PHYSICS 210A : STATISTICAL PHYSICS
HW ASSIGNMENT #3

(1) Consider an ultrarelativistic ideal gas in three space dimensions. The dispersion is $\varepsilon(\boldsymbol{p}) = pc$.

(a) Find E, T, p, and μ within the microcanonical ensemble (variables S, V, N).

(b) Find F, S, p, and μ within the ordinary canonical ensemble (variables T, V, N).

(c) Find Ω, S, p, and N within the grand canonical ensemble (variables T, V, μ).

(d) Find G, S, V, and μ within the Gibbs ensemble (variables T, p, N).

(e) Find H, T, V, and μ within the S-p-N ensemble. Here $\mathsf{H} = E + pV$ is the enthalpy.

(2) Consider a surface containing N_{s} adsorption sites which is in equilibrium with a two-component nonrelativistic ideal gas containing atoms of types A and B . (Their respective masses are m_{A} and m_{B}). Each adsorption site can be in one of three possible states: (i) vacant, (ii) occupied by an A atom, with energy $-\Delta_{\mathsf{A}}$, and (ii) occupied with a B atom, with energy $-\Delta_{\mathsf{B}}$.

(a) Find the grand partition function for the surface, $\Xi_{\text{surf}}(T, \mu_{\mathsf{A}}, \mu_{\mathsf{B}}, N_{\mathsf{s}})$.

(b) Suppose the number densities of the gas atoms are n_{A} and n_{B}. Find the fraction $f_{\mathsf{A}}(n_{\mathsf{A}}, n_{\mathsf{B}}, T)$ of adsorption sites with A atoms, and the fraction $f_0(n_{\mathsf{A}}, n_{\mathsf{B}}, T)$ of adsorption sites which are vacant.

(3) Consider a system composed of spin tetramers, each of which is described by the Hamiltonian

$$\hat{H} = -J(\sigma_1\sigma_2 + \sigma_1\sigma_3 + \sigma_1\sigma_4 + \sigma_2\sigma_3 + \sigma_2\sigma_4 + \sigma_3\sigma_4) - \mu_0 H(\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4) \ .$$

The individual tetramers are otherwise noninteracting.

(a) Find the single tetramer partition function ζ.

(b) Find the magnetization per tetramer $m = \mu_0 \langle \sigma_1 + \sigma_2 + \sigma_3 + \sigma_4 \rangle$.

(c) Suppose the tetramer number density is n_{t}. The magnetization density is $M = n_{\mathsf{t}} m$. Find the zero field susceptibility $\chi(T) = (\partial M / \partial H)_{H=0}$.