Physics 235

Notes 4

→ To $k \nu > 1$.

→ Recall had been concerned with transport and diffusion.

Focus: $O_m = \int d\epsilon \sum |d\beta_{\epsilon}|^2 e^{i \eta_{\epsilon k} \rho}$

→

$\sim \langle (\mathcal{O})^2 \rangle \approx k \nu$

Scattering: $k \nu = 0 \text{ means neutral}$

→ $k \nu \sim \frac{\langle \mathcal{O} \rangle d\beta_{\epsilon}}{\Delta \beta_{\epsilon}} \sim \frac{1}{2} \frac{d\beta_{\epsilon}}{\beta_{\epsilon}} \left[ \frac{d\beta_{\epsilon}}{\beta_{\epsilon}} \right]$
What happens for $Kn > 1$?

Recall:

Stochastic fields:

$$Kn \sim \frac{\Delta B/B}{\Delta Kn} \sim \frac{1}{\Delta Kn} \frac{\Delta B/B}{\Delta Kn}$$

~ Large ~

Ratio of auto-correlation to NL mixing length

~ Flow ~

$$Kn \sim \frac{U/\Delta}{\tau/\Delta} \sim \frac{\tau u/\tau}{\tau u/\tau} \sim \frac{\tau u/\tau}{\tau u/\tau} \sim \frac{\tau u/\tau}{\tau u/\tau}$$

Collisional NLF:

$$P_{ac} \sim \left( \frac{\Delta Kn}{\Delta Kn} \right)^{-1}$$

$$Kn \sim \frac{1}{\tau} (\Delta Kn^2)$$
2D GC Plasma - Simple/Compelling Example.

\( D_i \sim \sqrt{\frac{\tau}{\omega \Gamma_0 V_0}} \) 

\[ R(\tau) = e^{-c_0 (\omega - i \nu \tau)^2} = \frac{c}{1 + \frac{c}{2}} \quad \text{from upa} \]

\( \nu \gg 1 \) limit corresponds to:

\[ \rightarrow \nu \rightarrow 0 \quad \Rightarrow \quad \rightarrow \omega \rightarrow \infty \quad \Rightarrow \quad \text{time integral controlled by nonlinear scattering, not wave packet dispersion} \]

\[ \Rightarrow 2D GC Plasma/Fluid \]

\[ \frac{\partial \phi}{\partial t} + \nabla \times \nabla \cdot \nabla \phi = 0 \quad \text{(Taylor i.e. \( \text{Mac Name} \))} \]

\[ D_\phi = -4 \pi \phi \]

\[ \rightarrow 2D \text{ Fluid} \]

\[ \rightarrow GC \text{ Plasma} \]
Then generally:

$$A_{1} = \int d\tau \sum_{\text{P}} \left< \hat{\mathcal{P}} \right| e^{i \hat{H} \tau} \left| \text{P} \right> \left< \text{P} \right| e^{-i \hat{H} \tau}$$

but $$e^{-i \hat{H} \tau} = e^{i \hat{N} \tau}$$

$$\Rightarrow \text{ only evolution off of } \tau$$

$$\Rightarrow \text{ stochastic}$$

$$\Rightarrow \text{ need ensemble average}$$

$$A_{1} = \int d\tau \sum_{\text{P}} \left< \hat{\mathcal{P}} \right| e^{i \hat{H} \tau} \left| \text{P} \right>$$

$$\Rightarrow \int d\tau \sum_{\text{P}} \left< \hat{\mathcal{P}} \right| e^{-i \hat{H} \tau}$$

$$\Rightarrow \int d\tau \sum_{\text{P}} \left< \hat{\mathcal{P}} \right| e^{-i \hat{H} \tau}$$

$$(\text{turbulence itself controls cancel. time.})$$

$$\left< e^{i \hat{N} \tau} \right> = \left< (1 + \mathcal{N} \cdot \mathcal{N} \cdot \tau - \frac{\mathcal{N} \cdot \mathcal{N} \tau}{2}) \right>$$

$$= \left< 1 - \frac{\mathcal{N} \cdot \mathcal{N} \tau}{2} \right>$$

$$= \left< (1 - \mathcal{N} \cdot \mathcal{N} \tau) \right>$$

$$= e^{-\mathcal{N} \cdot \mathcal{N} \tau}$$
\[ Q = \int d\mathbf{p} \sum_{n} \left| \psi_n \right|^2 e^{-\frac{\mathbf{p}^2}{2m}} \]

\[ = \sum_{n} \left( \frac{\mathbf{p}_n^2}{m} \right)^{-\frac{1}{2}} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{1}{\left( \frac{\mathbf{p}^2}{m} \right)^{-\frac{1}{2}}} \]

Compare to dispersion!

**N.B.** Note recursive structure of laser rate!

\[ T_e \text{ in cutoff set by scattering} \quad \propto \sqrt{v_T D} \quad \text{From conservation} \quad \frac{v}{c} \]

\[ T_e \propto v \sim \frac{v}{c} \text{ Expt} \Rightarrow T_e \sim \frac{1}{B_0} \Rightarrow \text{Boltzmann} \]

\[ (Q_1)^2 = \sum \frac{\Delta n_p^2}{\Delta p^2} \quad \text{recursive definition} \]

But 2D assuming symmetric spectrum:

\[ Q^2 = \int d\mathbf{p} \Delta n_p^2 \frac{\Delta \mathbf{p}^2}{\mathbf{p}^2} \]
\[ D_l \sim \frac{c}{B} \left( \sum_{m=1}^{\infty} \frac{LE_m}{k_B T} \right)^{1/2} \]

Now, can explore different aspects:

- Thermal equilibrium:
  \[ LE_m = \frac{4\pi}{3} \frac{k_B T}{(1 + k_B T)^2} \]
  \[ \frac{g}{f} = \text{charge/length} \]

\[ \sim \text{Debye screening} \]

\[ \sim \frac{4\pi (e)}{e} \frac{k_B T}{(1 + k_B T)^2} \]

\[ D_l \sim \frac{c}{B} \left( \sum_{m=1}^{\infty} \frac{k_B T}{(1 + k_B T)^2} \right)^{1/2} \]

\[ \sim \frac{c k_B T}{eB} \left[ (m\lambda)^{-1} \ln (m\lambda) \right]^{1/2} \]
\[ n \sim D_B \left[ (A_B^2)^{-1} \ln (L_0/L) \right]^{3/2} \]

- recover basic Bohm scaling, even from thermal fields
- scaler weakly with \( L_0 \) \( \rightarrow \) not localized or intensive

\( \Rightarrow \) simple example of "non-locality"

- non-locality "appears from slow mode" i.e. \( \sqrt{\nu} \rightarrow \sim k_\perp^2 D_1 \) as \( \nu \rightarrow 0 \)

\( \rho \) is conserved \( \Rightarrow \) conserved order parameter \( W \sim k_\perp^2 D_1 \)

- if shear flow:

\[ RCH = \int dr \exp \left[ \frac{1}{2} \left( W - k_\perp D_1 \right) \right] \]

Interesting to note:

- can consider diffusion due to random array charges (intrinsic)
For spectrum:

\[ \frac{\partial E}{\partial \phi} = 4\pi \rho \]

\[ = 4\pi \sum \frac{\varepsilon_i}{\varepsilon} \sigma(r_i - x_i) \]

\[ c \cdot k_x \cdot E_y = \left( \frac{4\pi}{\varepsilon} \right) \sum z_i \cdot e^{-i k_x \cdot x_i} \]

Symmetry distribution \( \sum \text{random axis} \quad \text{correlated changes} \)

\[ |E_{n1}|^2 = \frac{1}{4\pi} \left( \frac{4\pi}{\varepsilon} \right)^2 \left\langle \sum_{j} \varepsilon_i z \cdot z_j \cdot e^{i k \cdot (x_j - x_i)} \right\rangle \]

\[ = \frac{16 \pi^2 \varepsilon_0^2}{k^2 \rho^2} \]

\[ \Rightarrow \quad 1 \cdot k_x^2 \]

\[ \Omega_{12} \sim \frac{\varepsilon_0^2}{k_2} \sum k_{11} \cdot \frac{16 \pi^2 \varepsilon_2^2}{k_1^2 \rho^2} \]

\[ \sim c^2 \frac{k_{\text{max}}}{k_2} \cdot \frac{9}{k_1^3 \rho} \cdot \frac{k_{\text{min}}}{k_2} \]

\[ \sim \frac{c^2}{k_1^3 \rho} \left( \frac{16 \pi^2 \varepsilon_2^2}{k_1^2 \rho^2} \right) \cdot \frac{1}{k_2} \]
\[ L_{\text{min}} \sim \frac{1}{b} \]

system size

\[ D_L = C \frac{4 \pi (1/2)^{1/2}}{B_0} \frac{1}{\sqrt{X}} \]

system size dependence on system size
Stochastic Fields - Toward High kHz Random Conductivity

Cont'd

- so far:
- reviewed theory of Hamiltonian chaos
- derived @ L Dm
- derived \( \lambda_e \) due stochastic fields in \( ku < 1 \) regime - diffusion
- focused on interaction of scattering (Heavis)
collisions, coarse graining
- discussed transport of GC plasma, as
example of \( T_a \to \infty \) regime.

**Observations**

- idea of resonance (small denominator problem)
  and resonance overlap fundamental to
  Hamiltonian chaos.
- \( ku \sim \frac{T_a}{T_{scatt}} \).
- might ask: unified treatment that
  combines \( ku < 1 \), \( ku > 1 \) regimes
  \( \Rightarrow \) renormalized response?
- on hydro treatment of \( \lambda_a \) what
  of nominal 3rd order contribution
  \( \lambda_a \), etc. See (4+1)!
- ir diffusive treatment of high kHz
  regime (as in Taylor + McNamar) invalid?
  See (4+1) Ribberng, Rubbo, papers.
- \( T_a \to \infty \) can recover strong 01 at modest kHz level.
Here:
- general analysis of diffusion
- aspects of percolation, large hy
- regime
- Dykhne method → conduction in random media.

⇒ Recall, \( k_{\text{in}} \sim \frac{d^2 B}{Q A} \)

- have considered how \( k_{\text{in}} \) with
  - Finite len
  - inhomogeneity in \( Z \)

⇒ now consider \( k_{\text{in}} \rightarrow \text{limit, appropriate} \)
  - random field, \( x \), \( y \)
  - homogeneous on \( Z \)

⇒ i.e., akin rods
\[ \frac{dx}{dz} = b \rho = \frac{\partial \phi}{\partial y} \]

\[ \frac{dy}{dz} = b \omega = -\frac{\partial \phi}{\partial x} \]

From:
\[ \frac{dt}{dz} = b \rho \]
\[ \Rightarrow \quad \dot{\rho} + \dot{\omega} = \frac{\partial \phi}{\partial z} + b \Theta \]
\[ \frac{\partial}{\partial z} = \frac{1}{T} \]

Equivalent of (Lebowitz) to F.C. Plasmas:
\[ \frac{dx}{dt} = -\frac{c}{B} \dot{y} \phi \]
\[ \frac{dy}{dt} = \frac{c}{B} \dot{x} \phi \]

(2) Motivates study of random media transport.
Formally can extend $D_m$ calculation to include resonance broadening

$$\frac{\partial}{\partial z} \mathcal{F} + b \cdot \nabla \mathcal{F} = -b \cdot \frac{\partial \mathcal{F}}{\partial t}$$

on localized model

$$D_m = \sum_{\mu, \nu} \left[ \left\langle \frac{\mu}{\nu} \right\rangle \frac{\epsilon}{\hbar \nu + i \hbar \nu^2 D_m} \right]$$

where

$$\frac{\hbar \nu^2 D_m}{\hbar \nu} \sim \hbar \nu$$

For $\hbar \nu < 1 

$$D_m = \sum_{\mu} \left\langle \left\langle \frac{\mu}{\nu} \right\rangle \right\rangle$$

For $\hbar \nu > 1$

$$D_m = \sum_{\mu} \left\langle \frac{\hbar \nu \mu}{\hbar \nu^2 D_m} \right\rangle$$

algorithm Taylor, McNamee.
\[ D_m = \left( \frac{\sum |A_m|^2}{h} \right)^{1/2} \sim b \Delta \]

\[ k u < 1 \Rightarrow D_m \sim b^2 \Delta \]
\[ k u > 1 \Rightarrow D_m \sim b \Delta \]

and transport \( \sim < A^2 >^{1/2} \)

\[ \text{But if } k u > 1 \text{ region really diffusive} \]

\[ \Rightarrow \text{recall: } \frac{dx}{dz} = 1A \times z \]

\[ \text{Taking 2D random media, for } \]
\[ \text{A index } z \]

\[ \Rightarrow \text{can view physically as: topographical map} \]
\[
\text{Map}
\]

(what with ambient diff?)

\[\text{lane} \quad \text{hills} \quad \text{levels}\]

Now, as \( \frac{dx}{dy} = \frac{-dy}{-dx} = \frac{d}{dx} \]

\[dy/dx = -dA/y dA\]

\[\text{DA} \cdot \text{dx} = 0\]

- Lines traverse constant \( A \) contours as on map

- \( \langle A \rangle = 0 \), \( \langle A^2 \rangle = A_0^2 \)

- Approximate height of "lakes" "hills" set by \( A_0 \)
most contours closed, isolated
=> little contribution to transport
but contours along "passer".

(i.e. 3) can take on long
path lengths.

Transport occurs primarily along
there.

=> parafraction, not diffusion

hot

\( a \)

\( b \)

cold

\( a + b \) transport
isolated along
contour C.

~ more like "lightning bolt" thin
diffusion. Heat channeled along C.

~ signature would be sharply
localized stroke mark (if \( 10 \rightarrow P \rightarrow \ell \))
and not periodic or (combs).

\( \infty \) = spread

strike

\( \text{spread} \)

\( 10 = \phi \)
- percolation
- extension of mean length as \( A \to 0 \)

\[ l \sim A^{-x} \]

Message:
- replaced concept of m.f.p.
- to understand \( hu > l \) regime, useful to examine
- transport in random media
- percolation theory