Schematics of Brownian Motion and Fluctuation - Dissipation Theory
- mixing decay relation

- entropy balance relation

Now have discussed structure of diffusion and its relation to underlying pdf. However, have not addressed physics i.e. relation to dynamics, i.e. what physics writes.

This brings us to \( \frac{\partial}{\partial t} \) \( \mathbf{\Phi} \), \( \mathbf{\Phi} \), etc.

**Brownian Motion**

- classic example of diffusion arises in random walk of particle, driven by thermal random kicks, and restricted by drag

\[
\frac{m \frac{dv}{dt}}{\text{drag}} = -Bv + \frac{\mathbf{f}}{m} \\
\text{while mass} \quad \text{Brownian force} \quad \text{random, additive noise}
\]

\( \text{small particle} \quad \lambda \to \text{scale} \quad \eta \to \rho \to \text{viscosity} \)
Random force → no memory

\[ \langle \tilde{f}(t_1) \tilde{f}(t_2) \rangle = \tilde{f}^2 \rho_{ac} \delta(t_2 - t_1) \]

\( \rho_{ac} \) → required for dimensioning

→ memory time of force, necessarily

short cut in problem

\( \Rightarrow \) delta-corr related force.

\[ \langle \tilde{f}^2 \rangle = \int e^{-i \omega (t_2 - t_1)} \langle \tilde{f}(t_1) \tilde{f}(t_2) \rangle \, d(t_2 - t_1) \]

dimension/ \( \omega \)

\[ = \tilde{f}^2 \rho_{ac} \rightarrow \sim \text{const} \rightarrow \text{white noise} \]

Now what is \( \tilde{f} \) relate to temperature

\( \rightarrow \) force strength.

\[ m \frac{d \tilde{v}}{dt} = -\beta \tilde{v} + \tilde{f} \]

\[ \langle \tilde{v}^2 \rangle \sim \langle \tilde{f}^2 \rangle \]

power dissipated by drag

power input by Brownian force.
\[ \beta \langle \vec{V}^2 \rangle \sim \langle \vec{F} \vec{V} \rangle \]

but \( m \langle \vec{V}^2 \rangle \sim T \rightarrow \text{bath is thermal reservoir!} \)

\[ \langle \vec{V}^2 \rangle \sim \frac{T}{m} \sim \frac{\langle \vec{F} \vec{V} \rangle}{m} \sim \frac{\vec{F}^2}{\beta^2} \]

\[ \Rightarrow \langle \vec{F}^2 \rangle \sim \beta^2 \frac{T}{m} \rightarrow \text{refer Brownian force.} \]

For diffusion of Brownian particle:

\[ D \sim \frac{(\Delta x)^2}{T} \sim \langle \vec{V}^2 \rangle T \]

spatial diffusion.

Velocity relaxer as \( \beta/m \) rate.

\[ D \sim \frac{\langle \vec{V}^2 \rangle m}{\beta} \sim \frac{T}{m} \frac{m}{\beta} \sim \frac{T}{\beta} \]
Fluctuation - Dissipation Thm.

→ Drag-induced energy dissipation

balances fluctuation work at steady state, to maintain temp. T.

→ Given any 2 of T, drag

fluctuations (force) can deduce third.

c.e. \( D = \frac{T}{\beta} \Leftrightarrow D = uT \)

but \( D \sim \frac{<\Delta v^2>/\rho}{\nu} \)

\( \sim \frac{<\Delta v^2>\gamma}{\nu} \)
$D \sim T / \sigma m L$

$D \sim T / \beta$

Recalls: Fano EV.

$D = m T$

$\sim \frac{1}{m (\sqrt{2} \pi \sigma)^{3/2}}$

Similarly, not conditional.

what of PDFs?

velocity, i.e. $P(u, t)$ for Brownian particle?

Recall, $m_p \frac{du}{dt} = -\alpha u + \tilde{\alpha}$

Or equivalently, 

$\frac{du}{dt} = -\alpha u + \tilde{\alpha}$

$\alpha = \frac{\beta}{m_p}$

$\tilde{\alpha} \equiv \text{accel.}$