3-8 Using \( E = hf \) with \( h = 4.136 \times 10^{-15} \text{ eV} \) gives

(a) for \( f = 5 \times 10^{14} \text{ Hz} \), \( E = 2.07 \text{ eV} \)

(b) for \( f = 10 \text{ GHz} \), \( E = 4.14 \times 10^{-5} \text{ eV} \)

(c) for \( f = 30 \text{ MHz} \), \( E = 1.24 \times 10^{-7} \text{ eV} \)

3-9 Use \( E = \frac{hc}{\lambda} \) or \( \lambda = \frac{hc}{E} \) (where \( hc = 1.240 \text{ eV nm} \)) to find

(a) \( \lambda = 600 \text{ nm} \)

(b) \( \lambda = 0.03 \text{ m} \)

(c) \( \lambda = 10 \text{ m} \)

3-10 The energy per photon, \( E = hf \) and the total energy \( E \) transmitted in a time \( t \) is \( Pt \) where power \( P = 100 \text{ kW} \). Since \( E = nhf \) where \( n \) is the total number of photons transmitted in the time \( t \), and \( f = 94 \text{ MHz} \), there results \( nhf = (100 \text{ kW})t = (10^5 \text{ W})t \), or

\[
\frac{n}{t} = \frac{10^5 \text{ W}}{hf} = \frac{10^5 \text{ J/s}}{6.63 \times 10^{-34} \text{ J s}} \left(94 \times 10^6 \text{ s}^{-1}\right) = 1.60 \times 10^{30} \text{ photons/s}.
\]

3-12 As in Problems 3-9 and 3-10,

\[
\frac{n}{t} = \frac{P}{hf} = \frac{P \lambda}{hc} = (10 \text{ W}) \frac{589 \times 10^{-9} \text{ m}}{1.99 \times 10^{-25} \text{ J m}} = 3.0 \times 10^{19} \text{ photons/s}.
\]

3-13 \( K = hf - \phi = \frac{hc}{\lambda - \phi} \)

\[
\phi = \frac{hc}{\lambda - K} = \frac{1.240 \text{ eV nm}}{250 \text{ nm}} = -2.92 \text{ eV} = 2.04 \text{ eV}
\]

3-14 (a) \( K = hf - \phi = \frac{hc}{\lambda - \phi} = \frac{1.240 \text{ eV nm}}{350 \text{ nm}} = -2.24 \text{ eV} = 1.30 \text{ eV} \)

(b) At \( \lambda = \lambda_c \), \( K = 0 \) and \( \lambda = \frac{hc}{\phi} = \frac{1.240 \text{ eV nm}}{2.24 \text{ eV}} = 554 \text{ nm} \)

3-15 (a) At the cut-off wavelength, \( K = 0 \) so \( \frac{hc}{\lambda} - \phi = 0 \), or

\[
\lambda_{\text{cut-off}} = \frac{hc}{\phi} = \frac{1.240 \text{ eV nm}}{4.2 \text{ eV}} = 300 \text{ nm} \). The threshold frequency, \( f_0 \) is given by
\[ f_0 = \frac{c}{\lambda_{\text{cut-off}}} = \frac{3.0 \times 10^8 \text{ m/s}}{3.0 \times 10^2 \times 10^{-9} \text{ m}} = 1.0 \times 10^{15} \text{ Hz}. \]

(b) \[ eV_s = K = \hbar f - \phi = \frac{\hbar c}{\lambda} - \phi \]
\[ V_s = \frac{\hbar c}{\lambda e} \frac{1.240 \text{ eV nm}}{200 \text{ nm e}} = -4.2 \text{ eV/e} = 2.0 \text{ V} \]

3-18 (a) \[ K_{\text{max}} = eV_s = s(0.45 \text{ V}) = 0.45 \text{ eV} \]

(b) \[ \phi = \frac{\hbar c}{\lambda} = \frac{1.240 \text{ eV nm}}{500 \text{ nm}} = 2.48 \text{ eV} = 2.03 \text{ eV} \]

(c) \[ \lambda_c = \frac{\hbar c}{\phi} = \frac{1.240 \text{ eV nm}}{2.03 \text{ eV}} = 612 \text{ nm} \]

3-20 \[ K_{\text{max}} = \hbar f - \phi = \frac{\hbar c}{\lambda} - \phi \Rightarrow \phi = \frac{\hbar c}{\lambda} - K_{\text{max}}; \]

First Source: \[ \phi = \frac{\hbar c}{\lambda} = 1.00 \text{ eV}. \]

Second Source: \[ \phi = \frac{\hbar c}{\lambda} - 4.00 \text{ eV} = \frac{2\hbar c}{\lambda} - 4.00 \text{ eV}. \]

As the work function is the same for both sources (a property of the metal),
\[ \frac{\hbar c}{\lambda} - 100 \text{ eV} = \frac{2\hbar c}{\lambda} - 4.00 \text{ eV} \Rightarrow \frac{\hbar c}{\lambda} = 3.00 \text{ eV and } \phi = \frac{\hbar c}{\lambda} - 1.00 \text{ eV} = 3.00 \text{ eV} - 1.00 \text{ eV} = 2.00 \text{ eV}. \]

3-25 \[ E = 300 \text{ keV}, \theta = 30^\circ \]

(a) \[ \Delta \lambda = \lambda' - \lambda_0 = \frac{h}{m_e c} \left[1 - \cos \theta \right] = \left[0.002 43 \text{ nm}\right]
\times \left[1 - \cos(30^\circ)\right] = 3.25 \times 10^{-13} \text{ m} \]
\[ = 3.25 \times 10^{-4} \text{ nm} \]

(b) \[ E = \frac{\hbar c}{\lambda_0} \Rightarrow \lambda_0 = \frac{\hbar c}{E_0} = \frac{\left(4.14 \times 10^{-15} \text{ eV s}\right)\left(3 \times 10^8 \text{ m/s}\right)}{300 \times 10^{-15} \text{ eV}} = 4.14 \times 10^{-12} \text{ m}; \text{ thus,} \]
\[ \lambda' = \lambda_0 + \Delta \lambda = 4.14 \times 10^{-12} \text{ m} + 0.325 \times 10^{-12} \text{ m} = 4.465 \times 10^{-12} \text{ m}, \text{ and} \]
\[ E' = \frac{\hbar c}{\lambda'} \Rightarrow E' = \frac{\left(4.14 \times 10^{-15} \text{ eV s}\right)\left(3 \times 10^8 \text{ m/s}\right)}{4.465 \times 10^{-12} \text{ m}} = 2.78 \times 10^5 \text{ eV}. \]

(c) \[ \frac{\hbar c}{\lambda_0} = \frac{\hbar c}{\lambda'} + K_e, \text{ (conservation of energy)} \]
\[ K_e = \hbar c \left( \frac{1}{\lambda_0} - \frac{1}{\lambda'} \right) = \frac{\left(4.14 \times 10^{-15} \text{ eV s}\right)\left(3 \times 10^8 \text{ m/s}\right)}{4.14 \times 10^{-12} \text{ m} - 4.465 \times 10^{-12} \text{ m}} = 22 \text{ keV} \]

3-28 (a) From conservation of energy we have \[ E_0 = E' + K_e = 120 \text{ keV} + 40 \text{ keV} = 160 \text{ keV}. \]

The photon energy can be written as \[ E_0 = \frac{\hbar c}{\lambda_0}. \] This gives
\[ \lambda_0 = \frac{\hbar c}{E_0} = \frac{1240 \text{ nm eV}}{160 \times 10^3 \text{ eV}} = 7.75 \times 10^{-3} \text{ nm} = 0.00775 \text{ nm}. \]

(b) Using the Compton scattering relation \( \lambda' - \lambda_0 = \lambda_c (1 - \cos \theta) \) where
\[
\lambda_c = \frac{\hbar}{m_e c} = 0.00243 \text{ nm} \quad \text{and} \quad \lambda' = \frac{\hbar c}{E'} = \frac{1240 \text{ nm eV}}{120 \times 10^3 \text{ eV}} = 10.3 \times 10^{-3} \text{ nm} = 0.0103 \text{ nm}.
\]
Solving the Compton equation for \( \cos \theta \), we find
\[
-\lambda_c \cos \theta = \lambda' - \lambda_0 - \lambda_c
\]
\[
\cos \theta = 1 - \frac{\lambda' - \lambda_0}{\lambda_c} = 1 - \frac{0.0103 \text{ nm} - 0.0075 \text{ nm}}{0.00243 \text{ nm}} = 1 - 1.049 = -0.049
\]
The principle angle is 87.2° or \( \theta = 92.8° \).

(c) Using the conservation of momentum Equations 3.30 and 3.31 one can solve for the recoil angle of the electron.
\[
p = p' \cos \theta + p_e \cos \phi
\]
\[p_e \sin \phi = p' \sin \theta; \text{ dividing these equations one can solve for the recoil angle of the electron}
\]
\[
\tan \phi = \frac{p' \sin \theta}{p - p' \cos \theta} = \frac{\sin \theta}{\frac{h}{\lambda'} - \frac{\hbar c}{E'} \cos \theta} = \frac{\hbar c}{\frac{\hbar c}{\lambda_0} - \frac{\hbar c}{\lambda'} \cos \theta}
\]
\[
= \frac{120 \text{ keV}(0.9988)}{160 \text{ keV} - 120 \text{ keV}(-0.049)} = 0.7232
\]
and \( \phi = 35.9° \).

3-30 Maximum energy transfer occurs when the scattering angle is 180 degrees. Assuming the electron is initially at rest, conservation of momentum gives
\[
hf + hf' = p_e c = \sqrt{\left(m_e c^2 + K\right)^2 - m_e^2 c^4} = \sqrt{\left(511 + 50\right)^2} = 178 \text{ keV}
\]
while conservation of energy gives \( hf - hf' = K = 30 \text{ keV} \). Solving the two equations gives \( E = hf = 104 \text{ keV} \) and \( hf' = 74 \text{ keV} \). (The wavelength of the incoming photon is \( \lambda = \frac{\hbar c}{E} = 0.0120 \text{ nm} \).

3-31 (a) \( E' = \frac{\hbar c}{\lambda'}, \lambda' = \lambda_0 + \Delta \lambda \)
\( \lambda_0 = \frac{\hbar c}{E_0} = \left( \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{0.1 \text{ MeV}} \right) = 1.243 \times 10^{-11} \text{ m} \)

\[ \Delta \lambda = \left( \frac{\hbar}{m_e c} \right) (1 - \cos \theta) = \left( \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{9.11 \times 10^{-34} \text{ kg} \left( 3 \times 10^8 \text{ m/s} \right)} \right) = 1.215 \times 10^{-12} \text{ m} \]

\( \lambda' = \lambda_0 + \Delta \lambda = 1.364 \times 10^{-11} \text{ m} \)

\[ E' = \frac{\hbar c}{\lambda'} = \left( \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{1.364 \times 10^{-11} \text{ m}} \right) = 9.11 \times 10^4 \text{ eV} \]

(b) \[ \frac{\hbar c}{\lambda_0} = \frac{\hbar c}{\lambda'} + K_e \]

\( K_e = 0.1 \text{ MeV} - 91.1 \text{ keV} = 8.90 \text{ keV} \)

(c) Conservation of momentum along \( x \): \[ \frac{\hbar}{\lambda_0} = \left( \frac{\hbar}{\lambda'} \right) \cos \theta + \frac{\gamma m_e v}{\sin \theta} \cos \phi \]

Conservation of momentum along \( y \): \[ \frac{\gamma m_e v}{\sin \theta} \cos \phi = \frac{\hbar}{\lambda} \cos \theta \]

So that \[ \tan \phi = \frac{\lambda_0 \sin \theta}{(\lambda' - \lambda_0) \cos \theta} \]

Here, \( \theta = 60^\circ \), \( \lambda_0 = 1.243 \times 10^{-11} \text{ m} \), and \( \lambda' = 1.364 \times 10^{-11} \text{ m} \). Consequently,

\[ \tan \phi = \frac{(1.24 \times 10^{-11} \text{ m})(\sin 60^\circ)}{(1.36 - 1.24 \cos 60^\circ) \times 10^{-11} \text{ m}} = 1.451 \]

\[ \phi = 55.4^\circ \]

3-33 Substituting equations 3-33 and 3-34 of the text, \( E_e = h(f_0 - f') + m_e c^2 \) and

\[ p_e^2 c^2 = h^2 \left( f'^2 + f_0^2 \right) - 2h^2 f' f_0 \cos \theta \]

into the relativistic energy expression \( E_e^2 = p_e^2 c^2 + (m_e c^2)^2 \) yields
\[ h^2 \left( f'^2 + f_0^2 - 2f_0f' \right) + m_e^2c^4 + 2h(f_0 - f')m_e c^2 = h^2 \left( f'^2 + f_0^2 \right) - 2h^2 f_0 f' \cos \theta^2 + \left( m_e c^2 \right)^2. \]

Canceling and combining there results

\[ \left( f'^2 + f_0^2 - 2f_0f' \right) + \frac{2m_e c^2(f_0 - f')}{h} = f'^2 + f_0^2 - 2f_0 f' \cos \theta \]

which reduces to \[ \frac{m_e c^2(f_0 - f')}{h} = f_0 f'(1 - \cos \theta). \] Using \( \lambda f = c \) one obtains

\[ \lambda' - \lambda_0 = \frac{h(1 - \cos \theta)}{m_ec}, \]

which is the Compton scattering or Compton shift relation.

3-43 (a) A 4000 Å wavelength photon is backscattered, \( \theta = \pi \) by an electron. The energy transferred to the electron is determined by using the Compton scattering formula \( \lambda' - \lambda_0 = \frac{h}{E_e}(1 - \cos \theta) \) where we take \( E_e = m_e c^2 \) for the rest energy of the electron

and so \( E_e = 0.511 \text{ MeV} \). Upon substitution, one obtains

\[ \Delta \lambda = 2 \left( 0.002 \text{ 43 nm} \right) = 0.004 \text{ 86 nm}. \]

The energy of a photon is related to its wavelength by the relation \( E = \frac{hc}{\lambda} \), so the change in energy associated with a corresponding change in wavelength is given by \( \Delta E = \left( \frac{hc}{\lambda^2} \right) \Delta \lambda \). Upon making substitutions one obtains the magnitude

\[ \Delta E = 6.037 \times 10^{-24} \text{ J} \] and using the conversion factor 1 Joule of energy is equivalent to \( 1.602 \times 10^{-19} \text{ eV} \). The result is \( \Delta E = 3.77 \times 10^{-5} \text{ eV} \).

(b) This may be compared to the energy that would be acquired by an electron in the photoelectric effect process. Here again the energy of a photon of wavelength \( \lambda \) is given by \( E = \frac{hc}{\lambda} \). With \( \lambda = 400 \text{ nm} \), one obtains

\[ E = \left( 6.626 \times 10^{-34} \text{ J} \cdot \text{s} \right) \left( 3.0 \times 10^8 \text{ m/s} \right) = 4.97 \times 10^{-19} \text{ J} \]

and upon converting to electron volts, \( E = 3.10 \text{ eV} \). \[ \frac{\Delta E}{E_{\text{photon}}} = 10^{-5} \]. The maximum energy transfer is about five orders of magnitude smaller than the energy necessary for the photoelectric effect.

(c) Could “a violet photon” eject an electron from a metal by Compton scattering? The answer is no, because the maximum energy transfer occurring at \( \theta = \pi \) is not sufficient.

3-44 Each emitted electron requires an energy
\[ hf = \frac{1}{2} mv^2 + \phi = \left( \frac{9.11 \times 10^{-31} \text{ kg}}{2} \right) \left( 4.2 \times 10^5 \text{ m/s} \right)^2 + (3.44 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV}) \]

\[ \Delta E = 6.3 \times 10^{-19} \text{ J per emitted electron} \]

Therefore, with an incident intensity of \( \frac{0.055 \text{ J/m}^2}{\text{s}} = \frac{5.5 \times 10^{-6} \text{ J/cm}^2}{\text{s}} \), the number of electrons emitted per cm\(^2\) per second is

\[
\text{electron flux} = \frac{5.5 \times 10^{-6} \text{ J/cm}^2}{\text{s}} \div 6.3 \times 10^{-19} \text{ J/emitted electron} = \frac{8.73 \times 10^{12}}{\text{cm}^2/\text{s}}.
\]