Problem 1
Speed of the particle: \( v = 0.8c \)

\[
\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - 0.8^2}} = \frac{5}{3}
\]

The time in the lab frame is, \( d \), distance traveled: \( d = 800 \text{ m} \)

\[
\Delta t = \frac{d}{v} = \frac{800}{0.8 \times 3 \times 10^8} = \frac{10}{3} \mu s = \frac{10}{3} \mu s
\]

The proper time in the particle is

\[
\Delta t_p = \Delta t \times \frac{\gamma}{\delta} = \frac{10}{3} \times 5 \mu s = \Delta t_p = 2 \mu s
\]

Problem 2
The time measured by me is the proper time, because it is measured at the same location: \( \Delta t_p = 15 \)

The time on the clock in the back of the train is improper time.

\[
\Delta t = \gamma \Delta t_p.
\]

For \( v = 0.6c \), \( \gamma = \frac{1}{\sqrt{1 - 0.6^2}} = 1.25 \)

\[
\Rightarrow \Delta t = 1.25 \text{ s}
\]
Problem 3

\[ \text{jobs} = \sqrt{\frac{1+u/c}{1-u/c}} \quad \text{f source} \Rightarrow \quad \text{f source} = \sqrt{\frac{1-u/c}{1+u/c}} \quad \text{jobs} \]

The passenger in the car on the left measures f source, I measure jobs = \( f \) = \( f \) = \( 0.5 \) \( f \)

\[ 0.5f \]

Problem 4

We can use \( \text{jobs} = \sqrt{\frac{1+u/c}{1-u/c}} \) f source, with f source = \( f \), assuming the source is "you", in the frequency observed by the car on the right

\[ \text{jobs} = \sqrt{\frac{1+0.6}{1-0.6}} \quad \text{f} = \sqrt{\frac{1.6}{0.4}} \quad \text{f} = 2 \text{f} \]

\[ 2f \]

Alternative solution: use the relative speed of the car on the right to the car on the left (see next problem)

Problem 5: let \( S' \) be the reference frame of the car on the left.

Then, \( M_x' = \frac{M_x-U}{1-M_xU/c^2} \), with \( U = 0.6c \), \( M_x = -U \) is the relative velocity

\[ M_x' = -2U \quad = -2 \times 0.6c = -0.8c \quad \text{speed} = 0.8c \]

Back to prob 4: frequency of source \( > 0.5f \), relative speed is 0.8c, so frequency measured by car on right \( < \)

\[ \text{jobs} = \sqrt{\frac{1+0.882}{1-0.882}} = \sqrt{\frac{1.882}{0.118}} \times 0.5f = 4 \times 0.5f = 2f \quad \text{agrees.} \]
Problem 6

Use Lorentz transformation: \( t = \gamma (t' + \frac{u}{c^2} x') \)

\( t_1 = \) time when bomb in lab
\( t_2 = \) time bomb left ship in lab
\( t_2 = t_1 + \Delta t \) according to problem \( 2 \)
\( t_1' = t_2' = \) time the events happened on ship
\( x_1' = \) back of the ship, \( x_2' = \) front of the ship
\( x_2' - x_1' = \) length of the ship as measured on the ship

\[ \gamma = \frac{1}{\sqrt{1 - 0.8^2}} = 1.667 = \frac{5}{3} \]

\[ t_1 = \gamma (t_1' + \frac{u}{c^2} x_1') \quad t_2 = t_1 + \frac{u}{c^2} (x_2' - x_1') \]

\[ t_2 = \gamma (t_1' + \frac{u}{c^2} x_2') \]

\[ x_2' - x_1' = \frac{c^2}{\gamma u} (t_2 - t_1) = \frac{c^2}{\frac{5}{3} \cdot \frac{4}{3}} = \frac{3}{4} t_0 c \]

\[ x_2' - x_1' = 0.75c t_0 \]