(1) Consider the gambler’s ruin problem in which two players A and B gamble repeatedly against each other by betting $1 on each trial. The probability of A winning in any given trial is $p$. Assume A starts with $n$ dollars and B with $N - n$ dollars, and that the game ends when one player runs out of money.

(i) What is the probability that player A eventually wins?
(ii) How long does the game last?
(iii) Given that A wins, how long does the game last?

(2) Consider aggregation with the kernel $K_{ij} = \alpha(i + j)$.

(i) Derive the dynamical system for the concentrations $c_n(t)$.
(ii) Obtain and solve the rate equations for the first four moments $\nu_k(t) = \sum_{n=1}^{\infty} n^k c_n(t)$, assuming initial conditions $c_n(0) = \kappa \delta_{n,1}$.
(iii) Assuming the same initial conditions, obtain the exact solution for $c_n(t)$. 