Events in $S'$: $(x_1', t_1')$; $(x_2', t_2')$; simultaneous $\Rightarrow t_1' = t_2'$. 

Events in $S$: $(x_1, t_1)$; $(x_2, t_2)$; $\Delta t = t_2 - t_1 = 1\mu s$

$L$ is length of ship seen from ground = 600 m

In time $t_2 - t_1$, ship travels distance $d = v \cdot (t_2 - t_1) = v \Delta t$

\[ x_2 = x_1 + L + d = x_1 + L + v \Delta t \]

Lorentz:

\[ t_1' = \gamma \left( t_1 - \frac{v}{c^2} x_1 \right) \]
\[ t_2' = \gamma \left( t_2 - \frac{v}{c^2} x_2 \right) \]

\[ t_2 - \frac{v}{c^2} x_2 = t_1 - \frac{v}{c^2} x_1 \Rightarrow t_2 - t_1 = \Delta t = \frac{v}{c^2} (x_2 - x_1) \]

\[ \Rightarrow \Delta t = \frac{v}{c^2} \left( L + v \Delta t \right) = \frac{v}{c} \cdot \frac{L}{c} + \left( \frac{v}{c} \right)^2 \Delta t \]

\[ \Rightarrow \Delta t \left( \frac{v}{c} \right)^2 + \frac{L}{c} \left( \frac{v}{c} \right) - \Delta t = 0. \text{ Solve quadratic eq. in } \frac{v}{c} \]

\[ \frac{v}{c} = \frac{\sqrt{\left( \frac{L}{2c^2\Delta t} \right)^2 + 1}}{2} = -1 + \sqrt{2} \Rightarrow \]

\[ \frac{v}{c} = 0.414 \quad (a) \]

\[ \frac{L}{2c\Delta t} = \frac{600 \text{ m}}{2 \times 3 \times 10^8 \text{ m/s} \times 10^{-6} \text{s}} = 1 \]
(b) Length as seen on spaceship is proper length $L_p$:

$$L_p = \gamma L \quad \gamma = \frac{1}{\sqrt{1 - 0.414^2}}$$

$$\Rightarrow L_p = 659.2 \text{ m}$$

(c) First find new $L = L_p / \gamma$ with new $\gamma = \frac{1}{\sqrt{1 - 0.8^2}} = 1.155$

$$= L = 395.5 \text{ m}$$

Then, from eqs. above:

$$t_1' - t_2' = \gamma (t_1 - t_2 + \frac{\gamma}{c^2} (x_2 - x_1)) =$$

$$= \gamma (t_1 - t_2 + \frac{\gamma}{c^2} (L + \gamma \Delta t)) =$$

$$= \frac{5}{3} \left(-10^{-6} s + \frac{0.8}{3 \times 10^8} \left(395.5 + 0.8 \times 3 \times 10^8 \times 10^{-6}\right)s\right) =$$

$$= \frac{5}{3} \left(-10^{-6}s + 10^{-6}s \cdot \frac{0.8 (395.5 + 240)}{100} \right) = +1.158 \times 10^{-6}$$

$$= \frac{5}{3} \left(-10^{-6}s + 10^{-6}s \cdot \frac{0.8 (395.5 + 240)}{100} \right) = +1.158 \times 10^{-6}$$

$$\Rightarrow \text{in spaceship, chicken (back) happens 1.158 ms after egg (front)}$$

Alternative (easier) solution: use reverse Lorentz transformation:

$$t_1 = \gamma (t_1' + \frac{\gamma}{c^2} x_1') \Rightarrow t_1' = \frac{t_1 - \gamma}{\gamma} \frac{\gamma}{c^2} x_1'$$

$$t_2 = \gamma (t_2' + \frac{\gamma}{c^2} x_2')$$

$$\Rightarrow t_1' - t_2' = \frac{1}{\gamma} (t_1 - t_2) + \frac{\gamma}{c^2} (x_2' - x_1') =$$

$$= \frac{3}{5} (-10^{-6}s) + \frac{0.8 \times 659.2 \times 10^{-6}s}{300} = 1.158 \times 10^{-6}s$$

$$= \frac{3}{5} (-10^{-6}s) + \frac{0.8 \times 659.2 \times 10^{-6}s}{300} = 1.158 \times 10^{-6}s$$

$$\Rightarrow \text{in spaceship, back event happens 1.158 ms after front event}$$
Problem 2

\( v = 0.6c \) - light from candle

\[ A, \, B \]

(a) B lights candle after 1 year in her frame \( \Rightarrow 1 \) year of proper time

\( \Delta t_p = 1 \) year, \( \Delta t = \gamma \Delta t_p \)

\[ \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = 1.25 \]

\( \Rightarrow \Delta t = 1.25 \) years \( \Rightarrow \) A is 21.25 years old when B lights candle (a)

(b) The distance that \( B \) traveled, in \( A \)'s reference frame, is

\[ d = v \Delta t = 0.6c \cdot \Delta t \]

Light emitted from \( B \)'s candle travels back to \( A \) in a time

\[ \Delta t_2 = \frac{d}{c} = 0.6 \Delta t = 0.75 \) years. \( \]

Therefore, light reaches \( A \) at time \( \Delta t + \Delta t_2 = 2 \) years after \( B \) departs

\( \Rightarrow \) A is exactly 22 years old when light from \( B \) reaches \( A \) (b)

(c) The situation is completely symmetric (ignoring acceleration)

\( \Rightarrow \) B is exactly 22 years old when light from \( A \) reaches \( B \) (c)
Problem 3

(a) Find relative speed.
Put frame $S'$ on source, find speed of observer in frame $S'$

$V = \text{speed of frame } S'$, \hspace{1cm} $M_x = 2U$

$M'_x = \frac{M_x - U}{1 - \frac{M_x U}{c^2}} = \frac{2U - U}{1 - \frac{2U^2}{c^2}} = \frac{U}{1 - \frac{2U^2}{c^2}} = M'_x$

(2) $M'_x = 0.2857c$

(b) Use Doppler formula with speed $V = -M'_x$:

$f' = f \sqrt{\frac{1 - \frac{M'_x}{c}}{1 + \frac{M'_x}{c}}}$

$1 - \frac{M'_x}{c} = 1 - \frac{2U^2 - U}{c^2} \frac{c}{U}$

$= f \sqrt{\frac{1 - \frac{2U^2 - U}{c^2} \frac{c}{U}}{1 + \frac{2U^2 - U}{c^2} \frac{c}{U}}}$

$= f \sqrt{\frac{1 - 0.2857}{1 + 0.2857}} = 0.745f$

(b) Or, simply $f' = f \sqrt{\frac{1 - 0.2857}{1 + 0.2857}} = 0.745f$

(c) $f_g = f \sqrt{\frac{1 + \frac{U}{c}}{1 - \frac{U}{c}}}$

$= f \sqrt{\frac{1.25}{0.75}} = 1.29f$ since source is approximately

then $f'' = f_g \sqrt{\frac{1 - 2U}{1 + 2U}} = f_g \sqrt{\frac{1 - 0.5}{1 + 0.5}} = 1.29 \times 0.577f = 0.745f$

$f'' = 0.745f$ (c)

result has to be same as in (b) since person on ground emits same frequency as it receives from source by assumption.