Lecture 2c - More on Boltzmann Eqn.

Recap:

- Derived Boltzmann Equation from BGK hierarchy
  
  \[ n \partial_t^3 \Phi \ll 1 \text{ to truncate at } F(1,2) \]
  
  \[ \Rightarrow F(1,2) = F(1) F(2) \text{ to complete} \]

- \( C(\Phi) = \int dp \int dp' \int dp'' \sum_{j, \alpha, k, A} \text{WC}(A, p, p', p'') (\Phi \Phi') (\Phi \Phi') - (\Phi \Phi') (\Phi \Phi') \)

- Proved H-Theorem using Lemma 7 and Stosszahlansatz

\[ \Sigma = \int dx \int dp \Phi \partial_t \Phi \]
\[ \frac{dS}{dt} = - \int \! \! \int \! \! \int \! \! \int c(\xi) \, d\xi \]

\[ \frac{dS}{dt} \geq 0. \]

Microscopically reversibility from microscopically reversible dynamics, \( W = \omega T \)

- When \( \frac{dS}{dt} = 0 \) \( \Rightarrow \) \( F \propto \frac{s}{T} \)

\[ c(\xi) = 0 \]

\( \Rightarrow \) identifies equilibrium distribution

Recall: \( \frac{dS}{dt} = \frac{3}{2} \int \! \! \int \! \! \int \! \! \int w(\xi, \eta) x \ln x \)

\[ x = \frac{f_1 f_2 / f'_1}{f_1} \]

\[ \frac{dS}{dt} = 0 \quad \text{at} \quad x = 1 \]

\[ f f'_1 = f'_1 f_1 \]
\[ \ln F + \ln F_i = \ln F' + \ln F_i' \]

\[ \Rightarrow \ln F + \ln F_i \text{ must be conserved in collision event} \]

\[ \ln F = c + x \cdot \mathbf{F} + \theta \mathbf{E} \]

\( \theta \) energy

\( \theta \) momentum

Now, \( F \) normalizable:

\[ \ln F = c + x \cdot \mathbf{F} - \theta \mathbf{E} \]

\( \theta > 0 \)

And \( \int d\mathbf{x} \) irrelevant, i.e. can specify \( C(f_{2\mathbf{x}}) = \theta \Rightarrow F_{2\mathbf{x}} = f_{2\mathbf{x}}(x) \), each \( x \)

\[ C = C(x) \Rightarrow \wedge C(x) \]

\[ \exists x = x(x) \Rightarrow \vee C(x) \]

\[ \exists x = 2 \mathbf{x} \Rightarrow \uparrow C(x) \]
\[
\text{normalization}
\]

\[
F_{eq} = \frac{c_{\text{max}}}{1} \exp \left[ \left( \mu - E \right) / T(x) \right]
\]

i.e. shifted Maxwellian, as expected!

- Local vs. Global?

\[ \int dy \text{ irrelevant} \]

i.e. \[ C(F_{eq}) = 0 \] at each \( x \)

Entropy produced \( \text{locally} \), on

\[ V \text{ time scale } (W \sim TV_{\text{rel}}) \]

\[ f_S = \frac{\mu_0}{h_m} \quad h_m = \frac{1}{\sqrt{n T}} \]

\[ \text{But : What of inhomogeneity in } \]

thermodynamic parameters?

\[ T(x) \]

\[ \Rightarrow \] Q \[ \Rightarrow \] \[ T(x) \]

The point:
\( f_{eq} = f_{eq}(T(x), V(x), \lambda(x)) \) does not satisfy Boltzmann Equation,

\[ \frac{\partial}{\partial t} f + \mathbf{v} \cdot \nabla f = C(f) \]

\[ \frac{\partial f_{eq}}{\partial t} + \mathbf{v} \cdot \nabla f_{eq} = C(f_{eq}) \rightarrow 0 \]

\[ \mathbf{v} \cdot \nabla f_{eq} = 0 \]

\[ \frac{\partial f}{\partial t} + \mathbf{F} \cdot \nabla f \rightarrow 0 (\text{large} / L) \]

Rate: \( r \left( \frac{\text{large}}{L} \right)^2 \int \text{induced collisional fluxes, etc.} \)

- How reconcile:

  - reversible Hamiltonian dynamics

  - \( \frac{dS}{dt} \geq 0 \) macro irreversibility
→ Coarse Graining ! → Partition

Recall Lyapunov Exponents

→ partition

ΔZ Δρ - Partition cell

Set resolution scale

S is integrated quantity

Why significant?

→ partition kills small details in phase volume evolution.

c.e

\[ f_0 \]

\[ A \]

\[ f_0 \]

\[ A \]

via Hamiltonian evolution

Exact
2 Time scales

with coarse graining:

\[ \rho A_0 = \text{Acc } \bar{f} \]
\[ \bar{f} = \frac{\rho A_0}{\text{Acc}} \]

coarse grained phase space density

A.R. Prediction of

close recurrence impossible as
partition sets resolution limit
What Next?

→ We have the Boltzmann Equation and H-Theorem. (Yay!)

→ What do we do with them?

N.B.: "The solution of the above equation [B.E.] as we will see shortly, is truly a gruesome task."

- Stewart Harris, in "An Introduction to the Theory of the Boltzmann Equation."

So?

Hint: Consider time scales...