Chapter 30

Easy

P30.1 An iron nucleus (in hemoglobin) has a few more neutrons than protons, but in a typical water molecule there are eight neutrons and ten protons.

So protons and neutrons are nearly equally numerous in your body, each contributing mass (say) 35 kg:

$$35\text{ kg}\left(\frac{1\text{ nucleon}}{1.67\times10^{-27}\text{ kg}}\right)\boxed{\sim10^{28}\text{ protons}}$$

and

$$\boxed{\sim10^{28}\text{ neutrons}}.$$

The electron number is precisely equal to the proton number,

$$\boxed{\sim10^{28}\text{ electrons}}.$$

P30.4 (a) $r=r_0A^{1/3}=\left(1.20\times10^{-15}\text{ m}\right)(4)^{1/3}=\boxed{1.90\times10^{-15}\text{ m}}$

(b) $r=r_0A^{1/3}=\left(1.20\times10^{-15}\text{ m}\right)(238)^{1/3}=\boxed{7.44\times10^{-15}\text{ m}}$

P30.5 The number of nucleons in a star of two solar masses is

$$A=\frac{2\left(1.99\times10^{30}\text{ kg}\right)}{1.67\times10^{-27}\text{ kg/nucleon}}=2.38\times10^{57}\text{ nucleons}.$$

Therefore $r=r_0A^{1/3}=\left(1.20\times10^{-15}\text{ m}\right)\left(2.38\times10^{57}\right)^{1/3}=\boxed{16.0\text{ km}}$.

P30.8 Using atomic masses as given in Table A.3,

(a) For ${}^2_1\text{H}$: $\dfrac{-2.014\,102+1(1.008\,665)+1(1.007\,825)}{2}$

$$\frac{E_b}{A}=(0.001\,194\text{ u})\left(\frac{931.5\text{ MeV}}{\text{u}}\right)=\boxed{1.11\text{ MeV/nucleon}}.$$

(b) For ${}^4_2\text{He}$: $\dfrac{2(1.008\,665)+2(1.007\,825)-4.002\,603}{4}$

$$\frac{E_b}{A}=0.007\,59\text{ u}c^2=\boxed{7.07\text{ MeV/nucleon}}.$$

(c) For ${}^{56}_{26}\text{Fe}$: $30(1.008\,665)+26(1.007\,825)-55.934\,942=0.528\text{ u}$

$$\frac{E_b}{A}=\frac{0.528}{56}=0.009\,44\text{ u}c^2=\boxed{8.79\text{ MeV/nucleon}}.$$

(d) For $^{238}_{92}\text{U}$: $146(1.008\,665)+92(1.007\,825)-238.050\,783=1.934\,2\text{ u}$

$$\frac{E_b}{A}=\frac{1.934\,2}{238}=0.008\,13\text{ u}c^2=\boxed{7.57\text{ MeV/nucleon}}.$$

P30.9 The binding energy of a nucleus is

$$E_b(\text{MeV})=\left[ZM(\text{H})+Nm_n-M\left(^A_Z\text{X}\right)\right](931.494\text{ MeV/u}).$$

For $^{15}_{8}\text{O}$:

$$E_b=[8(1.007\,825\text{ u})+7(1.008\,665\text{ u})-15.003\,065\text{ u}](931.494\text{ MeV/u})=111.96\text{ MeV}.$$

For $^{15}_{7}\text{N}$:

$$E_b=[7(1.007\,825\text{ u})+8(1.008\,665\text{ u})-15.000\,109\text{ u}](931.494\text{ MeV/u})=115.49\text{ MeV}.$$

Therefore, $\boxed{\text{the binding energy of } ^{15}_{7}\text{N is larger by 3.54 MeV}}$.

P30.12

$$R=R_0e^{-\lambda t}=(6.40\text{ mCi})e^{-(\ln 2/8.04\text{ d})(40.2\text{ d})}=(6.40\text{ mCi})\left(e^{-\ln 2}\right)^5=(6.40\text{ mCi})\left(\frac{1}{2^5}\right)=\boxed{0.200\text{ mCi}}$$

P30.13 (a) From $R=R_0e^{-\lambda t}$,

$$\lambda=\frac{1}{t}\ln\left(\frac{R_0}{R}\right)=\left(\frac{1}{4.00\text{ h}}\right)\ln\left(\frac{10.0}{8.00}\right)=5.58\times10^{-2}\text{ h}^{-1}=\boxed{1.55\times10^{-5}\text{ s}^{-1}}$$

$$T_{1/2}=\frac{\ln 2}{\lambda}=\boxed{12.4\text{ h}}$$

(b)
$$N_0=\frac{R_0}{\lambda}=\frac{10.0\times10^{-3}\text{ Ci}}{1.55\times10^{-5}/\text{s}}\left(\frac{3.70\times10^{10}/\text{s}}{1\text{ Ci}}\right)=\boxed{2.39\times10^{13}\text{ atoms}}$$

(c)
$$R=R_0e^{-\lambda t}=(10.0\text{ mCi})\exp\left(-5.58\times10^{-2}\times30.0\right)=\boxed{1.88\text{ mCi}}$$

P30.14 $\dfrac{dN}{dt}=-\lambda N$

so
$$\lambda=\frac{1}{N}\left(-\frac{dN}{dt}\right)=\left(1.00\times10^{-15}\right)\left(6.00\times10^{11}\right)=6.00\times10^{-4}\text{ s}^{-1}$$

$$T_{1/2}=\frac{\ln 2}{\lambda}=\boxed{1.16\times10^3\text{ s}}\ (=19.3\text{ min})$$

P30.20 (a) A gamma ray has zero charge and it contains no protons or neutrons. So for a gamma ray $Z=0$ and $A=0$. Keeping the total values of Z and A for the system conserved then requires $Z=28$ and $A=65$ for X. With this atomic number it must be nickel, and the nucleus must be in an exited state, so it is $\boxed{^{65}_{28}\text{Ni}^*}$.

(b) $\alpha={}^4_2\text{He}$ has $Z=2$ and $A=4$

so for X we require $Z=84-2=82$

for Pb and $A = 215 - 4 = 211$, $X = \boxed{{}^{211}_{82}\text{Pb}}$.

(c) A positron $e^+ = {}^0_1e$ has charge the same as a nucleus with $Z = 1$. A neutrino ${}^0_0\nu$ has no charge. Neither contains any protons or neutrons. So X must have by conservation $Z = 26 + 1 = 27$. It is Co. And $A = 55 + 0 = 55$. It is $\boxed{{}^{55}_{27}\text{Co}}$.

Similar reasoning about balancing the sums of Z and A across the reaction reveals:

(d) $\boxed{{}^{0}_{-1}\text{e}}$

(e) $\boxed{{}^{1}_{1}\text{H (or p)}}$. Note that this process is a nuclear reaction, rather than radioactive decay. We can solve it from the same principles, which are fundamentally conservation of charge and conservation of baryon number.

Medium

***P30.3** $E_\alpha = 7.70\text{ MeV} = \frac{1}{2}mv^2 = k_e\frac{2eZe}{d_{\min}}$

(a)

$$d_{\min} = \frac{4k_eZe^2}{mv^2} = \frac{2k_eZe^2}{E_\alpha} = \frac{2\left(8.99\times10^9\right)(79)\left(1.60\times10^{-19}\right)^2}{7.70\left(1.60\times10^{-13}\right)} = 29.5\times10^{-15}\text{ m} = \boxed{29.5\text{ fm}}$$

(b) The de Broglie wavelength of the α is

$$\lambda = \frac{h}{m_\alpha v_\alpha} = \frac{h}{\sqrt{2m_\alpha E_\alpha}} = \frac{6.626\times10^{-34}}{\sqrt{2\left(6.64\times10^{-27}\right)7.70\left(1.60\times10^{-13}\right)}} = 5.18\times10^{-15}\text{ m} = \boxed{5.18\text{ fm}}$$

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(c) Since $\boxed{\lambda\text{ is much less than the distance of closest approach}}$, the α may be considered a particle.

P30.10 (a) The neutron-to-proton ratio $\frac{A-Z}{Z}$ is greatest for $\boxed{{}^{139}_{55}\text{Cs}}$ and is equal to 1.53.

(b) $\boxed{{}^{139}\text{La}}$ has the largest binding energy per nucleon of 8.378 MeV.

(c) ${}^{139}\text{Cs}$ with a mass of 138.913 u. We locate the nuclei carefully on Figure 30.4, the neutron–proton plot of stable nuclei. $\boxed{\text{Cesium}}$ appears to be farther from the center of the zone of stability. Its instability means extra energy and extra mass.

P30.16 The number of nuclei which decay during the interval will be
$N_1 - N_2 = N_0\left(e^{-\lambda t_1} - e^{-\lambda t_2}\right)$.

First we find λ: $\lambda = \frac{\ln 2}{T_{1/2}}$

so $e^{-\lambda t} = e^{\ln 2\left(-t/T_{1/2}\right)} = 2^{-t/T_{1/2}}$

and $N_0 = \frac{R_0}{\lambda} = \frac{R_0 T_{1/2}}{\ln 2}$.

Substituting in these values

$$N_1 - N_2 = \frac{R_0 T_{1/2}}{\ln 2}\left(e^{-\lambda t_1} - e^{-\lambda t_2}\right) = \boxed{\frac{R_0 T_{1/2}}{\ln 2}\left(2^{-t_1/T_{1/2}} - 2^{-t_2/T_{1/2}}\right)}.$$

***P30.17** We have all this information: $N_x(0) = 2.50 N_y(0)$

$$N_x(3\text{d}) = 4.20 N_y(3\text{d})$$

$$N_x(0)e^{-\lambda_x 3\text{d}} = 4.20 N_y(0)e^{-\lambda_y 3\text{d}} = 4.20\frac{N_x(0)}{2.50}e^{-\lambda_y 3\text{d}}$$

$$e^{3\text{d}\lambda_x} = \frac{2.5}{4.2}e^{3\text{d}\lambda_y}$$

$$3\text{d}\lambda_x = \ln\left(\frac{2.5}{4.2}\right) + 3\text{d}\lambda_y$$

$$3\text{d}\frac{0.693}{T_{1/2\,x}} = \ln\left(\frac{2.5}{4.2}\right) + 3\text{d}\frac{0.693}{1.60\text{ d}} = 0.781$$

$$T_{1/2\,x} = \boxed{2.66\text{ d}}$$

P30.23 (a) $\boxed{e^- + p \to n + \nu}$

(b) For nuclei, $^{15}O + e^- \to {}^{15}N + \nu$.

Add seven electrons to both sides to obtain $\boxed{{}^{15}_{8}O \text{ atom} \to {}^{15}_{7}N \text{ atom} + \nu}$.

(c) From Table A.3, $m\left({}^{15}O\right) = m\left({}^{15}N\right) + \frac{Q}{c^2}$

$$\Delta m = 15.003\,065\text{ u} - 15.000\,109\text{ u} = 0.002\,956\text{ u}$$

$$Q = (931.5\text{ MeV/u})(0.002\,956\text{ u}) = \boxed{2.75\text{ MeV}}$$

P30.26 (a) For X, $A = 24 + 1 - 4 = 21$

and $Z = 12 + 0 - 2 = 10$, so X is $\boxed{{}^{21}_{10}Ne}$.

(b) $A = 235 + 1 - 90 - 2 = 144$

and $Z = 92 + 0 - 38 - 0 = 54$, so X is $\boxed{{}^{144}_{54}Xe}$.

(c) $A = 2 - 2 = 0$

and $Z = 2 - 1 = +1$, so X must be a positron.

As it is ejected, so is a neutrino: $\boxed{X = {}^{0}_{1}e^{+}}$ and $\boxed{X' = {}^{0}_{0}\nu}$.

Hard

P30.32 (a) $V = \left(317 \times 10^6 \text{ mi}^3\right)\left(\frac{1\,609 \text{ m}}{1 \text{ mi}}\right)^3 = 1.32 \times 10^{18} \text{ m}^3$

$$m_{\text{water}} = \rho V = \left(10^3 \text{ kg/m}^3\right)\left(1.32 \times 10^{18} \text{ m}^3\right) = 1.32 \times 10^{21} \text{ kg}$$

$$m_{H_2} = \left(\frac{M_{H_2}}{M_{H_2O}}\right) m_{H_2O} = \left(\frac{2.016}{18.015}\right)\left(1.32 \times 10^{21} \text{ kg}\right) = 1.48 \times 10^{20} \text{ kg}$$

$$m_{\text{Deuterium}} = (0.030\,0\%) m_{H_2} = \left(0.030\,0 \times 10^{-2}\right)\left(1.48 \times 10^{20} \text{ kg}\right) = 4.43 \times 10^{16} \text{ kg}$$

The number of deuterium nuclei in this mass is

$$N = \frac{m_{\text{Deuterium}}}{m_{\text{Deuteron}}} = \frac{4.43 \times 10^{16} \text{ kg}}{(2.014 \text{ u})\left(1.66 \times 10^{-27} \text{ kg/u}\right)} = 1.33 \times 10^{43}.$$

Since two deuterium nuclei are used per fusion, ${}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{4}_{2}He + Q$, the number of events is $\frac{N}{2} = 6.63 \times 10^{42}$.

The energy released per event is

$$Q = \left[M_{^2H} + M_{^2H} - M_{^4He}\right]c^2 = \left[2(2.014\,102) - 4.002\,603\right]\text{u}\,(931.5 \text{ MeV/u}) = 23.8 \text{ MeV}$$

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The total energy available is then

$$E = \left(\frac{N}{2}\right)Q = \left(6.63 \times 10^{42}\right)(23.8 \text{ MeV})\left(\frac{1.60 \times 10^{-13} \text{ J}}{1 \text{ MeV}}\right) = \boxed{2.53 \times 10^{31} \text{ J}}.$$

(b) The time this energy could possibly meet world requirements is

$$\Delta t = \frac{E}{P} = \frac{2.53 \times 10^{31} \text{ J}}{100\left(7.00 \times 10^{12} \text{ J/s}\right)} = \left(3.61 \times 10^{16} \text{ s}\right)\left(\frac{1 \text{ yr}}{3.16 \times 10^7 \text{ s}}\right) = \boxed{1.14 \times 10^9 \text{ yr}} \sim 1 \text{ billion years}$$

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