Homework 1

Due: August 5, 2011 (5 pm)

1. The initial position, velocity, and acceleration of an object moving in simple harmonic motion are x_i, v_i, and a_i; the angular frequency of oscillation is ω. (a) Show that the position and velocity of the object for all time can be written as

$$\begin{aligned} x(t) &= x_i \cos(\omega t) + \frac{v_i}{\omega} \sin(\omega t) \\ v(t) &= -x_i \omega \sin(\omega t) + v_i \cos(\omega t) \end{aligned}$$

   (b) Using A to represent the amplitude of the motion, show that

$$v^2 - ax = v_i^2 - a_i x_i = \omega^2 A^2$$

2. The amplitude of a system moving in simple harmonic motion is doubled. Determine the change in (a) the total energy, (b) the maximum speed, (c) the maximum acceleration, and (d) the period. what position does its speed equal half of its maximum speed?

3. A simple pendulum has a mass of 0.250 kg and a length of 1.00 m. It is displaced through an angle of 15.0° and then released. What are (a) the maximum speed, (b) the maximum angular acceleration, and (c) the maximum restoring force? Solve this problem once by using the simple harmonic motion model for the motion of the pendulum, and then solve the problem more precisely by using more general principles.

4. A very light rigid rod with a length of 0.500 m extends straight out from one end of a meter stick. The stick is suspended from a pivot at the far end of the rod and is set into oscillation. (a) Determine the period of oscillation. (Suggestion: Use the parallel-axis theorem from Section 10.11.) (b) By what percentage does the period differ from the period of a simple pendulum 1.00 m long?

5. Show that the time rate of change of mechanical energy for a damped, undriven oscillator is given by $dE/dt = bv^2$ and hence is always negative. (Suggestion: Differentiate the expression for the mechanical energy of an oscillator, $E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$, and use Equation 12.28.)

6. Considering an undamped, forced oscillator ($b = 0$), show that Equation 12.32 is a solution of Equation 12.31, with an amplitude given by Equation 12.33.

7. An object of mass $m_1 = 9.00\ kg$ is in equilibrium while connected to a light spring of constant k =100 N/m that is fastened to a wall as shown in Figure 1a. A second object, $m_2 = 7.00\ kg$, is slowly pushed up against m_1, compressing the spring by the amount $A = 0.200$ m (see Fig. 1b). The system is then released and both objects start moving to the right on the frictionless surface. (a) When m_1 reaches the equilibrium point, m_2 loses contact with m_1 (see Fig. 1c) and moves to the right with speed v. Determine the value of v. (b) How far apart are the objects when the spring is fully stretched for the first time (D in Fig. 1d)? (Suggestion: First determine the period of oscillation and the amplitude of the $m_1$spring system after m_2 loses contact with m1.)

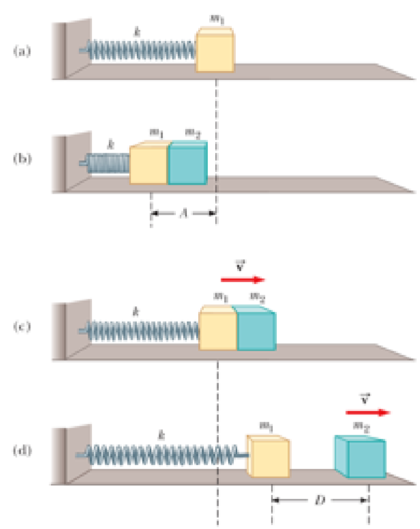

Figure 1:

8. A wave is described by $y = (2.00\ cm)\sin(kx - \omega t)$, where $k = 2.11$ rad/m, $\omega = 3.62\ rad/s$, x is in meters, and t is in seconds. Determine the amplitude, wavelength, frequency, and speed of the wave.

9. Transverse waves travel with a speed of 20.0 m/s in a string under a tension of 6.00 N. What tension is required for a wave speed of 30.0 m/s in the same string?

10. A series of pulses, each of amplitude 0.150 m, are sent down a string that is attached to a post at one end. The pulses are reflected at the post and travel back along the string without loss of amplitude. When two waves are present on the same string, the net displacement of a particular element of the string is the sum of the displacements of the individual waves at that point. What is the net displacement of an element at a point on the string where two pulses are crossing (a) if the string is rigidly attached to the post and (b) if the end at which reflection occurs is free to slide up and down?

11. It is found that a 6.00-m segment of a long string contains four complete waves and has a mass of 180 g. The string is vibrating sinusoidally with a frequency of 50.0 Hz and a peak-to-valley displacement of 15.0 cm. (The peak-to-valley distance is the vertical distance from the farthest positive position to the farthest negative position.) (a) Write the function that describes this wave traveling in the positive x direction. (b) Determine the power being supplied to the string.

12. An ultrasonic tape measure uses frequencies above 20 MHz to determine dimensions of structures such as buildings. It does so by emitting a pulse of ultrasound into air and then measuring the time interval for an echo to return from a reflecting surface whose distance away is to be measured. The distance is displayed as a digital readout. For a tape measure that emits a pulse of ultrasound with a frequency of 22.0 MHz, (a) what is the distance to an object from which the echo pulse returns after 24.0 ms when the air temperature is $26°C$? (b) What should be the duration of the emitted pulse if it is to include ten cycles of the ultrasonic wave? (c) What is the spatial length of such a pulse?

13. A block with a speaker bolted to it is connected to a spring having spring constant $k = 20.0\ N/m$ as shown in Figure 2. The total mass

of the block and speaker is 5.00 kg, and the amplitude of this units motion is 0.500 m. The speaker emits sound waves of frequency 440 Hz. Determine the highest and lowest frequencies heard by the person to the right of the speaker. Assume that the speed of sound is 343 m/s.

Figure 2:

14. Assume that an object of mass M is suspended from the bottom of the rope in Problem 13.47. (a) Show that the time interval for a transverse pulse to travel the length of the rope is

$$\Delta = 2\sqrt{\frac{L}{mg}}\left(\sqrt{M+m} - \sqrt{M}\right)$$

(b) Show that this expression reduces to the result of Problem 13.47 when $M = 0$. (c) Show that for $m << M$, the expression in part (a) reduces to

$$\Delta t = \sqrt{\frac{mL}{Mg}}$$