Formulas:

$F = k\frac{q_1 q_2}{r^2}$ Coulomb's law ; $k = \frac{1}{4\pi\varepsilon_0} = 8.99\times10^9 \mathrm{N\cdot m^2/C^2}$; $\varepsilon_0 = 8.85\times10^{-12} C^2/Nm^2$

Electric field: $\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r} = \frac{kq}{r^2}\hat{r}$; $\vec{E}(\vec{r}) = k\int \rho(\vec{r}\,')\frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|^3}d^3r'$; $\vec{F} = q_0\vec{E}$

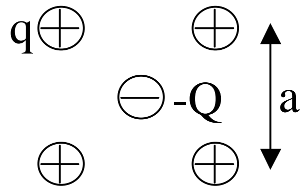
Linear, surface, volume charge density: $dq = \lambda\, d\ell$, $dq = \sigma\, da$, $dq = \rho\, dv$

Electric field of infinite: line of charge: $E = \frac{\lambda}{2\pi\varepsilon_0 r}$; sheet of charge: $E = \frac{\sigma}{2\varepsilon_0}$

Gauss law: $\Phi = \oint \vec{E}\cdot d\vec{a} = \frac{q_{enc}}{\varepsilon_0} = \frac{1}{\varepsilon_0}\int d^3r\, \rho(\vec{r})$; Φ=electric flux

Energy: $U = k\frac{q_1 q_2}{r_{12}}$ $\qquad U = \frac{\varepsilon_0}{2}\int E(\vec{r})^2 d^3r$

Problem 1 (10 pts)



The four positive charges q (q>0) are arranged at the corners of a square of side length a. The negative charge -Q (Q>0) is at the center of the square.

(a) Find the value of Q/q that makes the net force on the positive charges zero.
For the next points (b) and (c), assume Q has the value found in (a).

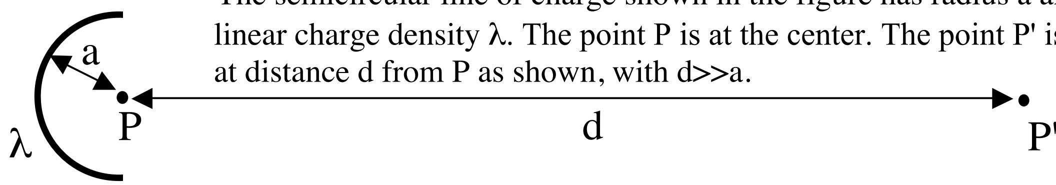
(b) Find how much work you have to do to bring the charge -Q from where it is to infinity. Give your answer in terms of q, a, k (Coulomb's constant), not in terms of Q. State whether the work you have to do is positive or negative.

(c) Find the total potential energy of this system of charges. Give your answer in terms of q, a, k (Coulomb's constant), not in terms of Q.

Hint: there two ways to get the answer to (c), either is ok, if you justify your answer clearly. Extra credit if you do it both ways and get the same (correct) answer.

Problem 2 (10 pts)

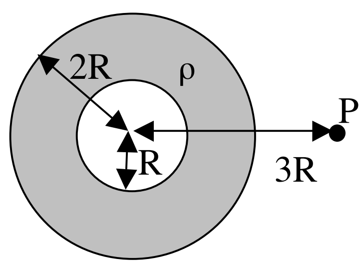
The semicircular line of charge shown in the figure has radius a and linear charge density λ. The point P is at the center. The point P' is at distance d from P as shown, with d>>a.



(a) Find the magnitude of the electric field at P, in terms of λ, a and ε_0.

(b) Find an approximate expression for the magnitude of the electric field at P', in terms of λ, a, d and ε_0.

Problem 3 (10 pts)



The spherical shell shown in the figure has inner radius R, outer radius 2R and uniform charge density ρ. The electric field produced by this shell at point P, that is at distance 3R from the center, has magnitude 20N/C.

(a) Find the magnitude of the electric field at the outer surface of the shell, in N/C.

(b) Find the magnitude of the electric field at distance 1.5R from the center, in N/C.

(c) Find the potential energy of this shell by building it up layer by layer of charge dq, using the fact that outside a spherical charge distribution of total charge q, the field is the same as if all the charge q were at the center. Give your answer in terms of ρ, ε_0 and R.