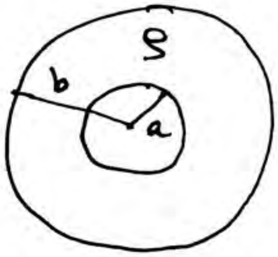
Physics 4C | Quiz 2 Solutions | Fall 2019

Problem 1



(a) Use $\phi(\vec{r}=0) = \frac{1}{4\pi\varepsilon_0}\int d^3r' \frac{\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}$. Since $\rho = \text{const}$,

$$\phi(0) = \frac{1}{4\pi\varepsilon_0}\rho \int_{\text{shell}} d^3r' \frac{1}{r'} = \frac{\rho}{4\pi\varepsilon_0}\cdot 4\pi \int_a^b dr' \frac{r'^2}{r'} \Rightarrow$$

$$\boxed{\phi(0) = \frac{\rho}{\varepsilon_0}\left.\frac{1}{2}r'^2\right]_a^b = \frac{\rho}{2\varepsilon_0}(b^2-a^2)}$$

Alternative solution to (a):

$$\phi(\infty) - \phi(0) = -\int_0^\infty \vec{E}\cdot d\vec{s} \Rightarrow \phi(0) = \int_0^\infty E(r)\,dr$$

(i) $0<r<a$: $E=0$. For $r>a$, use Gauss' law: $\oint \vec{E}\cdot d\vec{a} = \frac{q_{enc}}{\varepsilon_0}$

(ii) $a<r<b$: $E\cdot 4\pi r^2 = \frac{4\pi}{3\varepsilon_0}\rho(r^3-a^3) \Rightarrow \boxed{E = \frac{\rho}{3\varepsilon_0}\frac{r^3-a^3}{r^2}}$

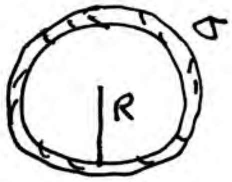
(iii) $r>b$: $\boxed{E = \frac{\rho}{3\varepsilon_0}\frac{b^3-a^3}{r^2} = \frac{Q}{4\pi\varepsilon_0 r^2}}$; $Q = \frac{4}{3}\pi(b^3-a^3)$ $\Rightarrow$

$$\phi(0) = \frac{\rho}{3\varepsilon_0}\left[\int_a^b dr\left(r - \frac{a^3}{r^2}\right) + \int_b^\infty dr\,\frac{b^3-a^3}{r^2}\right] =$$

$$= \frac{\rho}{3\varepsilon_0}\left[\frac{b^2}{2} - \frac{a^2}{2} + \cancel{\frac{a^3}{b}} - \frac{a^3}{a} + \frac{b^3}{b} - \cancel{\frac{a^3}{b}}\right] = \frac{\rho}{3\varepsilon_0}\left[\frac{3}{2}b^2 - \frac{3}{2}a^2\right] \Rightarrow$$

$$\Rightarrow \boxed{\phi(0) = \frac{\rho}{2\varepsilon_0}(b^2-a^2)}$$

Prob. 1 (b)



total charge is $Q = 4\pi R^2 \sigma$. Potential is same as for a point charge.

$$\boxed{\phi(R) = \frac{Q}{4\pi \varepsilon_0 R} = \frac{R\sigma}{\varepsilon_0}}$$

At the center of the spherical shell we have $\phi(0) = \phi(R)$ since the electric field is zero for $r < R$

(c) We had for (a) that the electric potential at the center of the shell is

$$\phi(0) = \frac{\rho}{2\varepsilon_0}(b^2 - a^2) = \frac{\rho}{2\varepsilon_0}(b-a)(b+a)$$

The thickness of the shell is $(b-a)$, so we have $\rho(b-a) = \sigma$, where σ is the surface charge density of the shell when $(b-a)$ is small.

$$\Rightarrow \boxed{\phi(0) = \frac{\sigma}{\varepsilon_0}\,\frac{b+a}{2}}$$

now $\frac{b+a}{2}$ is the average radius R. So this is the same as the result found in (b).

The potential at the center is the same as the potential at the radius of the spherical shell because the electric field is zero in the region $r < R$, or $r < a$.

Problem 2

$$\phi = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r-a} + \frac{1}{r} - \frac{2}{r+a}\right)$$

Using $\frac{1}{r-a} = \frac{1}{r}\left(1+\frac{a}{r}\right)$, etc

$$\phi = \frac{q}{4\pi\varepsilon_0 r}\left(1+\frac{a}{r}+1-2\left(1-\frac{a}{r}\right)\right) = \frac{q}{4\pi\varepsilon_0}\cdot\frac{3a}{r^2}$$

$$\Rightarrow \boxed{\phi = \frac{3qa}{4\pi\varepsilon_0 r^2}}$$

(b) The electric field at P clearly points along the line, to the right.

$$\boxed{E = -\frac{\partial\phi}{\partial r} = \frac{3qa}{2\pi\varepsilon_0 r^3}}$$

(c) From comparing with formula for electric potential of dipole,

$$\boxed{p = 3qa}$$

Problem 3

The total charge in the solid metallic tube is

$$Q = \lambda \cdot h$$

The total charge in the inner surface of the thin metallic shell has to be $-Q$, so that the field inside it is 0. So

$$-Q = 2\pi b h \sigma = -\lambda h \Rightarrow \boxed{\sigma = \frac{-\lambda}{2\pi b}}$$

(b) The electric field at the surface of a conductor is

$$E = \frac{\sigma}{\varepsilon_0} \Rightarrow \boxed{E = -\frac{\lambda}{2\pi\varepsilon_0 b}}$$ points towards the surface

(c) Difference in potential: electric field is

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$\phi(b) - \phi(a) = -\int_a^b E\,dr = -\frac{\lambda}{2\pi\varepsilon_0} \ln\frac{b}{a}$$

so $\boxed{\phi(b) - \phi(a) = -\frac{\lambda}{2\pi\varepsilon_0} \ln\frac{b}{a}}$

(d) Capacitance is defined by $Q = C\phi$, with ϕ potential difference.

$$\lambda = \frac{Q}{h} \Rightarrow \phi = \frac{Q}{2\pi\varepsilon_0 h} \ln\frac{b}{a} \Rightarrow$$

$$\boxed{C = \frac{2\pi\varepsilon_0 h}{\ln(b/a)}}$$