The Discovery of Superconducting Energy Gap

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Abstract. In this paper, a brief overview of the research on superconductivity is given, followed by a detailed description of the experiment leading to the discovery of the superconducting energy-gap and the relevant modeling. The paper is concluded with a discussion on the prominence of Tinkham's work and the profound influence on the modern spectroscopic study of superconductors.

Introduction

Superconductivity is a macroscopic quantum phenomenon that exhibits zero electrical resistance and diamagnetism in a certain class of material below critical temperature T_c. It was first discovered in mercury by Dutch physicist Dr. Heike Kamerlingh Onnes in 1911. This peculiar behavior evoked a wave of enthusiasm in searching for superconductors with higher Tc and satisfying theory of superconductivity. In the first half of the 20th century, tremendous efforts were made to explain the mechanism of superconductivity by theorists, including Einstein, Bohr, Kronig, Landau, Bloch, and Brillouin. Nevertheless, no theory can give a persuasive and complete explanation for the remarkable yet puzzling phenomenon of superconductivity. It was not until the year 1957 that three American theorists—Bardeen, Cooper and Schrieffer, often referred to as BCS,—proposed that the phonon-mediated electron pairs, i.e. Cooper pairs, are behind the scene of superconductivity. There are two significant experiments to support this epoch-making theory. One is the isotope effect on the critical temperature, while the other one is the energy gap, later know as superconducting gap, by Michael Tinkham who was recognized with/by Buckley prize. The experimental observation of superconducting gap will be the focus of this paper. It starts with a review of the original experiment, followed by the theoretical interpretation of the experimental results, and ends with a discussion of the profound influence on the future investigation of conventional and unconventional superconductivity.

Experiment

The first observation of an energy gap in superconductors was made by Tinkham *et. al.* using millimeter-wave and far-infrared spectroscopy. Before the work by Tinkham *et. al.*, several measurements were attempted to compare the optical property of normal and superconducting states, while none of these experiments reported a pronounced difference. The secret of Tinkham's success is using film, instead of bulk, sample, and measuring the optical properties over a broad spectral range that spans microwave and infrared frequencies.

Transmission measurements on a metal thin film allow one to sensitively probe any subtle change in optical conductivity of the sample. In contrast, the reflectivity measurement of a metallic bulk sample gives a unity reflectivity in both normal and superconducting states, and, consequently, it has poor sensitivity.

Fig. 1 shows the scheme of the far infrared experiment. Briefly, a quartz mercury arc was used to generate broadband light that spans 0.1 mm and 0.75 mm in wavelength. After collimated by a spherical mirror, the broadband infrared beam was then spatially dispersed and focused on the sample by a grating and spherical mirror pair for wavelength dependent

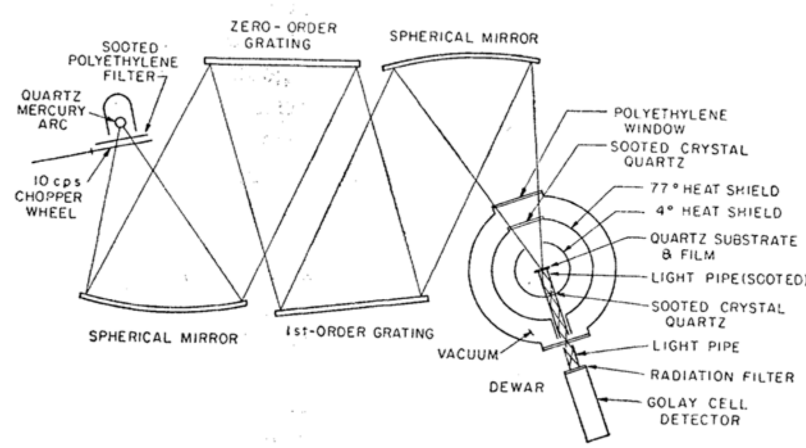

Fig.1 Schematic diagram of the apparatus for infrared measurement

measurement. To minimize the influence of ambient light, two long-pass filters were placed in front of the light source and the entrance window of the detector, respectively. The transmitted signal is measured by the Golay cell detector. The advantage of using the Golay cell detector is that it has an exact power law response, in the meantime, its sensitivity is comparable with that of a crystal detector at 3 mm. To cover the microwave regime, a crystal frequency multiplier served as source, and constricted sections of waveguide were utilized as long-pass filters.

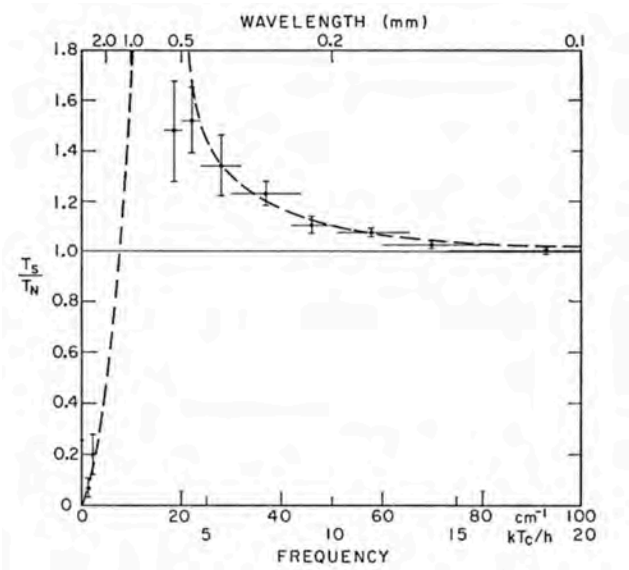

Fig.2 Experimental transmission ratios of superconducting and normal states of a typical lead film at T/Tc = 0.67 ± 0.03

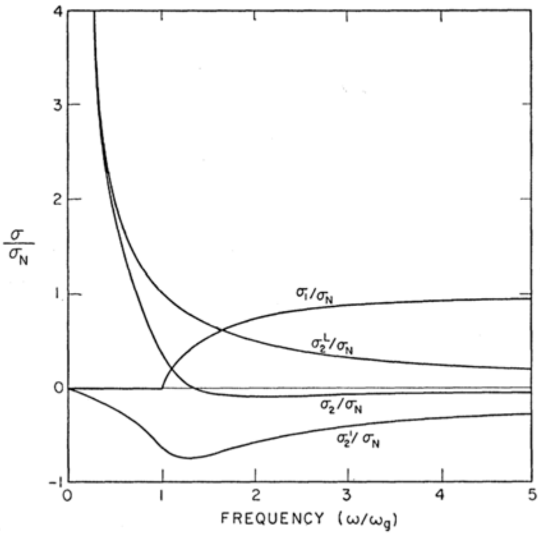

Fig.3 Frequency dependence of conductivity in energy-gap model at T=0 under assumption $\sigma_1/\sigma_N = 1 - \omega_g^2/\omega^2$ and $\sigma_2^L/\sigma_N = \omega_g/\omega$

Fig.2 shows the main result of this remarkable experiment. The low transmission ratio at low -end of the spectrum indicates the loss of resistance in the superconducting states while the high energy constant trending is consistent with the previous measurement. The transmission ratio changes rapidly at low frequency, suggesting a superconducting energy-gap which will be elaborated in the next section. The dash line is obtained by using the universal conductivity function for the thin superconducting films derived below (Eq.6). The microwave measurements only covers a narrow spectral range (1mm to 1m in wavelength) as indicated by the first two data points in the low energy range of Figure 2. Some follow-up experiments with elaborated scheme were performed to obtain precise data at low-frequency regime in greater detail.

Energy-gap Model

The experiment is simplified as a model where a free space plane wave incident onto an extremely thin film of thickness *d* on a substrate of refraction index *n*. In this scheme, we ignored edge effects as well as the multiple reflections inside the substrate. Subsequently, the transmission T can be expressed as $T = |1 + Z_0Y/(n+1)|^{-2}$, where Z_0 is free space impedance, which is a constant (377 Ω) and Y is the admittance. For normal metals, Y is simply $1/R_N$ or $\sigma_N d$, where R_N and σ_N is resistance and conductivity of normal state respectively. For superconducting state, Y is a frequency-dependent complex value, i.e., $Y_S = (\sigma_1 - i\sigma_2)d$, both σ_1 and σ_2 are dependent on frequency. Under these circumstances, the transmission ratio can be derived as follows:

(1) $T_N = (1 + \sigma_N d Z_0/(n+1))^{-2} \Rightarrow dZ_0/(n+1) = (T_N^{-1/2} - 1)/\sigma_N$

(2) $T_S/T_N = \dfrac{1}{T_N(1 + (\sigma_1 - i\sigma_2)dZ_0/(n+1))^2}$

(3) $T_S/T_N = \dfrac{1}{(T_N^{1/2})^2(1 + (\sigma_1 - i\sigma_2)(T_N^{-1/2} - 1)/\sigma_N)^2}$

(4) $T_S/T_N = \dfrac{1}{(T_N^{1/2} + (1 - T_N^{1/2})\sigma_1/\sigma_N)^2 + ((1 - T_N^{1/2})\sigma_2/\sigma_N)^2}$

In other words, the transmission ratio is also dependent on the complex conductivity of superconducting state. Together with the London type conductivities: $\sigma_1^L = \dfrac{c^2}{8\lambda^2}\delta(\omega - 0), \sigma_2^L = \dfrac{c^2}{4\pi\lambda^2\omega}$, we expect a gradually increasing transmission ratio as frequency increases and reaching 1 at high frequency. From Tinkham's measurement, only the high energy part fits the prediction, the discrepancy between the data and the theory at low energy part indicates the existence of energy-gap in the superconducting states.

Assuming there is an energy-gap of width $\hbar\omega_g$, only if the photon energy $\hbar\omega$ is larger than $\hbar\omega_g$ can σ_1 be finite value and increase gradually due to the quasi-particle excitations across the energy gap. The exclusion of states within the energy-gap results in a cut-off frequency for σ_1. Then from the Kramers-Kronig causality relation,

(5) $$\sigma_2(\omega) = -\frac{\omega}{\pi}\int_{-\infty}^{+\infty} \frac{\sigma_1(\omega_1)d\omega_1}{\omega_1^2 - \omega^2}$$

assuming $\sigma_1/\sigma_N = 1 - \omega_g^2/\omega^2$ and $\sigma_2^L/\sigma_N = \omega_g/\omega$, we can obtain

(6) $$\sigma_2'/\sigma_N = -(1/\pi)((1 - \omega_g^2/\omega^2)ln|\omega_g + \omega|/|\omega_g - \omega| + 2\omega_g/\omega)$$

which gives us dependance of σ_2'/σ_N on frequency as shown in fig.3. Now, from the plot it's clear that around $\omega = \omega_g$, both σ_1 and σ_2 are very small, giving rise to the sharp peak observed in the experiment.

As shown in fig.2, with the assumption of energy gap, the theoretical predication (dashed line) agrees with the experiment very well and, therefore, energy-gap model in superconductors is plausible.

Importance

Besides being an indispensable experimental evidence to support BCS theory, the observation of superconducting gap by Tinkham stood out as a new characteristic of the superconducting state and, subsequently, served as the order parameter of superconductivity. It had a long-standing influence over the spectroscopic studies of conventional and, more importantly, unconventional (high-Tc) superconductivity using modern spectroscopic techniques, e.g., FTIR or time-domain spectroscopy, Raman spectroscopy and photo-emission spectroscopy, which turned out to be key machinery to shed light on the symmetry of the superconducting gap, i.e. *d-wave* in cuprates and *s-wave* in pnictide. Although the pairing mechanism in high-Tc superconductor remains a mystery for almost three decades, the consensus is that elementary excitations involving lattice, spin and charge degrees of freedom are behind the scene. Spectroscopic techniques, in general, serve as the best tool to investigate the superconducting condensate and how it couples to those elementary excitations, and bear the promise to provide key discovery that leads to a satisfying theory for high-Tc superconductivity as it did for the BCS theory.

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