

PHYSICS 210A : STATISTICAL PHYSICS
HW ASSIGNMENT #7

(1) For each of the two cluster diagrams in Fig. 1, find the symmetry factor s_γ and write an expression for the cluster integral $b_\gamma(T)$.

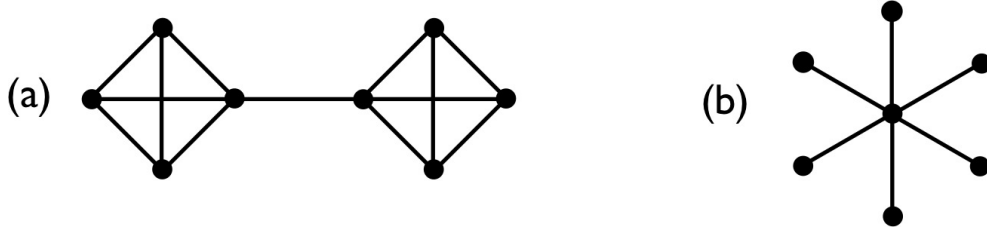


Figure 1: Mayer cluster expansion diagrams.

(2) Consider the one-dimensional Ising model with next-nearest neighbor interactions,

$$\hat{H} = -J \sum_n \sigma_n \sigma_{n+1} - K \sum_n \sigma_n \sigma_{n+2},$$

on a ring with N sites, where N is even. By considering consecutive pairs of sites, show that the partition function may be written in the form $Z = \text{Tr}(R^{N/2})$, where R is a 4×4 transfer matrix. Find R . *Hint:* It may be useful to think of the system as a railroad trestle, depicted in Fig. 2, with Hamiltonian

$$\hat{H} = - \sum_j \left[J \sigma_j \mu_j + J \mu_j \sigma_{j+1} + K \sigma_j \sigma_{j+1} + K \mu_j \mu_{j+1} \right].$$

Then $R = R_{(\sigma_j \mu_j), (\sigma_{j+1} \mu_{j+1})}$, with $(\sigma \mu)$ a composite index which takes one of four possible values $(++)$, $(+-)$, $(-+)$, or $(--)$.

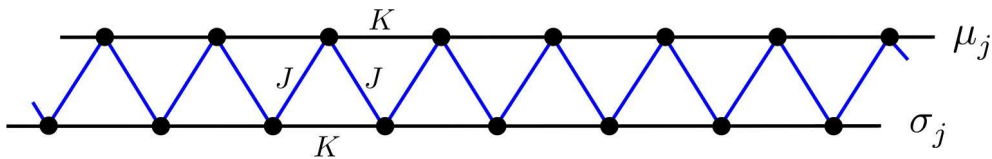


Figure 2: Railroad trestle representation of next-nearest neighbor chain.