Justify all your answers to all 3 problems. Write clearly.

Formulas:

Time dilation; Length contraction: \( \Delta t = \gamma \Delta t' = \gamma \Delta t \); \( L = L_p / \gamma \); \( c = 3 \times 10^8 \text{ m/s} \)

Lorentz transformation: \( x' = \gamma(x - vt) \); \( y' = y \); \( z' = z \); \( t' = \gamma(t - vx/c^2) \); inverse: \( v \rightarrow -v \)

Velocity transformation: \( u_x' = \frac{u_x - v}{1 - u_x v / c^2} \); \( u_y' = \frac{u_y}{\gamma(1 - u_x v / c^2)} \); inverse: \( v \rightarrow -v \)

Spacetime interval: \( (\Delta s)^2 = (c \Delta t)^2 - [\Delta x^2 + \Delta y^2 + \Delta z^2] \); \( \gamma = 1 / \sqrt{1 - v^2 / c^2} \)

Relativistic Doppler shift: \( f_{obs} = f_{source} \sqrt{1 + v/c} / \sqrt{1 - v/c} \)

Momentum: \( \vec{p} = \gamma m \vec{u} \); Energy: \( E = \gamma mc^2 \); Kinetic energy: \( K = (\gamma - 1)mc^2 \)

Rest energy: \( E_0 = mc^2 \); \( E = \sqrt{p^2 c^2 + m^2 c^4} \)

Electron: \( m_e = 0.511 \text{ MeV} / c^2 \); Proton: \( m_p = 938.27 \text{ MeV} / c^2 \); Neutron: \( m_n = 939.55 \text{ MeV} / c^2 \)

Atomic mass unit: \( 1 u = 931.5 \text{ MeV} / c^2 \); electron volt: \( 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J} \)

Stefan's law: \( e_{tot} = 0T^4 \); \( e_{tot} = \text{power/unit area} \); \( \sigma = 5.67 \times 10^{-8} \text{ W} / \text{m}^2 \text{K}^4 \)

\( e_{tot} = cU / 4 \), \( U = \text{energy density} = \int_0^\infty u(\lambda, T) d\lambda \) \( \text{Wien's law: } \lambda_m T = \frac{hc}{4.96 k_B} \)

Boltzmann distribution: \( P(E) = Ce^{E/(k_B T)} \)

Planck's law: \( u_\lambda(\lambda, T) = N_\lambda(\lambda) \times \overline{E}(\lambda, T) = \frac{8\pi}{\lambda^4} \times \frac{hc/\lambda}{e^{hc/\lambda k_B T} - 1} \); \( N(f) = \frac{8\pi f^2}{c^3} \)

Photons: \( E = hf = pc \); \( f = c / \lambda \); \( hc = 12,400 \text{ eV} \text{ Å} \); \( k_B = (1/11,600) \text{eV} / K \)

Photoelectric effect: \( eV_0 = K_{max} = hf - \phi \); \( \phi = \text{work function} \); \( 1 \)

Compton scattering: \( \lambda' - \lambda = \frac{h}{m_e c} (1 - \cos \theta) \); \( \frac{h}{m_e c} = 0.0243 \text{A} \); \( ke^2 = 14.4 \text{eV} \)

Coulomb force: \( F = \frac{k q_1 q_2}{r^2} \); Coulomb energy: \( U = \frac{k q_1 q_2}{r} \); Coulomb potential: \( V = \frac{k q}{r} \)

Force in electric and magnetic fields (Lorentz force): \( \vec{F} = q \vec{E} + q \vec{v} \times \vec{B} \)

Rutherford scattering: \( \Delta n(\theta) = \frac{C Z^2}{K^2 \sin^4 (\theta / 2)} \); \( b = \frac{k q_0 Q}{2K} \cot(\theta / 2) \)

Hydrogen: \( \frac{1}{\lambda_{nn}} = R \left( \frac{1}{m^2} - \frac{1}{n^2} \right) \); \( R = \frac{1}{911.8} \text{Å} \); \( \hbar c = 1973 \text{eV} \)

Bohr atom: \( E_n = -\frac{ke^2 Z}{2r_n} = -E_0 \frac{Z^2}{n^2} \); \( E_0 = \frac{ke^2}{2a_0} = \frac{m_e(k e^2)}{2\hbar^2} = 13.6 \text{eV} \); \( K = \frac{m_e v^2}{2} \); \( U = -\frac{ke^2 Z}{r} \)

\( hf = E_n - E_f \); \( r_n = r_0 n^2 \); \( r_0 = a_0 \frac{Z}{\lambda} \); \( a_0 = \frac{h^2}{m_e k e^2} = 0.529 \text{Å} \); \( L = m_e vr = nh \) \( \text{angular momentum} \)
**Problem 1** (10 points)
In a Compton scattering experiment, the incident photon has wavelength 0.5A and the scattered photon has wavelength 0.51215A.
(a) Find the angle at which the photon was scattered relative to the direction of incidence, in degrees.
(b) Find the kinetic energy of the scattered electron, in eV.
(c) Find the component of the electron momentum perpendicular to the direction of incidence, in eV/c.
(d) Find the component of the electron momentum parallel to the direction of incidence, in eV/c.
(e) Find the angle at which the electron was scattered relative to the direction of incidence, in degrees. Your answer should be accurate to at least one decimal point.

**Problem 2** (10 points)
In a Rutherford scattering experiment with α particles of kinetic energy 10MeV incident on a foil of zinc (Z=30), 1,777 α particles per minute are detected at angle 120°.
(a) How many α particles per minute do you expect will be detected at angles (i) 90° and (ii) 180°?
(b) Assume 800 α particles per minute are detected at angle 180°. What can you conclude about the radius of this nucleus?
(c) Assume you have at your disposal α particles of kinetic energy lower than 10MeV but not higher. Explain how you could get more information about the radius of this nucleus beyond what you learned from (b).

**Problem 3** (10 points)
In a hydrogen-like ion, the electron is in a stationary state where the radius of the orbit is 1.058A.
(a) What is the smallest possible value for Z (atomic number) for this ion? What is the next to smallest possible value for Z?
Assuming the smallest Z found in (a), answer (b) and (c):
(b) (i) What is the largest possible wavelength photon that this ion can absorb? (ii) What is the largest possible wavelength photon that this ion can emit? Give the answers in A.
(c) What is the magnitude of the linear momentum of the electron in this orbit, p? Give your answer in eV/c (equivalently, give the value of pc in eV).