"Splitting the atom is like trying to shoot a gnat in the Albert Hall at night and using ten million rounds of ammunition on the off chance of getting it. That should convince you that the atom will always be a sink of energy and never a reservoir of energy."

--Ernest Rutherford
Outline

- Last time – particle in a box & quantized energy
  wavefunctions and probability density
  tunneling & electron transfer
  emission spectra
- Hydrogen atom
- Atomic spectra - emission and absorption
- LASER (Townes, Basov, Prokhorov, Nobel Prize 1957)
In 1913, Neils Bohr explained atomic spectra by utilizing Rutherford’s Planetary model and quantization. In Bohr’s theory for the hydrogen atom, the electron moves in circular orbit around the proton. The Coulomb force provides the centripetal acceleration for continued motion.
Hydrogen Atom

Only certain electron orbits are stable.

In these orbits the atom does not emit energy in the form of electromagnetic radiation.

Radiation is only emitted by the atom when the electron “jumps” between stable orbits.
Hydrogen Atom

The electron will move from a more energetic initial state to less energetic final state.

The frequency of the photon emitted in the “jump” is related to the change in the atom’s energy:

\[ E_i - E_f = hf \]

If the electron is not “jumping” between allowed orbitals, then the energy of the atom remains constant.
Bohr then turned to conservation of energy of the atom in order to determine the allowed electron orbitals.

The total energy of the atom will be:

$$E_{\text{tot}} = KE + PE_{\text{elec}} = \frac{1}{2} m_e v^2 - k_e \frac{e^2}{r}$$

But the electron is undergoing centripetal acceleration (Newton’s second law):

$$F_{\text{cent}} = k_e \frac{e^2}{r^2} = m_e \frac{v^2}{r} = ma_{\text{cent}} \quad \quad m_e v^2 = k_e \frac{e^2}{r}$$
Angular Momentum

Recall from classical mechanics that there was this variable known as angular momentum, $L$.

Angular momentum, $L$, was defined as:
$$ L = I \omega $$
where $I$ was rotational inertia and $\omega$ was angular velocity.

For an electron orbiting a nucleus we have that:
$$ I = m_e r^2 $$
$$ \omega = \frac{v}{r} $$

Giving us:
$$ L = \left( m_e r^2 \right) \cdot \left( \frac{v}{r} \right) $$
$$ L = m_e v r $$
Bohr postulated that the electron’s **orbital angular momentum** must be quantized as well:

\[ L = m_e vr = n \frac{h}{2\pi} = n\hbar \]

where \( \hbar \) is defined to be \( \frac{h}{2\pi} \).

This gives us a velocity of:

\[ v = \frac{n\hbar}{m_e r} \]

Substituting into the last equation from two slides before:

\[ m_e v^2 = k_e \frac{e^2}{r} \quad \text{and} \quad m_e \left( \frac{n\hbar}{m_e r} \right)^2 = k_e \frac{e^2}{r} \]
Hydrogen Atom

Solving for the radii of Bohr’s orbits gives us:

\[
\frac{n^2\hbar^2}{m_e r_n^2} = k_e \frac{e^2}{r_n}
\]

\[
r_n = \frac{n^2\hbar^2}{m_e k_e e^2}
\]

The integer values of \( n = 1, 2, 3, \ldots \) give you the quantized Bohr orbits.

Electrons can only exist in certain allowed orbits determined by the integer \( n \).

When \( n = 1 \), the orbit has the smallest radius, called the **Bohr radius**, \( a_o \).

\( a_o = 0.0529 \text{nm} \)
Hydrogen Atom: Bohr’s Theory

We know that the radii of the Bohr orbits in a hydrogen atom are quantized:

\[ r_n = \frac{n^2 \hbar^2}{m_e k_e e^2} \]

Recall the Bohr radius \( a_0 = 0.0529 \text{nm} \).

So, in general we have: \( r_n = n^2 a_0 \)

The total energy of the atom can be expressed as:

\[ E_{tot} = KE + PE_{elec} = \frac{1}{2} m_e v^2 - k_e \frac{e^2}{r} \]
Hydrogen Atom

Plus, from centripetal acceleration we found that:

\[ m_e v^2 = k_e \frac{e^2}{r} \]

Putting this back into energy we get:

\[ E_{tot} = \frac{1}{2} m_e v^2 - k_e \frac{e^2}{r} = \frac{1}{2} \left( k_e \frac{e^2}{r} \right) - k_e \frac{e^2}{r} \]

\[ E_{tot} = -\frac{1}{2} \left( k_e \frac{e^2}{r} \right) \]

But we can go back to the result for the radius \( r_n = n^2 a_0 \) to get a numerical result.
Hydrogen Atom

\[ E_{tot} = -\frac{1}{2} \left( k_e \frac{e^2}{n^2 a_o} \right) = \frac{-13.6 \text{ eV}}{n^2} \]

- This is the energy of any quantum state (orbit). Please note the negative sign in the equation.
- When \( n = 1 \), the total energy is \(-13.6\text{eV}\).
- This is the lowest energy state and it is called the ground state.
- The **ionization energy** is the energy needed to completely remove the electron from the atom.
- The ionization energy for hydrogen is 13.6eV.
Hydrogen Atom

- So, a general expression for the radius of any orbit in a hydrogen atom is:

\[ r_n = n^2 a_0 \]

- Note that the orbits are not increasing linearly with \( n \).

- The energy of any orbit is:

\[ E_n = \frac{-13.6 \text{ eV}}{n^2} \]
What are the first four energy levels for the hydrogen atom?

- When \( n = 1 \)  \( \Rightarrow \)  \( E_1 = -13.6 \text{eV} \).
- When \( n = 2 \)  \( \Rightarrow \)  \( E_2 = -13.6 \text{eV}/2^2 = -3.40 \text{eV} \).
- When \( n = 3 \)  \( \Rightarrow \)  \( E_3 = -13.6 \text{eV}/3^2 = -1.51 \text{eV} \).
- When \( n = 4 \)  \( \Rightarrow \)  \( E_4 = -13.6 \text{eV}/4^2 = -0.850 \text{eV} \).

Note that the energy levels get closer together as \( n \) increases (similar to how the wavelengths got closer in atomic spectra).

When the atom releases a photon it will experience a transition from an initial higher energy level \( (n_i) \) to a final lower energy level \( (n_f) \).
Hydrogen Atom

- The energies can be compiled in an energy level diagram.
- As the atom is in a higher energy state and moves to a lower energy state it will release energy (in the form of a photon).
- The wavelength of this photon will be determined by the starting and ending energy levels.
The photon will have a wavelength $\lambda$ and a frequency $f$:

$$f = \frac{E_i - E_f}{h}$$

To find the wavelengths for an arbitrary transition from one orbit with $n_f$ to another orbit with $n_i$, we can generalize Rydberg’s formula:

$$\frac{1}{\lambda} = R_H \left( \frac{1}{n_i^2} - \frac{1}{n_f^2} \right)$$
The wavelength will be represented by a different series depending on your final energy level \((n_f)\).

- For \(n_f = 1\) it is called the **Lyman series** \((n_i = 2,3,4,...)\).
- For \(n_f = 2\) it is called the **Balmer series** \((n_i = 3,4,5,...)\).
- For \(n_f = 3\) it is called the **Paschen series** \((n_i = 4,5,...)\).
Atomic Spectra

**Example**

What are the first four wavelengths for the Lyman, Balmer, and Paschen series for the hydrogen atom?

**Answer**

The final energy level for either series will be \( n_f = 1 \) (Lyman), \( n_f = 2 \) (Balmer), and \( n_f = 3 \) (Paschen).
Atomic Spectra

Answer

Turn to the generalized Rydberg equation:

\[
\frac{1}{\lambda} = R_H \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)
\]

For the Lyman series we have:

\[
\frac{1}{\lambda} = R_H \left( 1 - \frac{1}{n_i^2} \right) = R_H \left( \frac{n_i^2}{n_i^2} - \frac{1}{n_i^2} \right)
\]

\[
\frac{1}{\lambda} = R_H \left( \frac{n_i^2 - 1}{n_i^2} \right)
\]

\[
\lambda_n = \frac{1}{R_H} \left( \frac{n_i^2}{n_i^2 - 1} \right) = \frac{1}{1.097 \times 10^7 \text{ m}^{-1}} \left( \frac{n_i^2}{n_i^2 - 1} \right)
\]
Atomic Spectra

Answer

Finally for the Lyman series:

\[ \lambda_1 = \frac{1}{1.097 \times 10^7 \text{ m}^{-1}} \left( \frac{2^2}{2^2 - 1} \right) = 121 \text{ nm} \]

\[ \lambda_2 = \frac{1}{1.097 \times 10^7 \text{ m}^{-1}} \left( \frac{3^2}{3^2 - 1} \right) = 103 \text{ nm} \]

\[ \lambda_3 = \frac{1}{1.097 \times 10^7 \text{ m}^{-1}} \left( \frac{4^2}{4^2 - 1} \right) = 97.2 \text{ nm} \]

\[ \lambda_4 = \frac{1}{1.097 \times 10^7 \text{ m}^{-1}} \left( \frac{5^2}{5^2 - 1} \right) = 95.0 \text{ nm} \]

For the Balmer series we have from before:

\[ \lambda_1 = 656 \text{ nm}, \lambda_2 = 486 \text{ nm}, \lambda_3 = 434 \text{ nm}, \lambda_4 = 410 \text{ nm}. \]
Atomic Spectra

Answer

For the Paschen series we have:

\[
\frac{1}{\lambda} = R_H \left( \frac{1}{9} - \frac{1}{n_i^2} \right) = R_H \left( \frac{n_i^2}{9n_i^2} - \frac{9}{9n_i^2} \right)
\]

\[
\lambda_n = \frac{1}{1.097 \times 10^7 \text{ m}^{-1}} \left( \frac{9n_i^2}{n_i^2 - 9} \right)
\]

\[
\lambda_1 = \frac{1}{1.097 \times 10^7 \text{ m}^{-1}} \left( \frac{9(4^2)}{4^2 - 9} \right) = 1880 \text{ nm}
\]

\[
\lambda_2 = \frac{1}{1.097 \times 10^7 \text{ m}^{-1}} \left( \frac{9(5^2)}{5^2 - 9} \right) = 1280 \text{ nm}
\]

\[
\lambda_3 = \frac{1}{1.097 \times 10^7 \text{ m}^{-1}} \left( \frac{9(6^2)}{6^2 - 9} \right) = 1090 \text{ nm}
\]

\[
\lambda_4 = \frac{1}{1.097 \times 10^7 \text{ m}^{-1}} \left( \frac{9(7^2)}{7^2 - 9} \right) = 1010 \text{ nm}
\]
The only series that lies in the **visible** range (390 – 750nm) is the Balmer series.

The Lyman series lies in the **ultraviolet** range and the Paschen series lies in the **infrared** range.

We can extend the Bohr hydrogen atom to fully describe atoms that are “close” to hydrogen.

These **hydrogen-like atoms** are those that only contain one electron. Examples: He+, Li++, Be+++

In those cases, when you have $Z$ as the atomic number of the element ($Z$ is the number of protons in the atom), you replace $e^2$ with $(Ze)^2$ in the hydrogen equations.
Laser

An acronym for Light Amplification by Stimulated Emission of Radiation.

A laser is a light source that produces a focused, collimated, monochromatic beam of light.

The laser operates using the principle of stimulated emission of light.

In 1917, Albert Einstein established the theoretic foundations for the laser.

Nobel Prize 1957 (Townes, Basov, Prokhorov)
Chapter 30 homework due Wednesday