Physics 1A

Class Review

Final Exam

- 3 hours and 30 multi-choice questions including
- ~ Uniformly covers all course material
- With some emphasis on Chapter 10
- How to prepare?
- Go through examples in Lectures
- Return to the Textbook as needed
- Highlights follow…

Unit Conversion

- **Always include units** for every quantity, and carry the units through the entire calculation
- Multiply original value by a **conversion factor** as needed ✔
- Example

$$15.0\,\text{in} = ?\,\text{cm}$$

$$15.0\,\text{in}\left(\frac{2.54\,\text{cm}}{1\,\text{in}}\right) = 38.1\,\text{cm}$$

Examples: scalar or vector?

- Mass ✔
- Momentum
- Angular velocity ✔
- Speed
- Torque ✔
- Moment of inertia
- Energy
- Work ✔
- Position
- Velocity ✔
- Acceleration
- Impulse
- Force
- Power
- Angular momentum

- scalar
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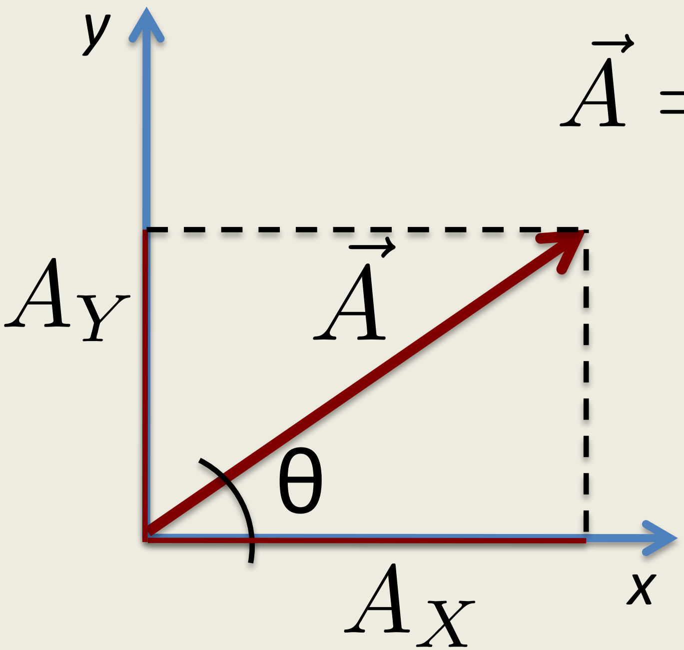
Vectors: notation & properties



$$\vec{A} = (A_X, A_Y) = A_X \hat{x} + A_Y \hat{y}$$

unit vectors

projection of vector along *X* and *Y* axes

$$\begin{cases} A_X = |\vec{A}| \cos\theta \\ A_Y = |\vec{A}| \sin\theta \end{cases}$$

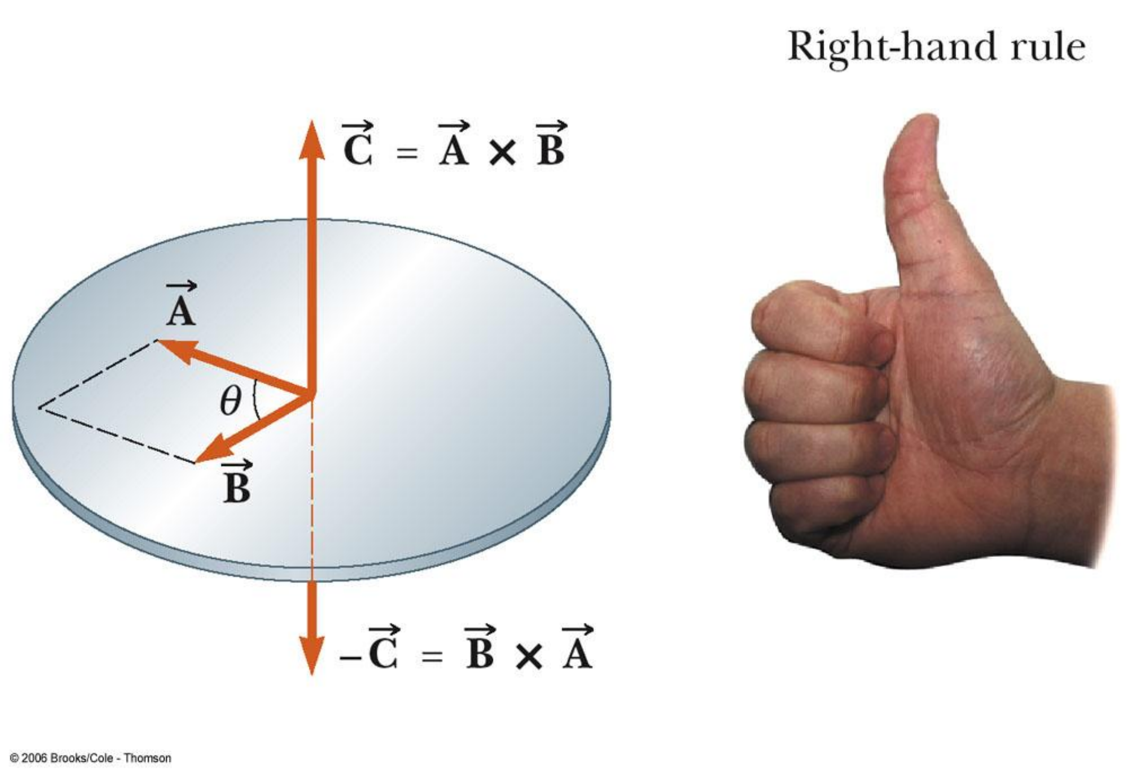
Magnitude: $|\vec{A}| = \sqrt{\vec{A} \cdot \vec{A}} = \sqrt{A_X^2 + A_Y^2}$

Angle: $\tan\theta = A_Y / A_X$

Vectors: cross product

– **Vector (cross) product of two vectors**:

result is a **vector** pointing perpendicular to ***A*** and ***B***



$$\vec{C} = \vec{A} \times \vec{B}$$

$$|\vec{C}| = AB \sin\theta$$

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

Equations of 1D Kinematics

If $a(t) = \text{constant},$ then:

2 initial conditions

$$v(t) = at + v_0$$

$$x(t) = \frac{1}{2}at^2 + v_0 t + x_0$$

$$v(t)^2 = v_i^2 + 2a\Delta x$$

The last relation follows from the two above and very helpful in problem solving ✔

time independent!

2D is the same as 1D, but twice the fun

1D	2D
$a(t) = \text{constant}$	$\vec{a}(t) = \text{constant}$
$v(t) = at + v_0$	$\vec{v}(t) = \vec{a}t + \vec{v}_0$
$x(t) = \frac{1}{2}at^2 + v_0 t + x_0$	$\vec{r}(t) = \frac{1}{2}\vec{a}t^2 + \vec{v}_0 t + \vec{r}_0$
$v(t)^2 = v_0^2 + 2a\Delta x$ ✔	$\lvert\vec{v}(t)\rvert^2 = \lvert\vec{v}_0\rvert^2 + 2\vec{a} \cdot \vec{\Delta x}$

$$\vec{r}(t) = \frac{1}{2}\vec{a}t^2 + \vec{v}_0 t + \vec{r}_0$$

Vector notation

$$\vec{r} = (x, y) \;\; \vec{a} = (a_x, a_y) \;\; \vec{v}_0 = (v_{0,x}, v_{0,y}) \;\; \vec{r}_0 = (x_0, y_0)$$

really means:

$$x = \frac{1}{2}a_x t^2 + v_{0,x} t + x_0$$

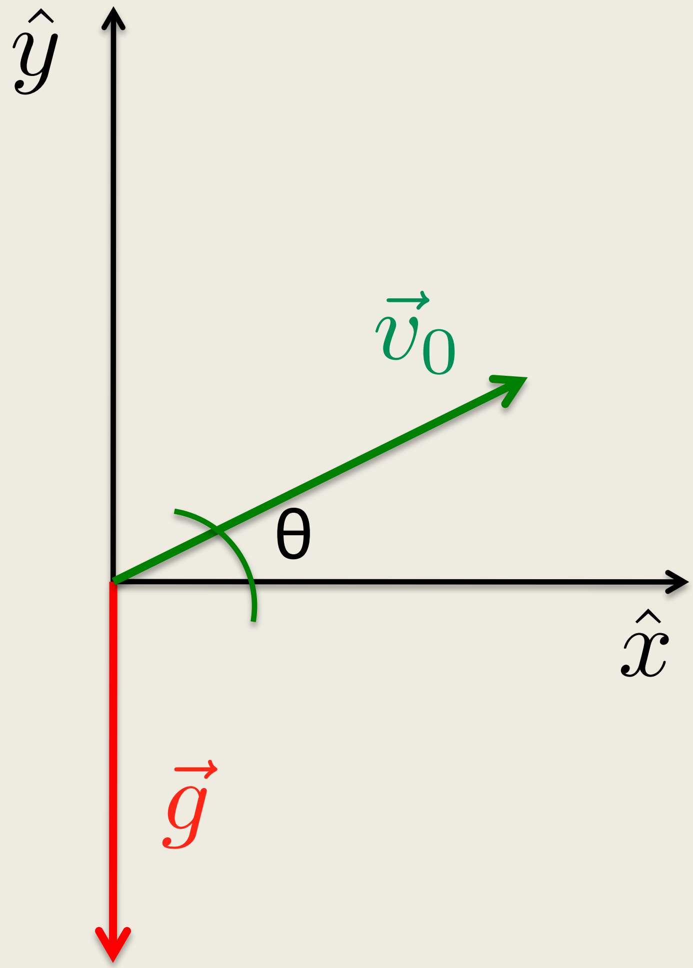
and

$$y = \frac{1}{2}a_y t^2 + v_{0,y} t + y_0$$

4 initial conditions!

these are two <u>independent</u> equations!

Ballistic motion ✔



$$x = v_{0,x}t + x_0$$

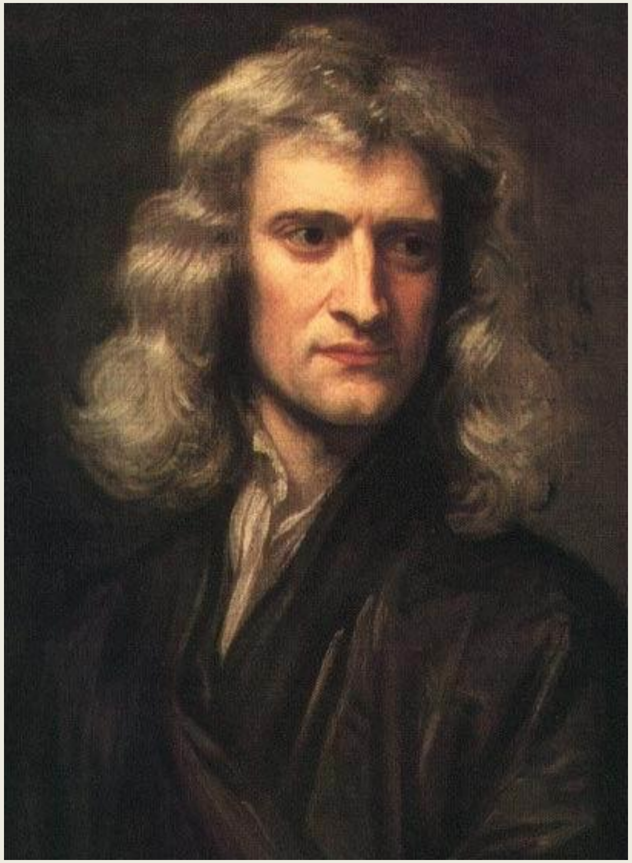
$$y = -\frac{1}{2}gt^2 + v_{0,y}t + y_0$$

$$v_{0,x} = v_0 \cos\theta$$

$$v_{0,y} = v_0 \sin\theta$$

Still 2 initial velocity conditions
(v_0 and θ instead of $v_{0,x}$ and $v_{0,y}$)

Newton’s Laws of Motion



1. without force, constant velocity (inertial motion)

$$\vec{F} = 0 \;\Rightarrow\; \vec{v} = \vec{v}_0 \text{ or } \vec{p} = \vec{p}_0$$

2. acceleration is proportional to the net force,

$$\Sigma\vec{F} = m\vec{a} \quad \text{or} \quad \Sigma\vec{F} = \frac{d\vec{p}}{dt}$$

3. every force has opposite and equal reaction force

$$\vec{F}_{12} = -\vec{F}_{21}$$

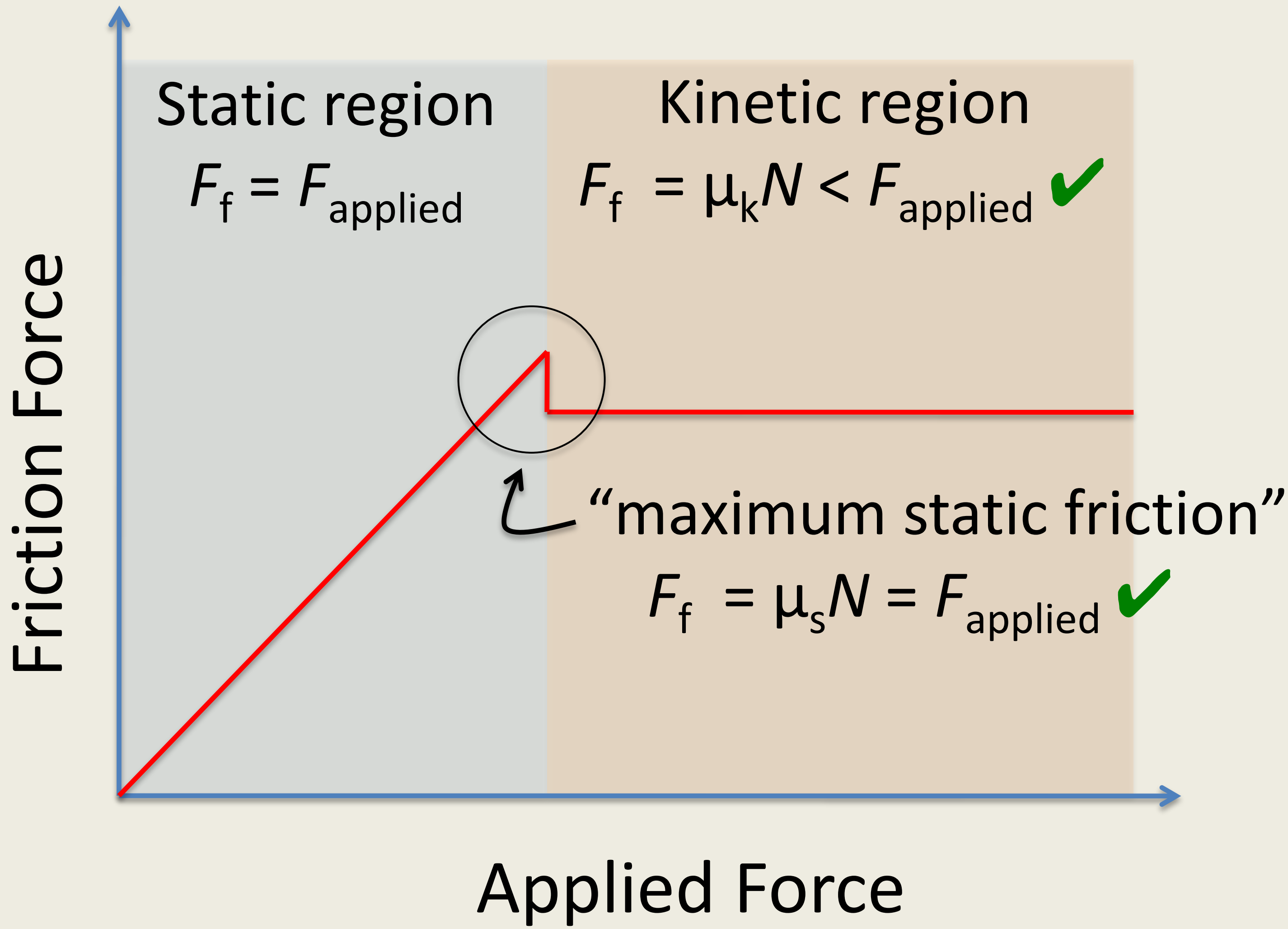
Mass is a **measure of inertia**

mass = net force / acceleration

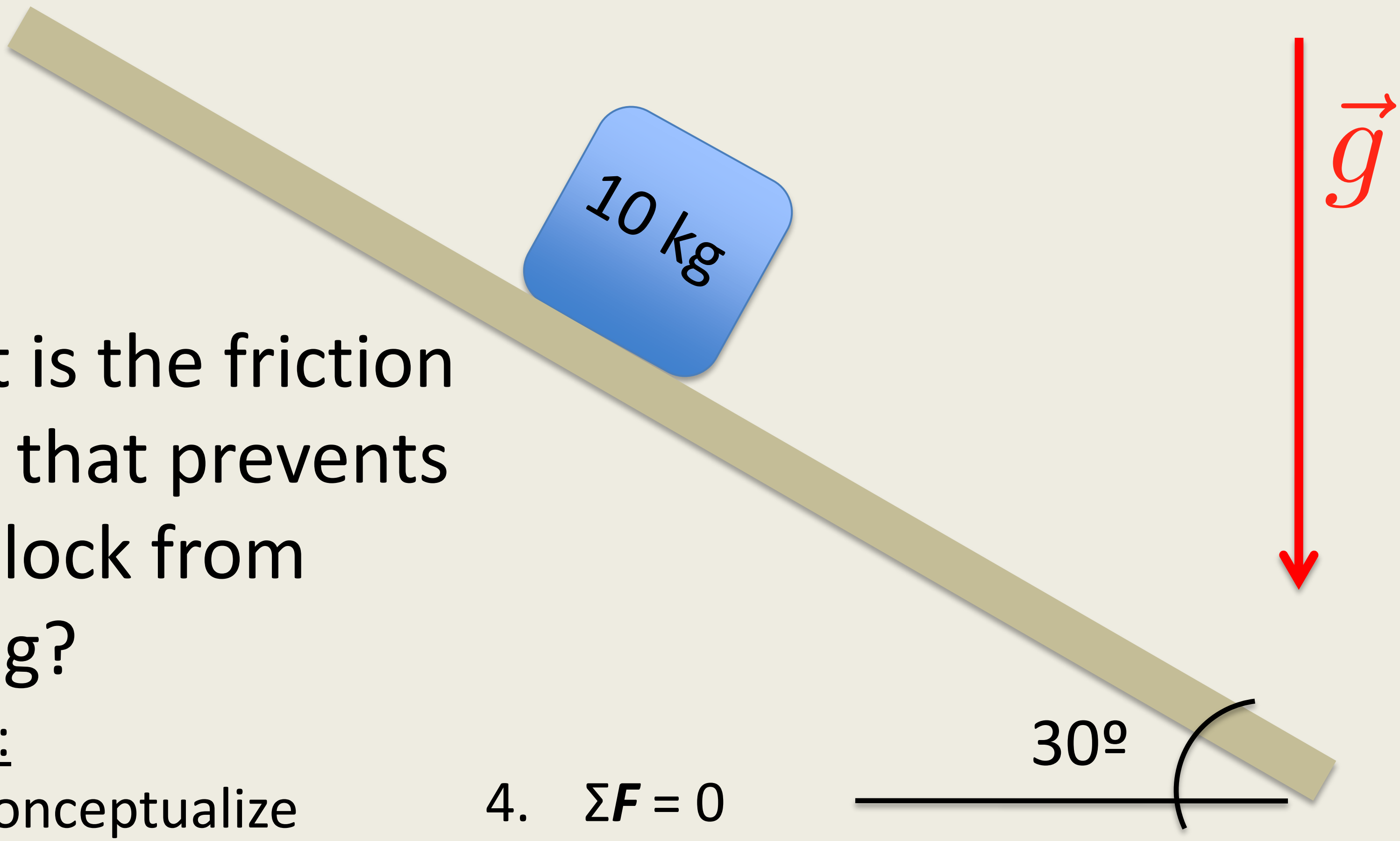
$$m = \frac{|\Sigma \vec{F}|}{|\vec{a}|}$$

Mass is a **scalar** quantity representing inherent properties of an object <u>independent of its surroundings</u> ✔

Coefficients of Friction



Example: Inclined Plane (static) ✔

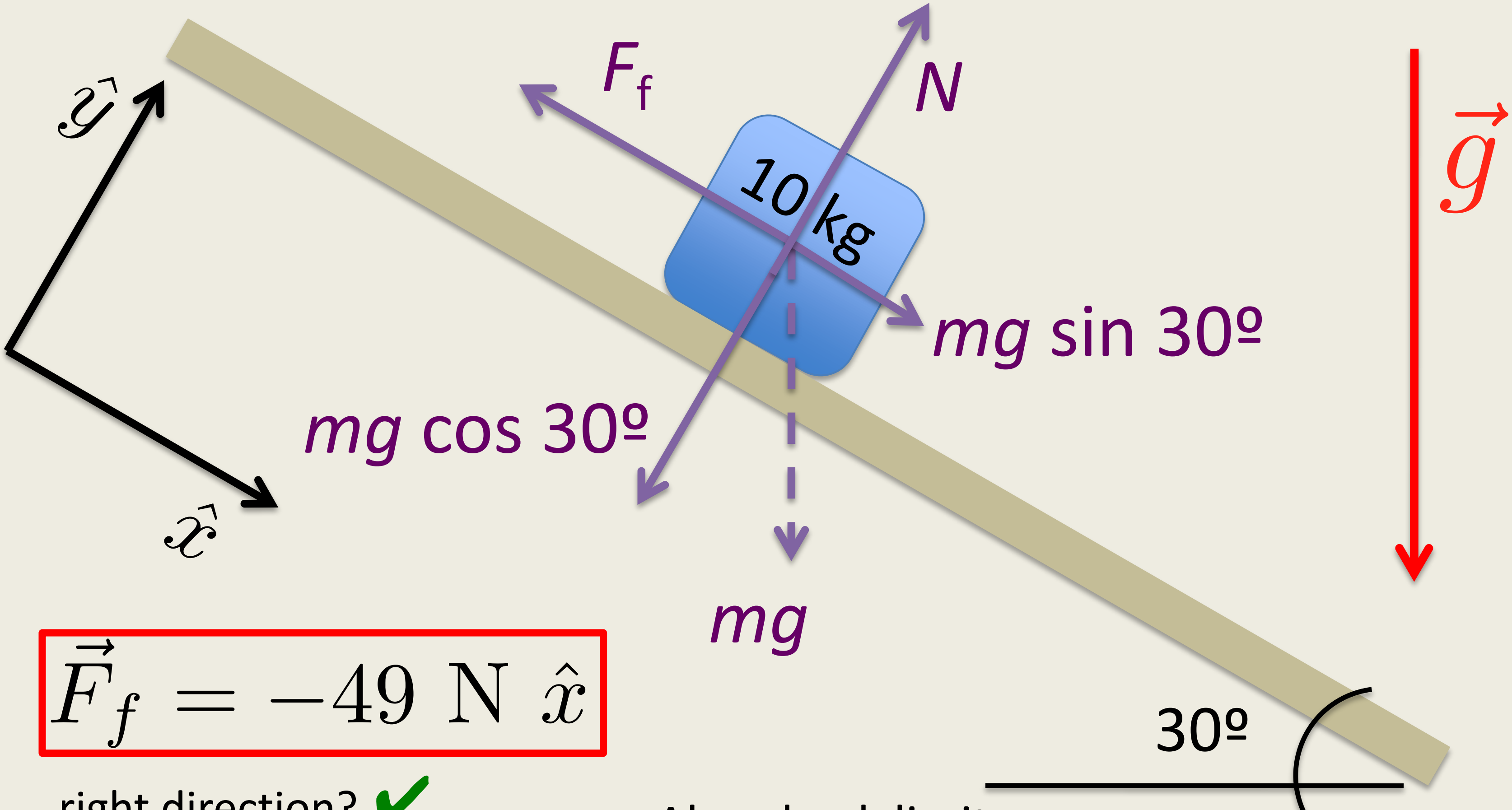


What is the friction force that prevents the block from sliding?

Steps:

1. conceptualize
2. draw forces
3. choose coordinates
4. $\Sigma \boldsymbol{F} = 0$
5. Solve for unknowns
6. Check solution

Step 6: Check your solution



$$\vec{F}_f = -49 \text{ N } \hat{x}$$

right direction? ✔

right dimensions/units? ✔

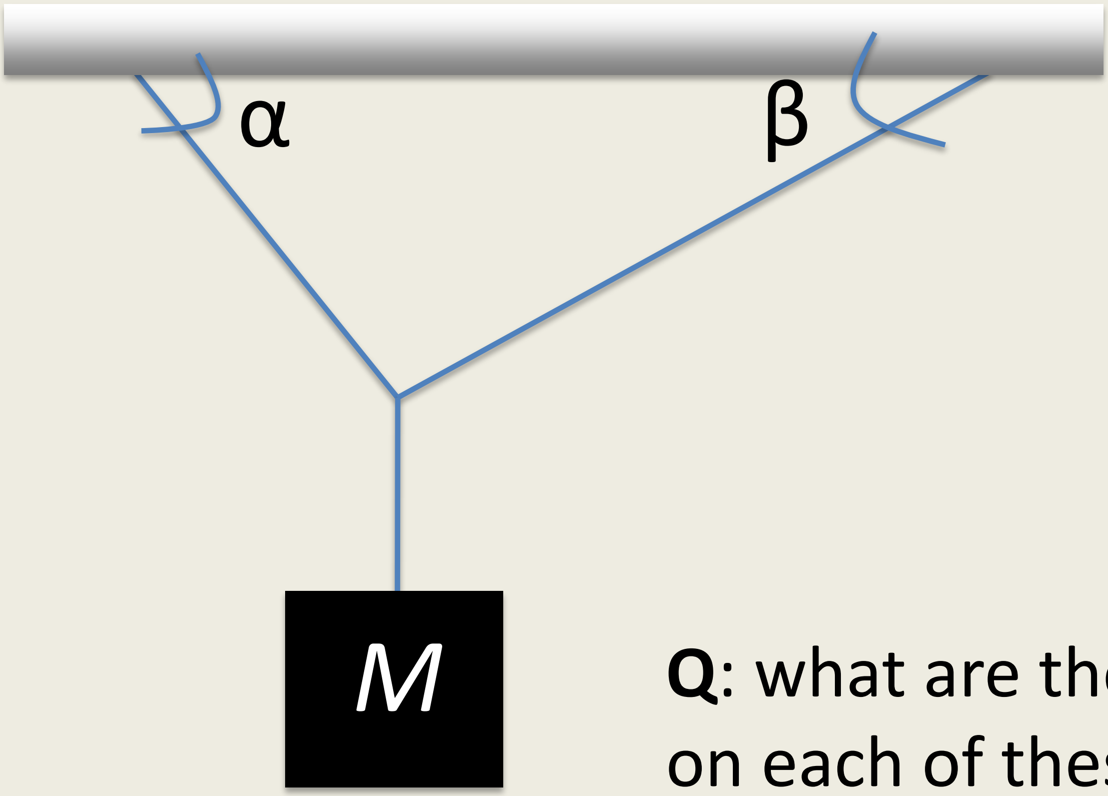
answer makes sense? ✔

Also check limits:

F_f = W sin 30º

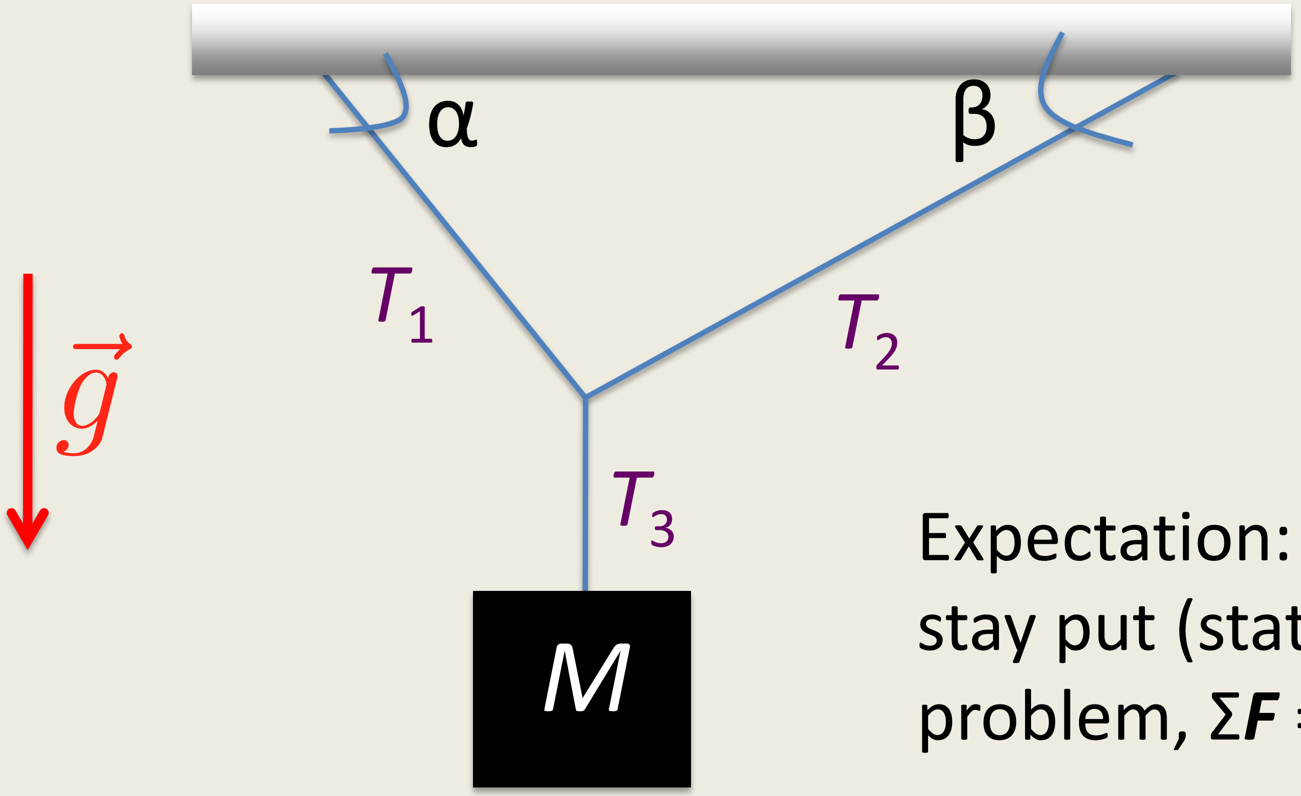
if angle = 0º, there would be no friction force ✔

Many-body static force balance: Mass on a rope



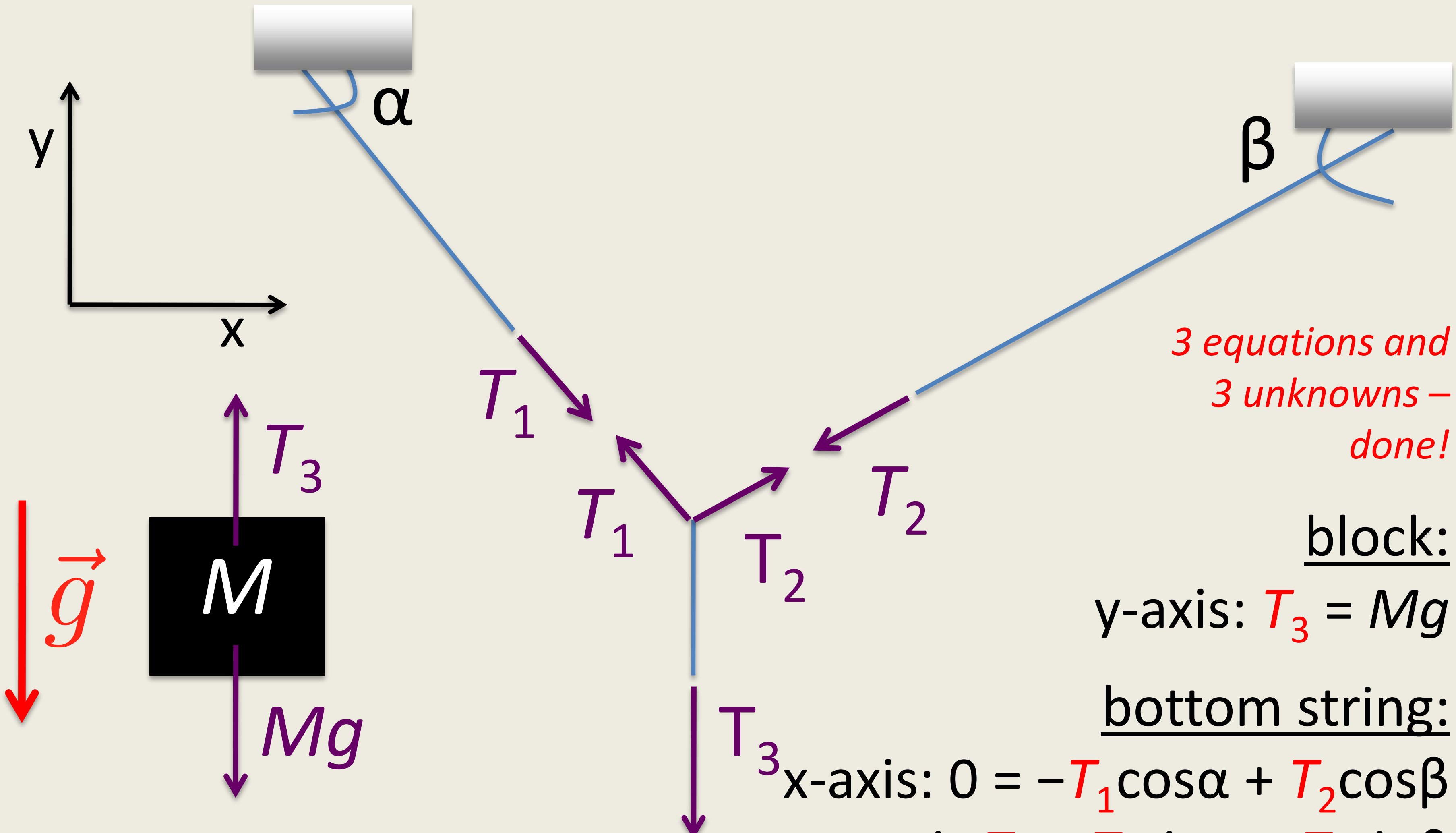
Q: what are the **tensions** on each of these rope segments? ✔

Step 1: Conceptualize & Set up



Expectation: mass will stay put (static problem, $\Sigma \boldsymbol{F} = 0$)

Step 5: Solve for unknowns & Step 6: Check solution



3 equations and 3 unknowns – done!

block:

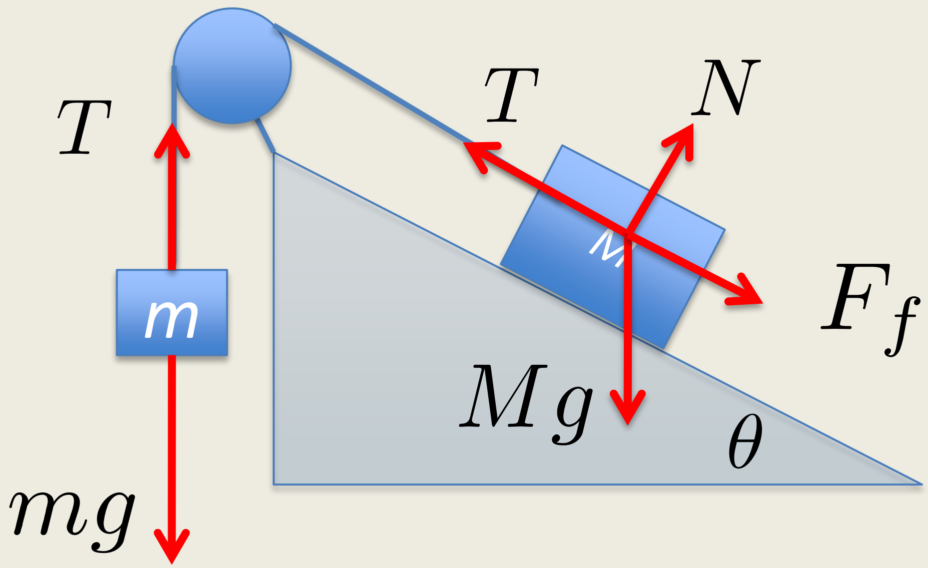
y-axis: $T_3 = Mg$

bottom string:

x-axis: $0 = -T_1\cos\alpha + T_2\cos\beta$

y-axis $T_3 = T_1\sin\alpha + T_2\sin\beta$

Suppose string moves to the left at **constant velocity**, and plane has friction coefficient μ_k



Q1: Find μ_k for given *m*, *M*, and θ

Q2: What happens if **acceleration** is **constant**? ✔

Kinetic Energy and Work

$$K = \frac{1}{2}mv^2$$

$$W_{\text{ext}} = K_f - K_i = \Delta K$$

depends on the mass of an object

depends on the **speed** squared

James Prescott Joules
(1818-1889)

- Kinetic energy is a **scalar** quantity associated with the motion of an object
- Dimensions: $M(L/T)^2 = ML^2T^{-2}$
- SI Units: $kg\ m^2/s^2$ = N m = **Joule** ✔

The work done by a **conservative force** is "stored" as a change in **potential energy**:

$$W_c = \int \vec{F}_c \cdot d\vec{r} \equiv -\Delta U$$

The **total mechanical energy** of an object or system is the sum of all kinetic and potential energies:

$$E_{\mathrm{mech}} = K + U$$

In a **closed system** without dissipative forces (e.g. friction) total mechanical energy remains **constant**. This is the **law of conservation of energy**

Linear Momentum

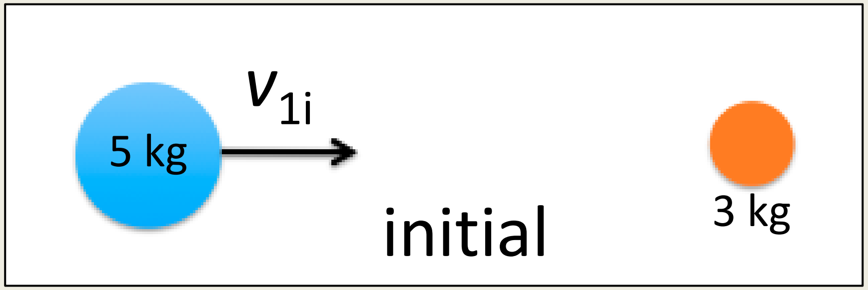
$$\vec{p} = m\vec{v}$$

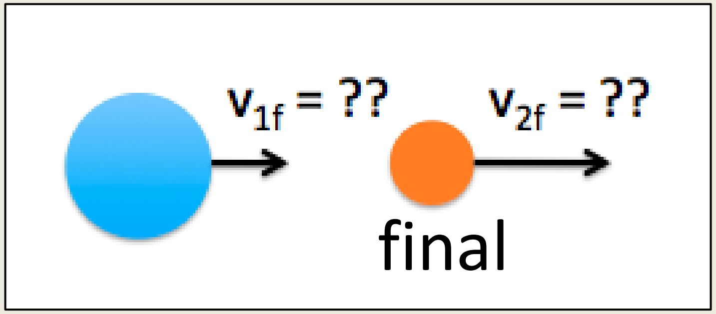
- **Vector** quantity
- Dimensions: [p] = M L T^{-1}
- SI Units: kg m/s
- First suggested by **Abu Ali Ibn Sina**

Abu Ali Ibn Sina
(AD 980-1037)

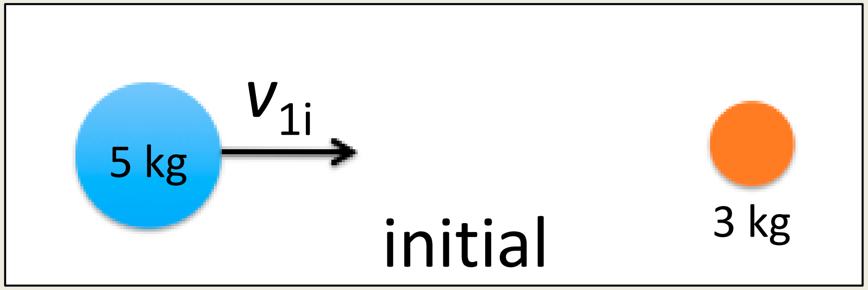
Types of Collisions

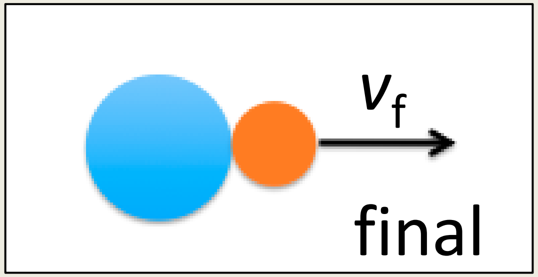
Elastic: Momentum and Kinetic Energy are **both conserved** ✔





Perfectly Inelastic: Objects **stick**, Momentum conserved, a fraction of **Kinetic Energy is lost** ✔





Center of Mass Motion

A net external force changes the velocity of the system's **center of mass**

total mass: $$M = \sum_i m_i$$

center of mass position: $$\vec{r}_{\mathrm{CM}} = \frac{\sum m_i \vec{r}_i}{M}$$

center of mass velocity: $$\vec{v}_{\mathrm{CM}} = \frac{\sum m_i \vec{v}_i}{M}$$

Center of Mass Motion

$$\frac{d\vec{p}_{\text{total}}}{dt} = M\frac{d\vec{v}_{\text{CM}}}{dt} = \vec{F}_{\text{ext}}$$

Center of mass of a system acts like a point particle acted on by the **net external force**

This is Newton's 2nd law applied to a system of particles

Kinematic equations for constant acceleration

Translation	Rotation
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$x = x_0 + v_0 t + \frac{1}{2}at^2$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a\Delta x$	$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$ ✔

Rotational kinematics are **identical** to 1D linear kinematics

Angular vs. Tangential Motion

Angular Motion × radius = Tangential Motion

$$\Delta\theta = \theta_f - \theta_i \qquad \Delta s = r\Delta\theta$$

$$\omega = \frac{d\theta}{dt} \qquad v_{tan} = r\frac{d\theta}{dt} = r\omega$$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} \qquad a_{tan} = r\frac{d\omega}{dt} = r\alpha$$

Torque and the Vector Product ✔

$$\vec{\tau} \equiv \vec{r} \times \vec{F} = I\vec{\alpha}$$

angular acceleration

position vector from center of mass (*or from a specified reference axis*) to interaction point

applied force

moment of inertia
rotational equivalent of mass

This is Newton's 2nd Law for Rotational Motion

Moment of Inertia

- Scalar quantity
- Dimensions: M L^2
- SI units: kg m^2 ✔

$$I = \int_V r^2 \rho dV$$

rigid body

$$I = \sum m_i r_i^2$$

system of particles

depends on: distance from axis, density of material

Parallel Axis Theorem

$$I = I_{CM} + Md^2$$

distance from center of mass to axis of rotation

Rotational Kinetic Energy & Angular Momentum

$$m, \vec{v} \to I, \vec{\omega}$$

Rotational kinetic energy

$$\Rightarrow \quad K = \frac{1}{2} m v^2 \to K_R = \frac{1}{2} I \omega^2$$

Angular momentum ✔

$$\Rightarrow \vec{p} = m\vec{v} \to \vec{L} = I\vec{\omega}$$

$$= \vec{r} \times \vec{p}$$

K_R has same dimensions as K

L has dimension: M L^2 T^{-1}, SI unit J-s

Rotational Equivalent to Momentum

Linear momentum

$$\vec{p} = m\vec{v}$$

$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$\Delta\vec{p} = \vec{F}_{avg}\Delta t$$

Conserved in absence of external **force**

Angular momentum

$$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$$

$$\vec{\tau} = \vec{r} \times \vec{F} = \frac{d\vec{L}}{dt}$$

$$\Delta\vec{L} = \vec{\tau}_{avg}\Delta t$$

Conserved in absence of external **torque**

Conditions for Translational & Rotational Equilibrium

$$\sum \vec{F} = 0 \Rightarrow \vec{a} = 0$$

object moves at constant velocity (can be zero)

$$\sum \vec{\tau} = 0 \Rightarrow \vec{\alpha} = 0$$

object rotates at constant angular velocity (can be zero)

Angular Momentum Conservation

$L_i = I_i \omega_i$ (wide)

With no external torques angular momentum is conserved: $L_f = L_i \Rightarrow \omega_f = \omega_i \frac{I_i}{I_f}$

$L_f = I_f \omega_f$ (narrow)

>1

$\Downarrow$

final angular speed is larger ✔ $\omega_f > \omega_i$

Good luck on your final!

- **Final exam**: Monday, Dec 9th, 11:30 am – 2:29 pm, York 2722
- **What to bring**: scantron, #2 pencil, calculator, student ID