**PHYSICS 210A : STATISTICAL PHYSICS**
**HW ASSIGNMENT #6**

**(1)** In our derivation of the low temperature phase of an ideal Bose condensate, we split off the lowest energy state $\varepsilon_0$ but treated the remainder as a continuum, taking $\mu = 0$ in all expressions relating to the overcondensate. Under what conditions is this justified? *I.e.* why are we not obligated to separately consider the contributions from the first excited state, *etc.*?

**(2)** Consider a three-dimensional Bose gas of particles which have two internal polarization states, labeled by $\sigma = \pm 1$. The single particle energies are given by

$$\varepsilon(\boldsymbol{p}, \sigma) = \frac{\boldsymbol{p}^2}{2m} + \sigma\Delta \ ,$$

where $\Delta > 0$.

(a) Find the density of states per unit volume $g(\varepsilon)$.

(b) Find an implicit expression for the condensation temperature $T_{\text{c}}(n, \Delta)$. When $\Delta \to \infty$, your expression should reduce to the familiar one derived in class.

(c) When $\Delta = \infty$, the condensation temperature should agree with the familiar result for three-dimensional Bose condensation. Assuming $\Delta \ll k_{\text{B}} T_{\text{c}}(n, \Delta = \infty)$, find analytically the leading order difference $T_{\text{c}}(n, \Delta) - T_{\text{c}}(n, \Delta = \infty)$.

**(3)** For an ideal Fermi gas in three dimensions,

(a) Find an expression for the isothermal compressibility $\kappa_{T,N}$ as a function of the temperature $T$ and fugacity $z$.

(b) Find an expression for the adiabatic compressibility $\kappa_{S,N}$ as a function of the temperature $T$ and fugacity $z$.

(c) Find an expression for the ratio $C_{p,N}/C_{V,N}$ as a function of the temperature $T$ and fugacity $z$.

**(4)** At low energies, the conduction electron states in graphene can be described as fourfold degenerate fermions with dispersion $\varepsilon(\boldsymbol{k}) = \hbar v_{\text{F}} |\boldsymbol{k}|$. Using the Sommerfeld expension,

(a) Find the density of single particle states $g(\varepsilon)$.

(b) Find the chemical potential $\mu(T, n)$ up to terms of order $T^4$.

(c) Find the energy density $\mathcal{E}(T, n) = E/V$ up to terms of order $T^4$.