Problem 1

\[ \psi(x) = \int a(k) e^{ikx} \, dk \]

\[ a(k) = A \quad \text{if} \quad k_0 - \Delta k < k < k_0 + \Delta k \]

\[ k_0 = 0.1 \text{ Å}^{-1}, \quad \Delta k = 0.02 \text{ Å}^{-1} \]

Uncertainty relations in wavepackets:

\[ \Delta x \Delta k \sim 1 \implies \Delta x = \frac{1}{\Delta k} = \frac{1}{0.02 \text{ Å}^{-1}} \]

\[ \implies \Delta x \approx 50 \text{ Å} \quad \text{(a)} \]

(b) \[ p = \hbar k, \quad \Delta p = \hbar \Delta k = \frac{\hbar c \Delta k}{c} = 19.73 \text{ eV Å} \times 0.02 \text{ Å}^{-1} \]

\[ \implies \Delta p \approx 39 \frac{\text{eV}}{c} \]

(c) The speed of the electron, assuming it's non-relativistic (check the letter) is

\[ \frac{U}{c} = \frac{\frac{\hbar k}{m_e}}{m_e c^2} \implies \frac{U}{c} = \frac{\frac{\hbar k_0}{m_e}}{m_e c^2} \]

\[ = \frac{19.73 \text{ eV Å} \times 0.1 \text{ Å}^{-1}}{51,000 \text{ eV}} = 3.9 \times 10^{-4} \approx 100 \text{ non-relativistic} \]

\[ \implies U = 3.9 \times 10^{-4} c = 116 \text{ km/s} \]

So after 1 s, the position of the electron \( x \approx 116 \text{ km} \)

(initially the electron is at \( x = 0 \))
Problem 2

Clearly, for \( n = 2 \) the electron is equally likely to be at \( x = 1 \) and at \( x = 3 \).

**I. Energy**

\[
E_n = \frac{\hbar^2 \pi^2}{2m L^2} \cdot 2^2 = 3.81 \text{ eV} \frac{\hbar^2 \pi^2}{8^2 \AA^2} \cdot 2^2 = 2.35 \text{ eV}
\]

(a) \( E = 2.35 \text{ eV}, \ n = 2 \)

(b) The wavefunction \( \Psi_n(x) = \sqrt{\frac{2}{L}} \sin \left( \frac{2\pi x}{L} \right) \)

\[
\frac{\mathcal{P}(2A)}{\mathcal{P}(1A)} = \frac{\Psi_n(2A)^2}{\Psi_n(1A)^2} = \frac{\sin^2 \frac{\pi}{2}}{\sin^2 \frac{\pi}{4}} = 2 = \text{twice as likely}
\]

(c) \[
\mathcal{P}(0.96 < x < 1.04) \approx |\Psi(x=1)|^2 \times (1.04 - 0.96) =
\]

\[
= |\Psi(x=1)|^2 \times 0.08 = \frac{2}{8} \times \sin^2 \frac{\pi}{2} \times 0.08 = \frac{1}{50} \times (\sqrt{2})^2 = \frac{1}{100}
\]

\[
\mathcal{P}(0.96 < x < 1.04) = 1\% = 0.01
\]

The classical value would be a uniform distribution:

\[
\mathcal{P}_{\text{cl}} = \Delta x \cdot \frac{1}{L} = \frac{0.08}{8} = 1\%
\]

\( \Rightarrow \) it is the same
Problem 3

\[ E_n = \frac{\hbar^2 \pi^2}{2mL^2} n^2 \]

\[ E_n = 1.504 \text{ eV} \cdot n^2 \]

For \( n = 5 \), \( E_5 = 37.6 \), for \( n = 6 \), \( E_6 = 54.15 \)

So in sure there are at least 5 states, and probably 6 states because the energy of the electron in the finite well is lower than in the \( \infty \) well, and the energy has to be \( < 50 \text{ eV} \).

(b) The wavefunction for \( x > 5 \text{ Å} \) is

\[ \Psi(x) \sim e^{-\alpha x}, \text{ with } \alpha = \sqrt{\frac{2m}{\hbar^2}} (U_0 - E) \]

for the lowest state. Assuming \( E_1 = \frac{\hbar^2 \pi^2}{2mL^2} \) = 1.504 eV

\[ \alpha = \sqrt{\frac{50 - 1.504}{3.81}} \text{ Å}^{-1} \Rightarrow \alpha = 3.57 \text{ Å}^{-1} \]

\[ \Psi(x) \sim e^{-\alpha x} = e^{-x/\delta}, \text{ electron penetrates a distance } \delta = \frac{1}{\alpha} = \frac{1}{3.57} \text{ Å}^{-1} = 0.28 \text{ Å} \]

(b)

(c) So the effective length of the well is

\[ L_eff = L + 2\delta = 5.56 \text{ Å} \], so the energy

\[ E'_1 = \frac{\hbar^2 \pi^2}{2m (L+2\delta)^2} = 1.22 \text{ eV} \]

So energy change by \[ E_1 - E'_1 \approx 0.29 \text{ eV} \]