Chapter 5

Force and Motion-I
Newton’s First Law:

If no force acts on a body, the body’s velocity cannot change

The purpose of Newton’s First Law is to introduce the special frames in which Newton’s Laws hold

We need not know what force is yet! All that is required is the absence of force. An isolated body experiences no force.
Inertial Reference Frames

An inertial reference frame is one in which Newton’s 1st law holds.

If a puck is sent sliding along a short strip of frictionless ice—the puck’s motion obeys Newton’s laws as observed from the Earth’s surface.

If the puck is sent sliding along a long ice strip extending from the north pole, and if it is viewed from a point on the Earth’s surface, the puck’s path is not a simple straight line.

The apparent deflection is not caused by a force, but by the fact that we see the puck from a rotating frame. In this situation, the ground is a noninertial frame.
5.4 Force

- When not in isolation a body may experience an acceleration (here and below we implicitly describe motion in inertial frames only).

- This effect is attributed to forces exerted by bodies on bodies.

- A force acting on a body produces an acceleration of that body.

- Forces are vectors: they have both magnitudes and directions.
Acceleration of a body is

- Larger the larger the applied force
- Smaller the larger the mass of the body
- Directed along the applied force

\[ \vec{a} = \frac{\vec{F}}{m} \]  

Newton’s 2nd (first version)
5.5 Mass

But what is \textit{mass}?

Mass is the amount of “stuff” in a body. It is an \textit{intrinsic} characteristic of the body.

It can be defined as the constant of proportionality that makes Newton’s law work.

It is not obvious from that definition that mass is \textit{additive}: two pieces of material of masses $m_1$ and $m_2$ make a bigger piece of mass $m_1 + m_2$

But the definition does give us a way to measure mass, relative to a fixed standard
For a standard of mass measure the acceleration produced by a given force:

\[ \vec{a}_0 = \frac{\vec{F}_0}{m_0} \]

The apply the same force to an object X of unknown mass and measure the resulting accelearation

\[ m_X \vec{a}_X = \vec{F}_0 = m_0 \vec{a}_0 \]

Therefore: The ratio of the masses of two bodies is equal to the inverse of the ratio of their accelerations when the same force is applied to both.

\[ \frac{m_X}{m_0} = \frac{a_0}{a_X} \]

This gives \( m_X \) in units of \( m_0 \)
The force that is exerted on a standard mass of 1 kg to produce an acceleration of 1 m/s² has a magnitude of 1 newton (abbreviated N)

\[ 1 \text{ N} = (1 \text{ kg}) \times (1 \text{ m/s}^2) = 1 \text{ kg} \cdot \text{m} / \text{s}^2 \]
**Principle of Superposition**

The net force on a body is the vector sum of all individual forces acting on the body.

\[
\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_N
\]

This is the most common case: there are almost always many bodies affecting any one body! But one can idealize the situation: imagine the individual force of each body, and... then the net force is the sum of the individual ones.
5.6 Newton’s Second Law

The net force on a body is equal to the product of the body’s mass and its acceleration.

\[ \vec{F}_{\text{net}} = m\vec{a} \]

Newton’s 2nd (second version)

This should be thought of as giving the acceleration produced by the net force acting on the body. But it can be used either way: given a measured acceleration we can infer the net force acting on the body that produced it.

It is a vector equation! In component form,

\[ F_{\text{net},x} = ma_x, \quad F_{\text{net},y} = ma_y, \quad \text{and} \quad F_{\text{net},z} = ma_z. \]

The acceleration component along a given axis is caused \textit{only by the sum of the force components along that same axis, and not by force components along any other axis.}
Sample Problem: Forces

Parts A, B, and C of Fig. 5-3 show three situations in which one or two forces act on a puck that moves over frictionless ice along an \( x \) axis, in one-dimensional motion. The puck’s mass is \( m = 0.20 \) kg. Forces \( \vec{F}_1 \) and \( \vec{F}_2 \) are directed along the axis and have magnitudes \( F_1 = 4.0 \) N and \( F_2 = 2.0 \) N. Force \( \vec{F}_3 \) is directed at angle \( \theta = 30^\circ \) and has magnitude \( F_3 = 1.0 \) N. In each situation, what is the acceleration of the puck?

Note: we do not know what caused the force: maybe a string tugging or a hand pushing. For now, we are just told the force is there.

Fig. 5-3 In three situations, forces act on a puck that moves along an \( x \) axis. Situations B and C in next slides.
Parts A, B, and C of Fig. 5-3 show three situations in which one or two forces act on a puck that moves over frictionless ice along an x axis, in one-dimensional motion. The puck’s mass is \( m = 0.20 \) kg. Forces \( \vec{F}_1 \) and \( \vec{F}_2 \) are directed along the axis and have magnitudes \( F_1 = 4.0 \) N and \( F_2 = 2.0 \) N. Force \( \vec{F}_3 \) is directed at angle \( \theta = 30^\circ \) and has magnitude \( F_3 = 1.0 \) N. In each situation, what is the acceleration of the puck?

**Situation A:** For Fig. 5-3b, where only one horizontal force acts, Eq. 5-4 gives us

\[
F_1 = ma_x,
\]

which, with given data, yields

\[
a_x = \frac{F_1}{m} = \frac{4.0 \text{ N}}{0.20 \text{ kg}} = 20 \text{ m/s}^2. \quad (Answer)
\]

The positive answer indicates that the acceleration is in the positive direction of the x axis.
Parts A, B, and C of Fig. 5-3 show three situations in which one or two forces act on a puck that moves over frictionless ice along an $x$ axis, in one-dimensional motion. The puck’s mass is $m = 0.20 \text{ kg}$. Forces $\vec{F}_1$ and $\vec{F}_2$ are directed along the axis and have magnitudes $F_1 = 4.0 \text{ N}$ and $F_2 = 2.0 \text{ N}$. Force $\vec{F}_3$ is directed at angle $\theta = 30^\circ$ and has magnitude $F_3 = 1.0 \text{ N}$. In each situation, what is the acceleration of the puck?
Sample Problem: Forces

Parts A, B, and C of Fig. 5-3 show three situations in which one or two forces act on a puck that moves over frictionless ice along an x axis, in one-dimensional motion. The puck’s mass is \( m = 0.20 \text{ kg} \). Forces \( \vec{F}_1 \) and \( \vec{F}_2 \) are directed along the axis and have magnitudes \( F_1 = 4.0 \text{ N} \) and \( F_2 = 2.0 \text{ N} \). Force \( \vec{F}_3 \) is directed at angle \( \theta = 30^\circ \) and has magnitude \( F_3 = 1.0 \text{ N} \). In each situation, what is the acceleration of the puck?

These forces compete. Their net force causes a horizontal acceleration.

**Situation B:** In Fig. 5-3d, two horizontal forces act on the puck, \( \vec{F}_1 \) in the positive direction of x and \( \vec{F}_2 \) in the negative direction. Now Eq. 5-4 gives us

\[
F_1 - F_2 = ma_x,
\]

which, with given data, yields

\[
a_x = \frac{F_1 - F_2}{m} = \frac{4.0 \text{ N} - 2.0 \text{ N}}{0.20 \text{ kg}} = 10 \text{ m/s}^2.
\]

(Answer)

Thus, the net force accelerates the puck in the positive direction of the x axis.
Parts A, B, and C of Fig. 5-3 show three situations in which one or two forces act on a puck that moves over frictionless ice along an x axis, in one-dimensional motion. The puck’s mass is \( m = 0.20 \text{ kg} \). Forces \( \vec{F}_1 \) and \( \vec{F}_2 \) are directed along the axis and have magnitudes \( F_1 = 4.0 \text{ N} \) and \( F_2 = 2.0 \text{ N} \). Force \( \vec{F}_3 \) is directed at angle \( \theta = 30^\circ \) and has magnitude \( F_3 = 1.0 \text{ N} \). In each situation, what is the acceleration of the puck?
Parts A, B, and C of Fig. 5-3 show three situations in which one or two forces act on a puck that moves over frictionless ice along an $x$ axis, in one-dimensional motion. The puck’s mass is $m = 0.20$ kg. Forces $\vec{F}_1$ and $\vec{F}_2$ are directed along the axis and have magnitudes $F_1 = 4.0$ N and $F_2 = 2.0$ N. Force $\vec{F}_3$ is directed at angle $\theta = 30^\circ$ and has magnitude $F_3 = 1.0$ N. In each situation, what is the acceleration of the puck?

**Situation C:** In Fig. 5-3f, force $\vec{F}_3$ is not directed along the direction of the puck’s acceleration; only $x$ component $F_{3,x}$ is. (Force $\vec{F}_3$ is two-dimensional but the motion is only one-dimensional.) Thus, we write Eq. 5-4 as

$$F_{3,x} - F_2 = ma_x. \quad (5-5)$$

From the figure, we see that $F_{3,x} = F_3 \cos \theta$. Solving for the acceleration and substituting for $F_{3,x}$ yield

$$a_x = \frac{F_{3,x} - F_2}{m} = \frac{F_3 \cos \theta - F_2}{m} = \frac{(1.0 \text{ N})(\cos 30^\circ) - 2.0 \text{ N}}{0.20 \text{ kg}} = -5.7 \text{ m/s}^2.$$ (Answer)
5.6 Newton’s Second Law: Drawing a free-body diagram

- In a free-body diagram, the only body shown is the one for which we are summing forces.

- The body is represented by a point.

- Each force on the body is drawn as a vector arrow with its tail on the body.

- A coordinate system is usually included, and the acceleration of the body is sometimes shown with a vector arrow (labeled as an acceleration).

The figure here shows two horizontal forces acting on a block on a frictionless floor and the corresponding free body diagram.
Example, 2-D forces:

In the overhead view of Fig. 5-4a, a 2.0 kg cookie tin is accelerated at 3.0 m/s² in the direction shown by $\vec{a}$, over a frictionless horizontal surface. The acceleration is caused by three horizontal forces, only two of which are shown: $\vec{F}_1$ of magnitude 10 N and $\vec{F}_2$ of magnitude 20 N. What is the third force $\vec{F}_3$ in unit-vector notation and in magnitude-angle notation?
In the overhead view of Fig. 5-4a, a 2.0 kg cookie tin is accelerated at 3.0 m/s² in the direction shown by \( \vec{a} \), over a frictionless horizontal surface. The acceleration is caused by three horizontal forces, only two of which are shown: \( \vec{F}_1 \) of magnitude 10 N and \( \vec{F}_2 \) of magnitude 20 N. What is the third force \( \vec{F}_3 \) in unit-vector notation and in magnitude-angle notation?

The net force \( \vec{F}_{\text{net}} \) on the tin is the sum of the three forces and is related to the acceleration \( \vec{a} \) via Newton’s second law \( (\vec{F}_{\text{net}} = m\vec{a}) \). Thus,

\[
\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = m\vec{a},
\]

which gives us

\[
\vec{F}_3 = m\vec{a} - \vec{F}_1 - \vec{F}_2.
\]

These are two of the three horizontal force vectors.

This is the resulting horizontal acceleration vector.

We draw the product of mass and acceleration as a vector.

Then we can add the three vectors to find the missing third force vector.
In the overhead view of Fig. 5-4a, a 2.0 kg cookie tin is accelerated at 3.0 m/s² in the direction shown by $\vec{a}$, over a frictionless horizontal surface. The acceleration is caused by three horizontal forces, only two of which are shown: $\vec{F}_1$ of magnitude 10 N and $\vec{F}_2$ of magnitude 20 N. What is the third force $\vec{F}_3$ in unit-vector notation and in magnitude-angle notation?

These are two of the three horizontal force vectors.

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We draw the product of mass and acceleration as a vector.

Then we can add the three vectors to find the missing third force vector.

The net force $\vec{F}_{\text{net}}$ on the tin is the sum of the three forces and is related to the acceleration $\vec{a}$ via Newton’s second law ($\vec{F}_{\text{net}} = m\vec{a}$). Thus,

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = m\vec{a},$$

which gives us

$$\vec{F}_3 = m\vec{a} - \vec{F}_1 - \vec{F}_2.$$

Calculations:

**x components:** Along the x axis we have

$$F_{3,x} = ma_x - F_{1,x} - F_{2,x}$$

$$= m(a \cos 50^\circ) - F_1 \cos(-150^\circ) - F_2 \cos 90^\circ.$$

Then, substituting known data, we find

$$F_{3,x} = (2.0 \text{ kg})(3.0 \text{ m/s}^2) \cos 50^\circ - (10 \text{ N}) \cos(-150^\circ) - (20 \text{ N}) \cos 90^\circ$$

$$= 12.5 \text{ N}.$$

**y components:** Similarly, along the y axis we find

$$F_{3,y} = ma_y - F_{1,y} - F_{2,y}$$

$$= m(a \sin 50^\circ) - F_1 \sin(-150^\circ) - F_2 \sin 90^\circ$$

$$= (2.0 \text{ kg})(3.0 \text{ m/s}^2) \sin 50^\circ - (10 \text{ N}) \sin(-150^\circ) - (20 \text{ N}) \sin 90^\circ$$

$$= -10.4 \text{ N}.$$

**Vector:** In unit-vector notation, we can write

$$\vec{F}_3 = F_{3,x}\hat{i} + F_{3,y}\hat{j} = (12.5 \text{ N})\hat{i} - (10.4 \text{ N})\hat{j}$$

$$\approx (13 \text{ N})\hat{i} - (10 \text{ N})\hat{j}.$$ (Answer)

We can now use a vector-capable calculator to get the magnitude and the angle of $\vec{F}_3$. We can also use Eq. 3-6 to obtain the magnitude and the angle (from the positive direction of the x axis) as

$$F_3 = \sqrt{F_{3,x}^2 + F_{3,y}^2} = 16 \text{ N}$$

and

$$\theta = \tan^{-1} \frac{F_{3,y}}{F_{3,x}} = -40^\circ.$$ (Answer)
Some Particular Forces

Gravitational Force:

- A gravitational force on a body is a certain type of pull that is directed toward a second body.

- On (or close to) the surface of the Earth this pull experienced by a body of mass \( m \) is directed downwards (towards the center of the Earth) and has magnitude

\[
F_g = mg
\]

We call this force the weight, \( W \), of a body. It is equal to the magnitude \( F_g \) of the gravitational force on the body on the surface of the Earth:

\[
W = mg \text{ (weight)},
\]
Normal Force:

When a body presses against a surface, the surface (even a seemingly rigid one) deforms and pushes on the body with a normal force, $F_N$, that is perpendicular to the surface.

In the figure, forces $F_g$ and $F_N$ are the only two forces on the block and they are both vertical. Thus, for the block we can write Newton’s second law for a positive-upward $y$-axis, $(F_{\text{net},y} = ma_y)$, as:

$$F_N - F_g = ma_y.$$  
$$F_N - mg = ma_y.$$  
$$F_N = mg + ma_y = m(g + a_y)$$

for any vertical acceleration $a_y$ of the table and block.

Fig. 5-7 (a) A block resting on a table experiences a normal force perpendicular to the tabletop. (b) The free-body diagram for the block.
Friction

If we either slide or attempt to slide a body over a surface, the motion is resisted by a bonding between the body and the surface.

The resistance is considered to be single force called the frictional force, \( f \). This force is directed along the surface, opposite the direction of the intended motion.

Detailed discussion postponed!!
Tension

When a cord is attached to a body and pulled taut, the cord pulls on the body with a force $T$ directed away from the body and along the cord.

Fig. 5-9 (a) The cord, pulled taut, is under tension. If its mass is negligible, the cord pulls on the body and the hand with force $T$, even if the cord runs around a massless, frictionless pulley as in (b) and (c).
5.8 Newton’s Third Law

When two bodies interact, the forces on the bodies from each other are always equal in magnitude and opposite in direction.

For the book and crate, we can write this law as the scalar relation

\[ F_{BC} = F_{CB} \]  

(equal magnitudes)

or as the vector relation

\[ \vec{F}_{BC} = -\vec{F}_{CB} \]  

(equal magnitudes and opposite directions),

- The minus sign means that these two forces are in opposite directions
- The forces between two interacting bodies are called a **third-law force pair**.
know how to identify third-law pairs!
5.9 Applying Newton’s Laws

Sample Problem

The Figure below shows a block $S$ (the *sliding block*) with mass $M = 3.3$ kg. The block is free to move along a horizontal frictionless surface and connected, by a cord that wraps over a frictionless pulley, to a second block $H$ (the *hanging block*), with mass $m = 2.1$ kg. The cord and pulley have negligible masses compared to the blocks (they are “massless”). The hanging block $H$ falls as the sliding block $S$ accelerates to the right. Find (a) the acceleration of block $S$, (b) the acceleration of block $H$, and (c) the tension in the cord.
The Figure showed a block $S$ with mass $M = 3.3$ kg. The block is free to move along a horizontal frictionless surface and connected, by a cord that wraps over a frictionless pulley, to a second block $H$, with mass $m = 2.1$ kg. The cord and pulley have negligible masses compared to the blocks. The hanging block $H$ falls as the sliding block $S$ accelerates to the right. Find (a) the acceleration of block $S$, (b) the acceleration of block $H$, and (c) the tension in the cord.

Key Ideas:
1. Forces, masses, and accelerations are involved: use Newton’s second law of motion: $\vec{F}_{\text{net}} = m\vec{a}$
2. This is a vector equation, write it as several component equations.
3. Identify the forces acting on each of the bodies and draw free body diagrams.
For the sliding block, $S$, which does not accelerate vertically ($a_y = 0$)

\[
F_{\text{net},y} = M a_y \quad \rightarrow \quad F_N - F_{gS} = 0 \quad \text{or} \quad F_N = F_{gS}.
\]

Also, for $S$, in the $x$ direction,

\[
F_{\text{net},x} = M a_x \quad \rightarrow \quad T = Ma.
\]

For the hanging block, because the acceleration is along the $y$ axis,

\[
T - F_{gH} = ma_y.
\]

With some algebra,

\[
T - mg = -ma.
\]

“Massless” cord and pulley give that $T$ is the same, both ends of cord.

\[
a = \frac{m}{M + m} g = \frac{2.1 \text{ kg}}{3.3 \text{ kg} + 2.1 \text{ kg}} (9.8 \text{ m/s}^2) = 3.8 \text{ m/s}^2
\]

\[
T = \frac{Mm}{M + m} g = \frac{(3.3 \text{ kg})(2.1 \text{ kg})}{3.3 \text{ kg} + 2.1 \text{ kg}} (9.8 \text{ m/s}^2) = 13 \text{ N}
\]
5.9 Applying Newton’s Laws

Sample Problem

In Fig. a, a cord pulls on a box of sea biscuits up along a frictionless plane inclined at $\theta = 30^\circ$. The box has mass $m = 5.00 \text{ kg}$, and the force from the cord has magnitude $T = 25.0 \text{ N}$. What is the box’s acceleration component $a$ along the inclined plane?

For convenience, we draw a coordinate system and a free-body diagram as shown in Fig. b. The positive direction of the $x$ axis is up the plane. Force from the cord is up the plane and has magnitude $T = 25.0 \text{ N}$. The gravitational force is downward and has magnitude $mg = (5.00 \text{ kg})(9.8 \text{ m/s}^2) = 49.0 \text{ N}$. Also, the component along the plane is down the plane and has magnitude $mg \sin \theta$ as indicated in the following figure. To indicate the direction, we can write the down-the-plane component as $-mg \sin \theta$.

Using Newton’s Second Law, we have:

$$T - mg \sin \theta = ma.$$ 

which gives:

$$a = 0.100 \text{ m/s}^2,$$

The positive result indicates that the box accelerates up the plane.
5.9 Applying Newton’s Laws

Sample Problem, Part a

In Fig. 5-17a, a passenger of mass $m = 72.2$ kg stands on a platform scale in an elevator cab. We are concerned with the scale readings when the cab is stationary and when it is moving up or down.

- The scale reading is equal to the magnitude of the normal force on the passenger from the scale.
- We can use Newton’s Second Law only in an inertial frame. If the cab accelerates, then it is not an inertial frame. So we choose the ground to be our inertial frame and make any measure of the passenger’s acceleration relative to it.

In Fig. 5-17b, we can use Newton’s second law written for $y$ components ($F_{\text{net},y} = ma_y$) to get:

$$F_N - F_g = ma$$

$$F_N = F_g + ma.$$  \hspace{1cm} \text{Calculations:}

This tells us that the scale reading, which is equal to $F_N$, depends on the vertical acceleration. Substituting $mg$ for $F_g$ gives us

$$F_N = m(g + a) \quad \text{Answer}$$  \hspace{1cm} (5-28)

for any choice of acceleration $a$.

(a) Find a general solution for the scale reading, whatever the vertical motion of the cab.
In Fig. 5-17a, a passenger of mass \( m = 72.2 \text{ kg} \) stands on a platform scale in an elevator cab. We are concerned with the scale readings when the cab is stationary and when it is moving up or down.

**(b)** What does the scale read if the cab is stationary or moving upward at a constant \( 0.50 \text{ m/s} \)?

For any constant velocity (zero or otherwise), the acceleration \( a \) of the passenger is zero.

**Calculation:** Substituting this and other known values into Eq. 5-28, we find

\[
F_N = (72.2 \text{ kg})(9.8 \text{ m/s}^2 + 0) = 708 \text{ N}.
\]

(Answer)

This is the weight of the passenger and is equal to the magnitude \( F_g \) of the gravitational force on him.
In Fig. 5-17a, a passenger of mass \( m = 72.2 \text{ kg} \) stands on a platform scale in an elevator cab. We are concerned with the scale readings when the cab is stationary and when it is moving up or down.

(c) What does the scale read if the cab accelerates upward at 3.20 m/s\(^2\) and downward at 3.20 m/s\(^2\)?

**Calculations:** For \( a = 3.20 \text{ m/s}^2 \), Eq. 5-28 gives

\[
F_N = (72.2 \text{ kg})(9.8 \text{ m/s}^2 + 3.20 \text{ m/s}^2) = 939 \text{ N},
\]

(Answer)

and for \( a = -3.20 \text{ m/s}^2 \), it gives

\[
F_N = (72.2 \text{ kg})(9.8 \text{ m/s}^2 - 3.20 \text{ m/s}^2) = 477 \text{ N}.
\]

(Answer)

(d) During the upward acceleration in part (c), what is the magnitude \( F_{\text{net}} \) of the net force on the passenger, and what is the magnitude \( a_{p,\text{cab}} \) of his acceleration as measured in the frame of the cab? Does \( \vec{F}_{\text{net}} = m\vec{a}_{p,\text{cab}} \)?

**Calculation:** The magnitude \( F_g \) of the gravitational force on the passenger does not depend on the motion of the passenger or the cab; so, from part (b), \( F_g \) is 708 N. From part (c), the magnitude \( F_N \) of the normal force on the passenger during the upward acceleration is the 939 N reading on the scale. Thus, the net force on the passenger is

\[
F_{\text{net}} = F_N - F_g = 939 \text{ N} - 708 \text{ N} = 231 \text{ N},
\]

(Answer) during the upward acceleration. However, his acceleration \( a_{p,\text{cab}} \) relative to the frame of the cab is zero. Thus, in the non-inertial frame of the accelerating cab, \( F_{\text{net}} \) is not equal to \( ma_{p,\text{cab}} \), and Newton's second law does not hold.