PHYSICS 211C : CONDENSED MATTER PHYSICS
HW ASSIGNMENT #1

(1) Consider an ion with a partially filled shell of angular momentum J, and Z additional electrons in filled shells. Show that the ratio of the Curie paramagnetic susceptibility to the Larmor diamagnetic susceptibility is

$$\frac{\chi^{\rm para}}{\chi^{\rm dia}} = -\frac{g_{\rm L}^2\, J(J+1)}{2Zk_{\rm B}T}\,\frac{\hbar^2}{m\langle r^2\rangle} \quad .$$

where $g_{\rm L}$ is the Landé g-factor. Estimate this ratio at room temperature.

(2) In an ideal paramagnet, the spins are noninteracting and the Hamiltonian is

$$\mathcal{H} = \sum_{i=1}^{N_{\rm p}} \gamma_i\, \boldsymbol{J}_i \cdot \boldsymbol{H} \quad ,$$

where $\gamma_i = g_i \mu_i/\hbar$ and $\boldsymbol{J}_i$ are the gyromagnetic factor and spin operator for the $i^{\rm th}$ paramagnetic ion, and $\boldsymbol{H}$ is the external magnetic field.

(a) Show that the free energy $F(H,T)$ can be written as

$$F(H,T) = T\,\varPhi(H/T) \quad .$$

If an ideal paramagnet is held at temperature $T_{\rm i}$ and field $H_{\rm i}\,\hat{\boldsymbol{z}}$, and the field $H_{\rm i}$ is *adiabatically* lowered to a value $H_{\rm f}$, compute the final temperature. This is called "adiabatic demagnetization".

(b) Show that, in an ideal paramagnet, the specific heat at constant field is related to the susceptibility by the equation

$$c_H = T\left(\frac{\partial s}{\partial T}\right)_H = \frac{H^2\,\chi}{T} \quad .$$

Further assuming all the paramagnetic ions to have spin J, and assuming Curie's law to be valid, this gives

$$c_H = \tfrac{1}{3} n_{\rm p} k_{\rm B} J(J+1) \left(\frac{g\mu_{\rm B} H}{k_{\rm B}T}\right)^2 \quad ,$$

where $n_{\rm p}$ is the density of paramagnetic ions. You are invited to compute the temperature T^* below which the specific heat due to lattice vibrations is smaller than the paramagnetic contribution. Recall the Debye result

$$c_V = \tfrac{12}{5}\pi^4\, n k_{\rm B} \left(\frac{T}{\varTheta_{\rm D}}\right)^3 \quad ,$$

where $n = 1/\varOmega$ is the inverse of the unit cell volume (*i.e.* the density of unit cells) and $\varTheta_{\rm D}$ is the Debye temperature. Compile a table of a few of your favorite insulating solids, and tabulate $\varTheta_D$ and T^* when 1% paramagnetic impurities are present, assuming $J = \frac{5}{2}$.