Turbulent Wakes, Thermal Boundary Layers

Here:

- turbulent wakes, completed wake story
- behind
- scaling
- eddy mixing

- Thermal BL / Heat Transfer

- behind, set up, types

- heat transfer problem

- heat transfer coeff
- Nu
- laminar, turbulent

- intro to temp fluctuation turbulence - passive scalar.
References: Boundary layers, wakes, heat transfer

Landau & Lifshitz: excellent, 'physicist style' treatment of these 'engineering' subjects

V. Ince: Good summary, many examples

H. Tennekes, J. Lumley: Basic discussion, good first course.

S. B. Pope: classic Engineering text, detailed analysis.
B.) Wakes - Simple Physics

Wake is:
- region of departure from potential flow behind object moving through water and experiencing drag.

- Wake is inextricably coupled to drag.

Message of wakes:
A little *forces a global adjustment of flow structure.*

- drag - thinking on DONE where object at rest, drag results from loss of flow momentum to object.
Take wake turbulence velocity given by advection not differentiable, not isotropic. 

W = \frac{v}{\lambda} \frac{\partial}{\partial x} \lambda \frac{\partial}{\partial y} 

\text{Turbulent Waves}

Re-arranged

\partial \alpha = \frac{\partial}{\partial x} \lambda \frac{\partial}{\partial y} 

\text{Ignore}
\[ W \sim x \frac{F_d}{
u w^2} \sim x \left( \frac{F_d}{
u w^2} \right) \]

\[ W^3 \sim F_d \frac{x}{
u w^2} \]

\[
W \sim \left( \frac{F_d}{
u w^2} \right)^{\frac{1}{3}} x^{\frac{1}{3}} \sim \left( C R^2 \right)^{\frac{1}{3}} x^{\frac{1}{3}}
\]

Then, comparing widths:

Laminar: \[ \frac{w}{R} \sim \left( \frac{x}{R} \right)^{\frac{1}{2}} \quad \text{Re} \sim \frac{\nu R}{\nu} \]

Turbulent: \[ \frac{w}{R} \sim \left( \frac{x}{R} \right)^{\frac{1}{3}} C_0^{\frac{1}{3}} \]

Interestingly, laminar wake expands with downstream length more rapidly.
Also observe: wake velocity with

\[ \bar{y} \Rightarrow \frac{x}{\Delta x} \]

Reynolds number (Re)

\[ \frac{x}{\Delta x} \]

\[ \frac{v}{u} \]

\[ \frac{R}{d} \]

\[ \frac{d}{\Delta x} \]

Turbulence can relax IV behind

\[ \frac{u}{\Delta x} \]

\[ \frac{d}{\Delta x} \]

object (due separation)

\[ \frac{R}{d} \]

\[ \frac{x}{\Delta x} \]

\[ \frac{R}{d} \]

\[ \frac{x}{\Delta x} \]

\[ \frac{R}{d} \]
\[ \text{Re}(x) \sim \text{Re}_e \left( \frac{R}{x} \right)^{1/3} \]

and \[ \text{Re}(x) \rightarrow 0 \text{ at} \]

\[ x \sim R \left( \text{Re}_e \right)^3 \]

distance behind host where turbulent wake transitions to laminae.

i.e. thin led: transition from turbulent mixing to viscous mixing

N.B. In wake, vertical/rotational region can expand into inviscid region, but never reverse! i.e. would really violate H-Thm...
Wake buoyant, supplement with ship

Revisit turbulent wake using turbulent viscosity, i.e.

\[ W \sim \left( \frac{r_x}{u} \right)^{1/2} \quad (r \to 0) \]

\[ \rightarrow \left( \frac{D_x}{u} \right)^{1/2} \]

i.e. is width of turbulent wake set by turbulent diffusivity following Blasius Law.

but \( D_x \sim W \) \& turbulent viscosity at mixing length level.

\[ \sim W \left( \frac{F_l}{\rho u_w^2} \right) \]

\[ \sim F_d / \rho u_w \sim \text{const} / W \]

\[ \Rightarrow W \sim \left( \frac{F_d x}{\rho u^2 w} \right)^{1/2} \]

\[ \sim \left( \frac{F_d}{\rho u^2} \right)^{1/2} x^{1/2} \sim (C_R^2)^{1/2} x^{1/2} \]

\[ W \sim (C_0)^{1/3} R^{1/3} \sim \frac{C_0}{\rho u^2} R x^{1/2} \]
\[ \frac{W}{L} \sim C_0 \left( \frac{x}{L} \right)^{\frac{1}{3}} \]

Now, \( D \sim \sqrt{W} \)

\[ \sim \frac{(\sqrt{W^2})}{W} \]

\[ \sim \frac{pu \sqrt{W^2}}{puW} = \frac{Q}{w} \sim \frac{Q}{w} \left( \frac{x}{L} \right)^{\frac{1}{3}} \]

The point is that turbulent viscosity mixing drops downstream relative to constant viscosity mixing.

- Follows from \( \sqrt{W} \sim \frac{Q}{w} \)
- Explains why turbulent wake spreads more slowly than laminar wake.
→ Some Observations re: Wake Flows

→ note,

\[ F_x = -\rho U \int_{-\infty}^{\infty} v_x \, dy \, dz \]

Wake

Now \[ Q = \rho \int_{-\infty}^{\infty} v_x \, dy \, dz \]

\[ \Delta \text{ mass flow due to def} \]

\[ \Delta \text{ deflect} \]

→ but, if encircle body

\[ \rho \int v_x \, dA = 0 \quad \text{i.e. continuity} \]

Now total \[ V \rightarrow \{ \text{velocity field} \}

\[ \text{departure from } V \]

\[ \text{Vertical} \]

\[ = \text{Wake flow + potential flow} \]
so, must have $\nabla \cdot \text{rot} \mathbf{v}$
flow

$$\int \mathbf{v} \cdot d\mathbf{a} = \frac{Q}{\rho}$$

then, for area at $r$:

$$v \pi r^2 = \frac{Q}{\rho}$$

$$\Rightarrow v = \frac{Q}{\pi r^2}$$

$$\phi = \frac{Q}{r}$$

\{ \text{global adjustment in potential flow due to viscous/viscous localized} \}

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Message:

A little $v$ forces a global adjustment in flow structure.
Thermal Boundary Layer & Heat Transfer

Consider stationary flow & heat conduction

\[ \frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \kappa \nabla^2 T \]

- thermal diffusion

\[ \kappa = \frac{\kappa}{\rho c_p} \]

\[ \rho \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{\partial P}{\partial x} + \sigma \nabla \mathbf{u} + \frac{\mathbf{g}}{\rho} \]

So: dimensionless # for now exclude buoyancy

- \( \text{Re} \), as usual

- \( \text{Pr} = \nu / \kappa \)

n.b. if buoyancy, \( Ra = \frac{\alpha (ET) L^3}{\nu X} \)

Rayleigh #.

Now general problem.
- body scale $l$, at temp $T_1$
- incoming flow $U$, at $T_0$

Q. what is temp field?

i.e. can flow cool body?

\[
\frac{T-T_0}{T_1-T_0} = f \left( \frac{U}{l}, \frac{1}{Pr}, Pr \right)
\]

\[
\frac{U}{u} = f \left( \frac{1}{Re}, Re \right)
\]

is scaling of result.
Further ways of keeping score:

→ if concerned with cooling body to surface heat flux of body.

\[ h = \alpha = \frac{q}{(T_i - T_a)} \]

Heat transfer coefficient

→ dim-less ratio:

\[ N = \frac{h \cdot l}{k} = \frac{D}_{\text{eddy Thermal}} - \frac{D}_{\text{eddy}} \]

Nusselt #

\[ N = F(Re, Pr) \text{ for B-L heat transfer.} \]
N.B.: Note trade-offs in cooling problem
i.e. resistance of pipe heat transfer.

\[ \frac{d}{L} \]

\[ T \,

\[ T_i \]

\[ \text{How does Nu scale in laminar BL?} \]

\[ Q = -K \frac{dT}{dx} \]

\[ \text{How effective is laminar flow in cooling?} \]

\[ \approx K \frac{(T_i - T_o)}{d} \]

\[ \text{Surface heat flux} \]

\[ \text{but we know for laminar BL} \]

\[ \Delta \sim \frac{1}{(Re)^{1/2}} \]

\[ \text{i.e. Blasius} \]

\[ \omega \text{ for Pad}. \]
\[ Nu \sim \frac{h \ell}{\frac{\ell}{K}} \sim \left( \frac{2}{T_i - T_o} \right) \frac{\ell}{K} \sim \frac{K (T_i - T_o)}{\ell} \sim \sqrt{Re} \]

\[ Nusselt \ Number \]

\[ h \sim \frac{K \sqrt{Re}}{\ell} \]

\[ heat \ transfer \ coeff. \]

\[ \sim \text{(note size scaling)} \]

\[ \sim \text{note } C_p \text{ importance!} \]

2. Turbulent B.L.

Sufficient to calculate temp field in flow.
\[ z = - \frac{u_T}{T} \frac{dT}{dy} \]

thermal boundary layer

\[ K_T = \rho c_p U_* y \]

\[ \frac{dT}{dy} = \frac{I}{\rho c_p U_*} / y \]  

turb. boundary layer for Temp Field.

\[ T = \frac{\rho_0}{\rho c_p u_*} \ln \left( \frac{y}{y_0} \right) + f(P) \]

\[ y_0 = \sqrt{\frac{y}{U_*}} \]

additional edd. coeff. may enter.

\[(P \sim 1)\]
Any flow is turbulent, with temp fluctuations.

Production: \[ \frac{\partial \tilde{T} \tilde{\rho}}{\partial t} + \frac{\partial}{\partial x} \tilde{u} \tilde{T} \tilde{\rho} = \frac{\partial}{\partial x} \left( \tilde{u} \tilde{T} \tilde{\rho} \right) \]

\[ \sim \left( \frac{\tilde{u}}{\tilde{L}} \right)^2 \frac{\nu}{\tilde{L}} \]

So,

\[ \chi = \frac{u(x)}{l} \left( \frac{\tilde{u}}{\tilde{L}} \right)^2 \]

\[ \sim \frac{\nu^{1/3}}{l^{2/3}} \tilde{u}^2 \Rightarrow \tilde{u} \sim l^{1/3} \frac{x}{\nu^{1/6}} \]

\[ \tilde{u}^2 \sim \left( \frac{x}{\nu^{1/6}} \right)^{4/3} l \rightarrow \nu^{-5/3} \]

i.e. scaling for \( \tilde{T}/\tilde{\rho} \) fluctuation.

But? \( Pr \ll 1 \rightarrow \text{how reconcile dissipation ranges?} \) One field may see other smooth...