3-16. $\lambda_m T = 2.898\times10^{-3} m\cdot K$ (a) $T = \dfrac{2.898\times10^{-3} m\cdot K}{700\times10^{-9} m} = 4140\,K$

(b) $T = \dfrac{2.898\times10^{-3} m\cdot K}{3\times10^{-2} m} = 9.66\times10^{-2}\,K$ (c) $T = \dfrac{2.898\times10^{-3} m\cdot K}{3 m} = 9.66\times10^{-4}\,K$

3-17. Equation 3-10: $R_1 = \sigma T_1^4$ $R_2 = \sigma T_2^4 = \sigma(2T_1)^4 = 16\sigma T_1^4 = 16R_1$

3-18. (a) Equation 3-23: $\overline{E} = \dfrac{hc/\lambda}{e^{hc/\lambda kT} - 1} = \dfrac{hc/(10hc/kT)}{e^{(hc/kT)/(10hc/kT)} - 1} = \dfrac{0.1kT}{e^{0.1} - 1} = 0.951\,kT$

(b) $\overline{E} = \dfrac{hc/\lambda}{e^{hc/\lambda kT} - 1} = \dfrac{hc/(0.1hc/kT)}{e^{(hc/kT)/(0.1hc/kT)} - 1} = \dfrac{10kT}{e^{10} - 1} = 4.59\times10^{-4}\,kT.$

Equipartition theorem predicts $\overline{E} = kT.$ The long wavelength value is very close to kT, but the short wavelength value is much smaller than the classical prediction.

3-19. (a) $\lambda_m T = 2.898\times10^{-3} m\cdot K \therefore T_1 = \dfrac{2.898\times10^{-3} m\cdot K}{27.0\times10^{-6} m} = 107\,K$

$$R_1 = \sigma T_1^4 \text{ and } R_2 = \sigma T_2^4 = 2R_1 = 2\sigma T_1^4$$

$$\therefore T_2^4 = 2T_1^4 \text{ or } T_2 = 2^{1/4}T_1 = (2^{1/4})(107\,K) = 128\,K$$

(b) $\lambda_m = \dfrac{2.898\times10^{-3} m\cdot K}{128\,K} = 23\times10^{-6}\,m$

3-20. $\lambda_m = 2.898\times10^{-3}\,m\cdot K$ (Equation 3-20)

$$\lambda_m = \frac{2.898\times10^{-3} m\cdot K}{2\times10^4\,K} = 1.45\times10^{-7} m = 145\,nm$$

3-21. Equation 3-10:

$R = \sigma T^4$

$P_{abs} = (1.36\times 10^3 W/m^2)(\pi R_E^2 m^2)$ where R_E = radius of Earth

$p_{emit} = (R\,W/m^2)(4\pi R_E^2) = (1.36\times 10^3 W/m^2)(\pi R_E^2 m^2)$

$$R = (1.36\times 10^3 W/m^2)\left(\frac{\pi R_E^2}{4\pi R_E^2}\right) = \frac{1.36\times 10^3}{4}\frac{W}{m^2} = \sigma T^4$$

$$T^4 = \frac{1.36\times 10^3\ W/m^2}{4(5.67\times 10^{-8} W/m^2\cdot K^4)} \quad \therefore\ T = 278.3K = 5.3C$$

3-22. (a) $\lambda_m T = 2.898\times 10^{-3} m\cdot K \ \therefore\ \lambda_m = \dfrac{2.898\times 10^{-3} m\cdot K}{3300K} = 8.78\times 10^{-7} m = 878\ nm$

$$f_m = c/\lambda_m = \frac{3.00\times 10^8 m/s}{8.78\times 10^{-7} m} = 3.42\times 10^{14}\ Hz$$

(b) Each photon has average energy *E = hf* and *NE = 40 J/s*.

$$N = \frac{40J/s}{hf_m} = \frac{40J/s}{(6.63\times 10^{-34} J\cdot s)(3.42\times 10^{14} Hz)} = 1.77\times 10^{20}\ photons/s$$

(c) At 5m from the lamp N photons are distributed uniformly over an area $A = 4\pi r^2 = 100\pi m^2$. The density of photons on that sphere is $(N/A)/s\cdot m^2$. The area of the pupil of the eye is $\pi(2.5\times 10^{-3} m)^2$, so the number *n* of photons entering the eye per second is

$$n = (N/A)(\pi)(2.5\times 10^{-3} m)^2 = \frac{(1.77\times 10^{20}/s)(\pi)(2.5\times 10^{-3} m)^2}{100\pi m^2}$$

$$= (1.77\times 10^{18}/s)(2.5\times 10^{-3})^2 = 1.10\times 10^{13}\ photons/s$$

Chapter 8 – Statistical Physics

8-1. (a) $v_{rms} = \sqrt{\dfrac{3RT}{M}} = \left[\dfrac{3(8.31\,J/mole{\cdot}K)(300K)}{2(1.0079\times10^{-3}\,kg/mole)}\right]^{1/2} = 1930\,m/s$

(b) $T = \dfrac{Mv_{rms}^2}{3R} = \dfrac{2(1.0079\times10^{-3}\,kg/mole)(11.2\times10^3\,m/s)^2}{3(8.31\,J/mole{\cdot}K)} = 1.01\times10^4\,K$

8-2. (a) $\overline{E_K} = \dfrac{3}{2}kT \quad \therefore\ T = \dfrac{2\overline{E_K}}{3k} = \dfrac{2(13.6\,eV)}{3(8.617\times10^{-5}\,eV/K)} = 1.05\times10^5\,K$

(b) $\overline{E_K} = \dfrac{3}{2}kT = \dfrac{3}{2}(8.617\times10^{-5}\,eV/K)(10^7\,K) = 1.29\,keV$

8-3. $v_{rms} = \sqrt{\dfrac{3RT}{M}}$ (Equation 8-12)

(a) For O_2: $v_{rms} = \sqrt{\dfrac{3(8.31\,J/K{\cdot}mol)(273\,K)}{32\times10^{-3}\,kg/mol}} = 461\,m/s$

(b) For H_2: $v_{rms} = \sqrt{\dfrac{3(8.31\,J/K{\cdot}mol)(273\,K)}{2\times10^{-3}\,kg/mol}} = 1840\,m/s$

8-4. $\left[\dfrac{3RT}{M}\right]^{1/2} = \left[\dfrac{(J/mole{\cdot}K)(K)}{kg/mole}\right]^{1/2} = \left[\dfrac{kg{\cdot}m^2/s^2}{kg}\right]^{1/2} = m/s$

8-5. (a) $E_K = n \cdot \frac{3}{2} RT = (1\ mole)\frac{3}{2}(8.31\ J/mole{\cdot}K)(273) = 3400\ J$

(b) One mole of any gas has the same translational kinetic energy at the same temperature.

8-6. $\langle v^2 \rangle = \frac{1}{N}\int_0^\infty v^2 n(v)\,dv = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\int_0^\infty v^4 e^{-\lambda v^2}\,dv$ where $\lambda = m/2kT$

$\langle v^2 \rangle = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2} I_4$ where I_4 is given in Table B1-1.

$$I_4 = \frac{3}{8}\pi^{1/2}\lambda^{-5/2} = \frac{3}{8}\pi^{1/2}(m/2kT)^{-5/2}$$

$$\langle v^2 \rangle = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\left(\frac{3}{8}\right)\pi^{1/2}\left(\frac{2kT}{m}\right)^{5/2} = \frac{3kT}{m} = \frac{3RT}{mN_A} = \frac{3RT}{M}$$

$$v_{rms} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3RT}{M}}$$

8-7. $\langle v \rangle = \sqrt{\frac{8kT}{\pi m}} = \left[\frac{8(1.381\times10^{-23}\,J/K)(300\,K)}{\pi(1.009\,u)(1.66\times10^{-27}\,kg/u)}\right]^{1/2} = 2510\ m/s$

$$v_m = \sqrt{\frac{2kT}{m}} = \left[\frac{2(1.381\times10^{-23}\,J/K)(300\,K)}{\pi(1.009\,u)(1.66\times10^{-27}\,kg/u)}\right]^{1/2} = 2220\ m/s$$

$n(v) = 4\pi N(m/2\pi kT)^{3/2} v^2 e^{-mv^2/kT}$ (Equation 8 – 28)

(Problem 8-7 continued)

At the maximum: $\dfrac{dn}{dv} = 0 = 4\pi N(m/2\pi kT)^{3/2}\{2v + v^2(-mv/kT)\}e^{-mv^2/2kT}$

$$0 = ve^{-mv^2/2kT}(2 - mv^2/kT)$$

The maximum corresponds to the vanishing of the last factor. (The other two factors give minima at v = 0 and v = ∞.) So $2 - mv^2/kT = 0$ and $v_m = (2kT/m)^{1/2}$.

8-8. (a) $$f(v_x) = (m/2\pi kT)^{1/2}e^{-mv_x^2/2kT} \quad \text{(Equation 8-20)}$$

$$= \frac{1}{\sqrt{2\pi}}\sqrt{\frac{m}{kT}}\,e^{-(m/kT)(v_x^2/2)}$$

$$= (2\pi)^{-1/2}v_0^{-1}e^{-v_x^2/2v_0^2} \quad \textit{where } v_0 = v_{x,\,rms} = (kT/m)^{1/2}$$

(b) $$\Delta N = Nf(v_x)\Delta v_x = N_A f(v_x)(0.01v_0)$$

$$= (6.02\times 10^{23})(2\pi)^{-1/2}v_0^{-1}e^{-0}(0.01v_0) = 2.40\times 10^{21}$$

(c) $\Delta N = 6.02\times 10^{23}(2\pi)^{-1/2}v_o^{-1}e^{-1/2}(0.01v_0) = 1.46\times 10^{21}$

(d) $\Delta N = 6.02\times 10^{23}(2\pi)^{-1/2}v_o^{-1}e^{-2}(0.01v_0) = 3.25\times 10^{20}$

(e) $\Delta N = 6.02\times 10^{23}(2\pi)^{-1/2}v_o^{-1}e^{-32}(0.01v_0) = 3.04\times 10^{7}$

8-9. $$m(v)dv = 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2}v^2e^{-mv^2/2kT}dv \quad \text{(Equation 8-28)}$$

$\dfrac{du}{dv} = A\left[v^2\left(-\dfrac{2vm}{2kT}\right) + 2v\right]e^{-mv^2/2kT}$ The v for which $dn/dv = 0$ is v_m.

(Problem 8-9 continued)

$$A\left[-\frac{2mv^3}{2kT} + 2v\right]e^{-mv^2/2kT} = 0$$

Because A = constant and the exponential term is only zero for $v = \infty$, only the quantity in [] can be zero, so $-\frac{2mv^3}{2kT} + 2v = 0$

or $v^2 = \frac{2kT}{m} \rightarrow v_m = \sqrt{\frac{2kT}{m}}$ (Equation 8-29)

8-10. The number of molecules N in 1 liter at 1 atm, 20°C is:

$$N = 1\ell(1\,g\cdot mol/22.4\,\ell)(N_A\,molecules/g\cdot mol)$$

Each molecule has, on the average, $3kT/2$ kinetic energy, so the total translational kinetic energy in one liter is: $KE = \frac{6.02\times10^{23}}{22.4}\left[\frac{3(1.381\times10^{-23}J/K)(293K)}{2}\right] = 163\,J$

8-11.

$$\frac{n_2}{n_1} = \frac{g_2\,e^{-E_2/kT}}{g_1\,e^{-E_1/kT}} = \frac{g_2}{g_1}\,e^{-(E_2-E_1)/kT}$$

$$e^{(E_2-E_1)/kT} = \frac{g_2}{g_1}\cdot\frac{n_1}{n_2} = (E_2 - E_1)/kT = \ln\left(\frac{g_2}{g_1}\cdot\frac{n_1}{n_2}\right)$$

$$T = \frac{E_2 - E_1}{k\ln[(g_2/g_1)(n_1/n_2)]} = \frac{10.2\,eV}{(8.617\times10^{-5}eV/K)\ln(4\times10^6)} = 7790\,K$$

8-12. $$\frac{n_2}{n_1} = \frac{g_2}{g_1}e^{-(E_1-E_2)/kT} = \frac{3}{1}\,e^{-\left[\frac{4\times10^{-3}eV}{(8.617\times10^{-5}eV/K)(300K)}\right]} = 0.155$$

8-43. $f_{FD} = \dfrac{1}{e^{\alpha} e^{E/kT} + 1} = \dfrac{1}{e^{(E-E_F)/kT} + 1}$ where $\alpha = -\dfrac{E_F}{kT}$

For $E >> E_F$, $e^{(E-E_F)/kT} >> 1$ and

$$f_{FD} \approx \frac{1}{e^{(E-E_F)/kT}} = \frac{1}{e^{-E_F/kT} \cdot e^{E/kT}} = \frac{1}{e^{\alpha} e^{E/kT}} = f_B$$

8-44. $N = e^{-\alpha} \dfrac{4\pi (2m_e)^{3/2} V}{h^3} \int_0^{\infty} E^{1/2} e^{-E/kT}\, dE$ (Equation 8-67)

Considering the integral, we change the variable: $E/kT = u^2$, then

$E = kTu^2$, $E^{1/2} = (kT)^{1/2} u$, and $dE = kT(2u)du$. So,