Problem Set 1: Due 1/23

1) Complete the calculation of the induced mass of potential flow around a sphere, which was begun and discussed in class. In particular, show the energy of potential flow is

\[ E = \rho \left[ 4\pi (A \cdot u) - V_0 \frac{u^2}{2} \right] = m_{ik} \frac{u_i u_k}{2} , \]

where \( A \) is the dipole moment of the flow and \( V_0 \) is the volume of the body in motion at \( u \). Compute \( m_{ik} \), the induced mass tensor. What is its value for a sphere?

2) Consider a small body immersed in a fluid flow which oscillates. Derive the general relation between the velocity of the body and that of the fluid. What is the result for a spherical body of density \( \rho_0 \)?

3) Derive the energy relation

\[ \frac{\partial}{\partial t} \left( \frac{\rho v^2}{2} + \rho \epsilon \right) = -\nabla \cdot \left( \rho v \left( \frac{v^2}{2} + \omega \right) \right) \]

from the continuity, Euler and energy equations. Here, \( \omega \) is the enthalpy density.

4) a) Derive the dispersion relation for an azimuthally symmetric wave propagating along the \( \hat{z} \) axis and in radius in an ideal incompressible, unbounded fluid rotating at \( \Omega = \Omega_0 \hat{z} \).

b) Now assume the fluid is bounded by a cylindrical wall at \( r = R \). What is the profile of radial velocity?