Problem 1

Events in $S'$: $(x_1', t_1')$; $(x_2', t_2')$; simultaneous $\Rightarrow t_1' = t_2' = t'$

Events in $S$: $(x_1, t_1)$; $(x_2, t_2)$

Given: $t_1$ is $10^{-6}$ s earlier than $t_2 \Rightarrow t_2 - t_1 = 10^{-6}$ s
and $x_2 - x_1 = 600$ m

Using Lorentz:

$t_1' = \gamma (t_1 - \frac{V}{c^2} x_1) \Rightarrow t_2 - t_1 = \frac{V}{c^2} (x_2 - x_1) \Rightarrow$
$t_2' = \gamma (t_2 - \frac{V}{c^2} x_2)$

$\Rightarrow \frac{V}{c} = \frac{c (t_2 - t_1)}{x_2 - x_1} = \frac{3 \times 10^8 \times 10^{-6}}{600} = 0.5 \Rightarrow \boxed{\frac{V}{c} = 0.5}$ (a)

(b) Length as seen on spaceship is proper length $L_p$:

$L_p = \gamma L$, $L = 600$ m, $\gamma = \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}} = 1.155 \Rightarrow$

$\Rightarrow \boxed{L_p = 693}$ m (b)

(c) First find $L = \frac{L_p}{\gamma}$, now $\gamma = \frac{1}{\sqrt{1 - \frac{0.8^2}{c^2}}} = \frac{5}{3} = 1.667$

$\Rightarrow L = 415.8$ m

then from eqs. above, $t_1' - t_2' = \gamma (t_1 - t_2 + \frac{V}{c^2} (x_2 - x_1)) \Rightarrow$

$\Rightarrow t_1' - t_2' = \frac{5}{3} \left(-10^{-6} s + \frac{0.8 \times 415.8 \times 10^{-6}}{300} s\right) = \frac{5}{3} \times 0.088 \mu s = 0.18 \mu s$

$\Rightarrow$ in spaceship, chicken (bed event) happened $0.18 \mu s$ after egg (hen event)

(c)
(c) Alternative solution: use reverse Lorentz transformation and Lp

\[ t_1 = \gamma (t_1' + \frac{u}{c^2} x_1') \Rightarrow \quad t_1 - t_2 = \gamma (t_1' - t_2') + \frac{u}{c^2} (x_1' - x_2') \Rightarrow \]

\[ t_2 = \gamma (t_2' + \frac{u}{c^2} x_2') \]

\[ \Rightarrow \quad t_1' - t_2' = \frac{1}{\gamma} (t_1 - t_2) + \frac{u}{c^2} (x_2' - x_1') \Rightarrow \]

\[ t_1' - t_2' = \frac{5}{3} (-10^{-6}s) + \frac{0.8 \times 693 \times 10^{-6}s}{300} = 0.18 \times 10^{-6}s \]

\[ = 1 \text{ microsecond} \]

chicken happened 0.18 µs after egg happened as seen from space ship
Problem 2

A, B

light from candle

(a) B lights candle after 1 year in her frame \( \Rightarrow \) 1 year \( \Rightarrow \) proper time

\[ \Delta t_p = 1 \text{ year}, \quad \Delta t = \gamma \Delta t_p \quad . \quad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = 1.25 \]

\( \Rightarrow \) \( \Delta t = 1.25 \text{ years} \Rightarrow \) A is 21.25 years old when B lights candle \( \text{(a)} \)

(b) The distance that B traveled, in A's reference frame, is

\[ d = v \Delta t = 0.6c \cdot \Delta t \]

Light emitted from B's candle travels back to A in a time

\[ \Delta t_2 = \frac{d}{c} = 0.6 \Delta t = 0.75 \text{ years}. \]

Therefore, light reaches A at time \( \Delta t + \Delta t_2 = 2 \text{ years} \) after B departed

\( \Rightarrow \) A is exactly 22 years old when light from B reaches A \( \text{(b)} \)

(c) The situation is completely symmetric (ignoring acceleration)

\( \Rightarrow \) B is exactly 22 years old when light from A reaches B \( \text{(c)} \)
Problem 3

(a) Find relative speed.
Put frame $S'$ on source, find speed of observer in frame $S'$
$U = \text{speed of frame } S'$, $M_x = 2U$

$$M'_x = \frac{M_x - U}{1 - \frac{M_x U}{c^2}} = \frac{2U - U}{1 - \frac{2U^2}{c^2}} = \frac{U}{1 - \frac{2U^2}{c^2}} = M'_x$$

$\text{Case 1: } M'_x = 0.2857c$

(b) Use Doppler formula with speed $U = -M'_x$

$$f' = f \sqrt{\frac{1 - \frac{M'_x}{c}}{1 + \frac{M'_x}{c}}}$$

$$\frac{1 - \frac{M'_x}{c}}{1 + \frac{M'_x}{c}} = \frac{1 - \frac{2U^2}{c^2} - \frac{U}{c}}{1 - \frac{2U^2}{c^2} + \frac{U}{c}}$$

$$f' = f \sqrt{\frac{1 - \frac{2U^2}{c^2} - \frac{U}{c}}{1 - \frac{2U^2}{c^2} + \frac{U}{c}}} = f \sqrt{\frac{1 - 2 \times 0.25^2 - 0.25}{1 - 2 \times 0.25^2 + 0.25}} = 0.745 f$$

$\boxed{f' = 0.745 f}$

(b) Or, simply

$$f' = f \sqrt{1 - \frac{0.2857}{1 + 0.2857}} = 0.745 f$$

(c) $f_g = f \sqrt{\frac{1 + \frac{U}{c}}{1 - \frac{U}{c}}} = f \sqrt{\frac{1.25}{0.75}} = 1.29 f$ since source is approaching

then $f'' = f_g \sqrt{\frac{1 - \frac{2U}{c}}{1 + \frac{2U}{c}}} = f_g \sqrt{\frac{1 - 0.5}{1 + 0.5}} = 1.29 \times 0.577 f = 0.745 f$

$\boxed{f'' = 0.745 f}$

result has to be same as in (b) since person on ground emits same frequency as it receives from source by assumption.