\left(9.11\times10^{-31} \text{ kg}\right)(9)\left(5.29\times10^{-11} \text{ m}\right)\right]^{1/2}}{(2)(3.14)(9)\left(5.29\times10^{-11} \text{ m}\right)}}$$

$$f_{\text{rev}} = 2.44\times10^{14} \text{ Hz } (n=3)$$

$$f_{\text{rev}} = 1.03\times10^{14} \text{ Hz } (n=4)$$

Thus the photon frequency is about halfway between the two frequencies of the revolution.

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4-32 (a)

$$\mu_{1H} = \frac{m_e M}{m_e + M} = \frac{(9.109\,390\times10^{-31}\text{ kg})(1.672\,63\times10^{-27}\text{ kg})}{(9.109\,390\times10^{-31}\text{ kg})+(1.672\,63\times10^{-27}\text{ kg})}$$

$$= \frac{(9.109\,390\times10^{-31}\text{ kg})(1.672\,63\times10^{-27}\text{ kg})}{(0.000\,910\,939\,0\times10^{-27}\text{ kg})+(1.672\,63\times10^{-27}\text{ kg})} = 9.104\,431\times10^{-31}\text{ kg}$$

$$\frac{1}{\lambda} = \frac{\mu}{m_e}(k)\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right) = \left(\frac{9.104\,431\,6\times10^{-31}\text{ kg}}{9.109\,390\times10^{-31}\text{ kg}}\right)(1.097\,315\,3\times10^{7}\text{ m}^{-1})\left(\frac{1}{2^2}-\frac{1}{3^2}\right)$$

$$\lambda_{1H} = 656.469\,1\text{ nm}$$

(b) Similarly, we find $\lambda_{2H} = 656.292\,5$ nm .

(c) $\lambda_{3H} = 656.232\,5$ nm

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4-36 (a)

$$f_{\text{revolution}} = \frac{v}{2\pi r} = \frac{\frac{n\hbar}{m_e r}}{2\pi r} = \frac{n\hbar}{2\pi m_e r^2} \text{ as } r^2 = n^4 (a_0)^2,$$

$$f_{\text{revolution}} = \frac{\hbar}{2\pi m_e}(a_0)^2 n^3 = \frac{6.58\times10^{15}}{n^3 \text{ Hz}}. \text{ Thus:}$$

$$\text{for } n = 100,\ f_{\text{revolution}} = 6.62\times10^{9} \text{ Hz}$$
$$\text{for } n = 1\,000,\ f_{\text{revolution}} = 6.62\times10^{6} \text{ Hz}$$
$$\text{for } n = 10\,000,\ f_{\text{revolution}} = 6.62\times10^{3} \text{ Hz}$$

using $r_n = n^2 a_0 = n^2\left(0.529\times10^{-10} \text{ m}\right)$,

$$r_{100} = 0.529\times10^{-6} \text{ m}$$
$$r_{1\,000} = 0.529\times10^{-4} \text{ m}$$
$$r_{10\,000} = 0.529\times10^{-2} \text{ m} \cong \frac{1}{2} \text{ cm}$$

(b) $$f_{\text{photon}} = \frac{\Delta E}{h} = \frac{13.6Z^2}{h}\left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right) = \left(\frac{(13.6)(1)^2 \text{ eV}}{4.14\times10^{-15} \text{ eV s}}\right)\left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right)$$
$$= 3.285\times10^{15}\left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right)\text{Hz}$$

$$\text{for } n = 100,\ f_{\text{photon}} = 6.579\,8\times10^{9} \text{ Hz}$$
$$\text{for } n = 1\,000,\ f_{\text{photon}} = 6.579\,8\times10^{6} \text{ Hz}$$
$$\text{for } n = 10\,000,\ f_{\text{photon}} = 6.579\,8\times10^{3} \text{ Hz}$$

Thus, the difference in frequency, $\Delta f = f_{\text{revolution}} - f_{\text{photon}}$, are

$$n = 100,\ \Delta f = -0.05\times10^{9} \text{ Hz}$$
$$n = 1\,000,\ \Delta f = -0.04\times10^{6} \text{ Hz}$$
$$n = 10\,000,\ \Delta f = -0.05\times10^{3} \text{ Hz}$$

(c) These results show that f_{photon} tends to $f_{\text{revolution}}$ in the limit of large quantum numbers $(n = 10\,000)$ and macroscopic sizes $\left(r \sim \frac{1}{2} \text{ cm}\right)$.

4-37 $hf = \Delta E = \frac{4\pi^2 m_e k^2 e^4}{2h^2}\left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right)$, $f = \left(\frac{2\pi^2 m_e k^2 e^4}{h^3}\right)\left(\frac{2n-1}{(n-1)^2\left(n^2\right)}\right)$ as $n \to \infty$,

$f \to \left(\frac{2\pi^2 m_e k^2 e^4}{h^3}\right)\left(\frac{2}{n^3}\right)$. The revolution frequency is $f = \frac{v}{2\pi r} = \left(\frac{1}{2\pi}\right)\left(\frac{ke^2}{m_e}\right)^{1/2}\left(\frac{1}{r^{3/2}}\right)$ where

$r = \frac{n^2 h^2}{4\pi^2 m_e k e^2}$ substituting for r, $f = \left(\frac{1}{2\pi}\right)\left(\frac{ke^2}{m_e}\right)^{1/2}\left(\frac{8\pi^3 m_e k e^3 (m_e k)^{1/2}}{n^3 h^3}\right) = \frac{4\pi^2 m_e k^2 e^4}{h^3 n^3}$.

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