PHYSICS 2D - Relativity and Quantum Mechanics
Summer Session II, 2011
Final exam

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Please read carefully before beginning!

- You have 180 minutes to complete the exam.
- Write your name **and** 3-digit code number on your bluebook.
- You are not allowed to have any materials other than a bluebook, a pen and/or a pencil, and a calculator. Equations are provided below.
- Write clearly and show your work. You will not receive credit for your answers unless you show your work, even if the answers are correct. Integrals taken with calculators will not receive credit.
- Even if you are unable to solve a problem completely, do as much work as you can. You will receive partial credit for all the work that is relevant to the problem (just copying the relevant equation from the equation sheet will not get any credit).
- Each problem is worth 5 points.

Best of luck!

Useful equations:

Length contraction:
$L = L_p\sqrt{1-u^2/c^2}$

Time dilation:
$t = \frac{t_p}{\sqrt{1-u^2/c^2}}$

Doppler effect:
$f_{obs} = f_{source}\frac{\sqrt{c+v}}{\sqrt{c-v}}$
where $v > 0$ when approaching

Lorentz transformations:

$t = \frac{t'+x'u/c^2}{\sqrt{1-u^2/c^2}}$

$x = \frac{x'+ut'}{\sqrt{1-u^2/c^2}}$

$y = y'$

$z = z'$

$v_x = \frac{v'_x+u}{1+v'_x u/c^2}$

$v_y = \frac{v'_y\sqrt{1-u^2/c^2}}{1+v'_x u/c^2}$

$v_z = \frac{v'_z\sqrt{1-u^2/c^2}}{1+v'_x u/c^2}$

$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2$

Energy and momentum:

$\vec{p} = \frac{m\vec{v}}{\sqrt{1-v^2/c^2}}$

$E = \frac{mc^2}{\sqrt{1-v^2/c^2}}$

$K = E - mc^2$

$E^2 = p^2c^2 + m^2c^4$

$\vec{v} = \frac{\vec{p}}{E}c^2$

$Mc^2 + BE = \sum_{i=1}^{n} m_i c^2$

General relativity:

$\Delta f = f_0\frac{gH}{c^2}$

$\Delta f = \frac{Gf_0}{c^2}\left(\frac{M_{earth}}{R_{earth}} - \frac{M_\star}{R_\star}\right)$

$\Delta t = t_0\frac{gH}{c^2}$

Blackbody radiation:	$u(\lambda, T) = \frac{8\pi hc}{\lambda^5\left(e^{hc/\lambda k_B T} - 1\right)}$	$e = u\frac{c}{4}$
$e_{total} = \sigma T^4$	$\lambda_{max} T = 2.892 \times 10^{-3} m \cdot K$	
Photoelectric effect:	$eV_s = K_{max} = hf - \phi$	$f_0 = \frac{\phi}{h}$
Compton effect:	$\lambda' - \lambda_0 = \frac{h}{m_e c}(1 - \cos\theta)$	
Rutherford scattering:	$\Delta n \propto \sin^{-4}(\phi/2)$	$K_\alpha = k\frac{2eZe}{d_{min}}$
Bohr model:	$L_n = n\hbar$	$n = 1, 2, 3, ...$
$E_n = \frac{1}{2}U_n$	$r_n = \frac{n^2 a_0}{Z}$	$a_0 = \frac{\hbar^2}{m_e k e^2} = 0.0529nm$
$E_n = -\frac{Z^2 E_0}{n^2}$	$E_0 = \frac{m_e k^2 e^4}{2\hbar^2} = 13.6eV$	$\frac{1}{\lambda} = Z^2 R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$
$R = \frac{E_0}{hc} \approx 1.097 \times 10^7 m^{-1}$	Lyman: $n_f = 1$	Balmer: $n_f = 2$
Reduced mass:	$\mu = \frac{m_e M}{m_e + M}$	
Wave-particle duality:	$E = hf$	$p = \frac{h}{\lambda}$
Group and phase velocity:	$v_g = \frac{\Delta\omega}{\Delta k} = \frac{d\omega}{dk}$	$v_p = \frac{\omega}{k}$
Uncertainty principle:	$\Delta k \Delta x \geq \frac{1}{2}$	$\Delta\omega\Delta t \geq \frac{1}{2}$
	$\Delta p \Delta x \geq \frac{\hbar}{2}$	$\Delta E \Delta t \geq \frac{\hbar}{2}$
Wavefunction:	$P(x) = \lvert\psi(x)\rvert^2$	$\int_{-\infty}^{\infty} \lvert\psi(x)\rvert^2 dx = 1$
Operators:	$[x] = x$	$[p] = -i\hbar\frac{d}{dx}$
Average and uncertainty:	$< O >= \int_{-\infty}^{\infty} dx \psi^*(x)[O]\psi(x)$	$\Delta O = \sqrt{< O^2 > - < O >^2}$
Eigenstate:	$[O]\psi(x) = O_0\psi(x)$	
Schrödinger equation:	$[H]\psi(x) = E\psi(x)$	$-\frac{\hbar^2}{2m}\psi''(x) + U(x)\psi(x) = E\psi(x)$
Infinite square well:	$E_n = \frac{\pi^2 n^2 \hbar^2}{2mL^2}$	$\psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{\pi n}{L}x\right)$
		$n = 1, 2, 3, ...$
Harmonic oscillator:	$E_n = \hbar\omega\left(n + \frac{1}{2}\right)$	$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega x^2}{2\hbar}}$
		$n = 0, 1, 2, ...$
Time dependence:	$[H]\Psi(x, t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$	$\Psi_n(x, t) = \psi_n(x)e^{-\frac{iE_n t}{\hbar}}$
Step potential:	$k_1 = \frac{\sqrt{2mE}}{\hbar}$	$k_2 = \frac{\sqrt{2m(E-U_0)}}{\hbar}$
	$R = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2}$	$T = 1 - R$
Tunneling:	$\alpha = \frac{\sqrt{2m(U_0 - E)}}{\hbar}$	$T \sim e^{-2\alpha l}$
	$T \sim e^{-\frac{2\sqrt{2m}}{\hbar}\int_a^b \sqrt{U(x)-E}dx}$	$U(a) = U(b) = E$
$3D$ square well:	$E = \frac{\pi^2\hbar^2}{2m}\left(\frac{n_1^2}{L_1^2} + \frac{n_2^2}{L_2^2} + \frac{n_3^2}{L_3^2}\right)$	$\psi(x, y, z) = \sqrt{\frac{2}{L_1}}\sqrt{\frac{2}{L_2}}\sqrt{\frac{2}{L_3}}\sin\left(\frac{\pi n_1}{L_1}x\right)$
		$\times\sin\left(\frac{\pi n_2}{L_2}y\right)\sin\left(\frac{\pi n_3}{L_3}z\right)$

Angular momentum:	$[L_z] = -i\hbar\frac{\partial}{\partial\phi}$	$[L]^2 = -\hbar^2\left(\frac{\partial^2}{\partial\theta^2} + \cot\theta\frac{\partial}{\partial\theta} + \csc^2\theta\frac{\partial^2}{\partial\phi^2}\right)$
	$L_z = m_l\hbar$	$\lvert L\rvert = \sqrt{l(l+1)}\hbar$
Radial wavefunction:	$\int_0^\infty r^2\lvert R(r)\rvert^2\, dr = 1$	$Prob(a,b) = \int_a^b r^2\lvert R(r)\rvert^2\, dr$
	$< f(r) >= \int_0^\infty r^2\lvert R(r)\rvert^2 f(r)\, dr$	
Hydrogen atom:	$U(r) = -\frac{kZe^2}{r}$	
	$E_n = -\frac{Z^2E_0}{n^2}$	$E_0 = \frac{ke^2}{2a_0} = \frac{m_ek^2e^4}{2\hbar^2} = 13.6eV$
Zeeman effect:	$\Delta E = \mu_B B m_l$	$\mu_B = \frac{e\hbar}{2m_e} = 5.8\times10^{-5}eV/T$
Spin:	$\lvert S\rvert = \sqrt{s(s+1)}\hbar$	$S_z = m_s\hbar$
	$\Delta E_s = 2\mu_B B m_s$ (electron)	
Periodic table:	$1s < 2s < 2p < 3s < 3p < 4s...$	
$c = 3\times10^8 m/s$	$h = 6.63\times10^{-34}m^2kg/s$	$G = 6.67\times10^{-11}m^3kg^{-1}s^{-2}$
$k_B = 1.38\times10^{-23}J/K$	$\sigma = 5.67\times10^{-8}W\cdot m^{-2}\cdot K^{-4}$	
$hc = 1240eV\cdot nm$	$ke^2 = 1.44eV\cdot nm$	$\hbar = \frac{h}{2\pi}$