PHYSICS 152B/232
Spring 2017
Homework Assignment #4

[1] *Atomic physics* – Consider an ion with a partially filled shell of angular momentum J, and Z additional electrons in filled shells. Show that the ratio of the Curie paramagnetic susceptibility to the Larmor diamagnetic susceptibility is

$$\frac{\chi^{\rm para}}{\chi^{\rm dia}} = -\frac{g_{\rm L}^2\, J(J+1)}{2Zk_{\rm B}T}\,\frac{\hbar^2}{m\langle r^2\rangle} \ .$$

where $g_{\rm L}$ is the Landé g-factor. Estimate this ratio at room temperature.

[2] *Adiabatic demagnetization* – In an ideal paramagnet, the spins are noninteracting and the Hamiltonian is

$$\mathcal{H} = \sum_{i=1}^{N_{\rm p}} \gamma_i\, \boldsymbol{J}_i \cdot \boldsymbol{H}$$

where $\gamma_i = g_i \mu_i/\hbar$ and $\boldsymbol{J}_i$ are the gyromagnetic factor and spin operator for the $i^{\rm th}$ paramagnetic ion, and $\boldsymbol{H}$ is the external magnetic field.

(a) Show that the free energy $F(H,T)$ can be written as

$$F(H,T) = T\,\Phi(H/T) \ .$$

If an ideal paramagnet is held at temperature $T_{\rm i}$ and field $H_{\rm i}\,\hat{\boldsymbol{z}}$, and the field $H_{\rm i}$ is *adiabatically* lowered to a value $H_{\rm f}$, compute the final temperature. This is called "adiabatic demagnetization".

(b) Show that, in an ideal paramagnet, the specific heat at constant field is related to the susceptibility by the equation

$$c_H = T\left(\frac{\partial s}{\partial T}\right)_H = \frac{H^2\,\chi}{T} \ .$$

Further assuming all the paramagnetic ions to have spin J, and assuming Curie's law to be valid, this gives

$$c_H = \tfrac{1}{3} n_{\rm p} k_{\rm B} J(J+1)\left(\frac{g\mu_{\rm B} H}{k_{\rm B}T}\right)^2 \ ,$$

where $n_{\rm p}$ is the density of paramagnetic ions. You are invited to compute the temperature T^* below which the specific heat due to lattice vibrations is smaller than the paramagnetic contribution. Recall the Debye result

$$c_V = \tfrac{12}{5}\pi^4\, n k_{\rm B}\left(\frac{T}{\Theta_{\rm D}}\right)^3 \ ,$$

where $n = 1/\Omega$ is the inverse of the unit cell volume (*i.e.* the density of unit cells) and $\Theta_{\rm D}$ is the Debye temperature. Compile a table of a few of your favorite insulating solids, and tabulate Θ_D and T^* when 1% paramagnetic impurities are present, assuming $J = \frac{5}{2}$.

[3] *Ferrimagnetism* – A *ferrimagnet* is a magnetic structure in which there are different types of spins present. Consider a sodium chloride structure in which the A sublattice spins have magnitude $S_{\rm A}$ and the B sublattice spins have magnitude $S_{\rm B}$ with $S_{\rm B} < S_{\rm A}$ (*e.g.* $S = 1$ for the A sublattice but $S = \frac{1}{2}$ for the B sublattice). The Hamiltonian is

$$\mathcal{H} = J \sum_{\langle ij \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + g_{\rm A} \mu_\circ H \sum_{i \in {\rm A}} S_i^z + g_{\rm B} \mu_\circ H \sum_{j \in {\rm B}} S_j^z$$

where $J > 0$, so the interactions are antiferromagnetic.

Work out the mean field theory for this model. Assume that the spins on the A and B sublattices fluctuate about the mean values

$$\langle \boldsymbol{S}_{\rm A} \rangle = m_{\rm A}\, \hat{\boldsymbol{z}} \qquad , \qquad \langle \boldsymbol{S}_{\rm B} \rangle = m_{\rm B}\, \hat{\boldsymbol{z}}$$

and derive a set of coupled mean field equations of the form

$$\begin{aligned} m_{\rm A} &= F_{\rm A}(\beta g_{\rm A} \mu_\circ H + \beta J z m_{\rm B}) \\ m_{\rm B} &= F_{\rm B}(\beta g_{\rm B} \mu_\circ H + \beta J z m_{\rm A}) \end{aligned}$$

where z is the lattice coordination number ($z = 6$ for NaCl) and $F_{\rm A}(x)$ and $F_{\rm B}(x)$ are related to Brillouin functions. Show graphically that a solution exists, and fund the criterion for broken symmetry solutions to exist when $H = 0$, *i.e.* find $T_{\rm c}$. Then linearize, expanding for small $m_{\rm A}$, $m_{\rm B}$, and H, and solve for $m_{\rm A}(T)$ and $m_{\rm B}(T)$ and the susceptibility

$$\chi(T) = -\frac{1}{2} \frac{\partial}{\partial H} (g_{\rm A} \mu_\circ m_{\rm A} + g_{\rm B} \mu_\circ m_{\rm B})$$

in the region $T > T_{\rm c}$. Does your $T_{\rm c}$ depend on the sign of J? Why or why not?

[4] *Let's all do the spin flop* – In real solids crystal field effects often lead to anisotropic spin-spin interactions. Consider the anisotropic Heisenberg antiferromagnet in a uniform magnetic field,

$$\mathcal{H} = J \sum_{\langle ij \rangle} (S_i^x\, S_j^x + S_i^y\, S_j^y + \Delta\, S_i^z\, S_j^z) + h \sum_i S_i^z$$

where the field is parallel to the direction of anisotropy. Assume $\delta \geq 0$ and a bipartite lattice.

Consider the case of classical spins In a small external field, show that if the anisotropy Δ is not too large that the lowest energy configuration has the spins on the two sublattices lying predominantly in the (x, y) plane and antiparallel, with a small parallel component along the direction of the field. This is called a canted, or 'spin-flop' structure. What is the angle $\theta_{\rm c}$ by which the spins cant out of the (x, y) plane? What do I mean by not too large? (You may assume that the lowest energy configuration is a two sublattice structure, rather than something nasty like a four sublattice structure or an incommensurate one.)