Chapter 30

1896 → discovery of radioactivity

Rutherford → 3 radiation types:

- α (He nucleus)
- β (e^-)
- γ (high-E photons)

1911 → Rutherford, Geiger & Marsden

→ scattering exp.

* Nuc. → point mass, point charge
* Most atomic mass in nucleus
* Nucleus force → new force

Nuclei:

$P + N$ ← neutral

+ ch. → P

$Z = \#P$

Atomic #, charge #

$N \equiv$ # neutrons

neutron number

$A = Z + N$

mass #

Nucleon → either n or p

mass # $\neq$ total mass

${}^{A}_{Z}X$ ← chemical sym.

mass #

${}^{56}_{26}Fe \rightarrow$ 26 protons

$30 = 56 - 26$ neutrons

A. #

* Z can be omitted → given by element

* Nuclei of ~~all~~ atom of same element have same Z

* A might be diff.
  → diff. # n, N
  → Isotopes   same Z, diff N & A

$$^{11}_{6}C \qquad ^{12}_{6}C \qquad ^{13}_{6}C \qquad ^{14}_{6}C$$

$$\left.\begin{array}{l} \text{Charge of } p = +e \\ \text{" " } e^- = -e \\ \text{" " } n = 0 \end{array}\right\} e = 1.60\times10^{-19}\ C$$

Mass:

$1\ u = 1.66 \times 10^{-27}$ kg

atomic mass unit

(C-12 $\rightarrow$ 12 u)

$E = mc^2 \longrightarrow$ can use MeV/c^2

$\Rightarrow\ 1\ u = 931.494\ MeV/c^2$

Size:

Rutherford $\rightarrow$ scattering exp.

found an expression for how close α-particles get to nucleus before they turn around.

$E = K$ $\qquad\qquad$ $E = U$

$$d = \frac{4 k_e Z e^2}{m v^2}$$

dist. of closest approach

d gives an upp. lim. for the size of the nucleus

e.g. gold 3.2×10^{-14} m

silver 2×10^{-14} m

→ conc.: + charge concentrated in a sphere w $r < 10^{-15}$ m

$= 1$ fm

He called it nucleus

Also called fm 1 fermi

* Most nuclei are approx. sph.

* $\boxed{r \approx r_0 A^{1/3}}$

  where $r_0 = 1.2 \times 10^{-15}$ m

Density of Nuclei:

* proportional to # of nucleons

* All nuclei → approx. same density

Stability:

* Large electrostatic repulsion between protons

  ← ⊕ ⊕ →

*

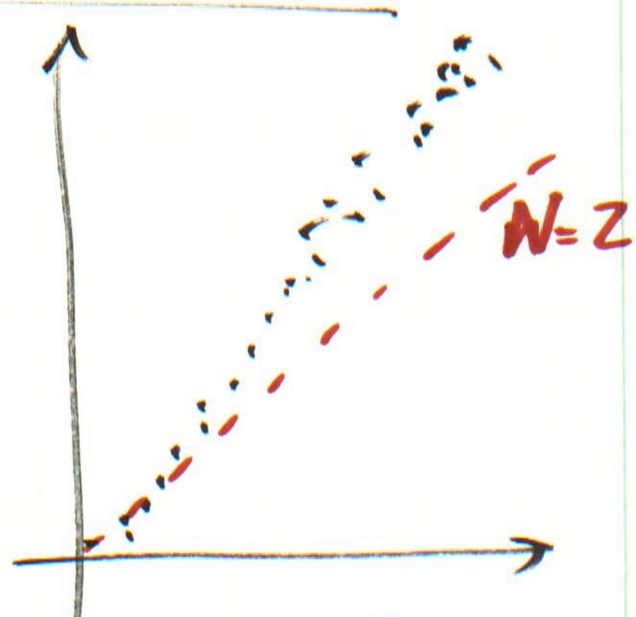
* Short range force → nuclear force

  - attractive force → nucleus
  - stronger than e.s. repulsion in short range

Nuclear Force

* Strongest force in nature
* Very short range
  - falls to zero when $r \gtrsim 1$ fm
* Ind. of charge
  (p-p, p-n, n-n)
* Does not affect e^-

Stability:



light nuclei → N=Z

Heavy nuclei N>Z
(Z>20)

more p → larger Coulomb repulsion
→ more n need to keep the nucleus stable (All unst. Z>83)

Magic #s:

Stable nuclei:

- even A
- Certain val. of N & Z

$N/Z = 2, 8, 20, 28, 50, 82, 126$

Nuclear Spin

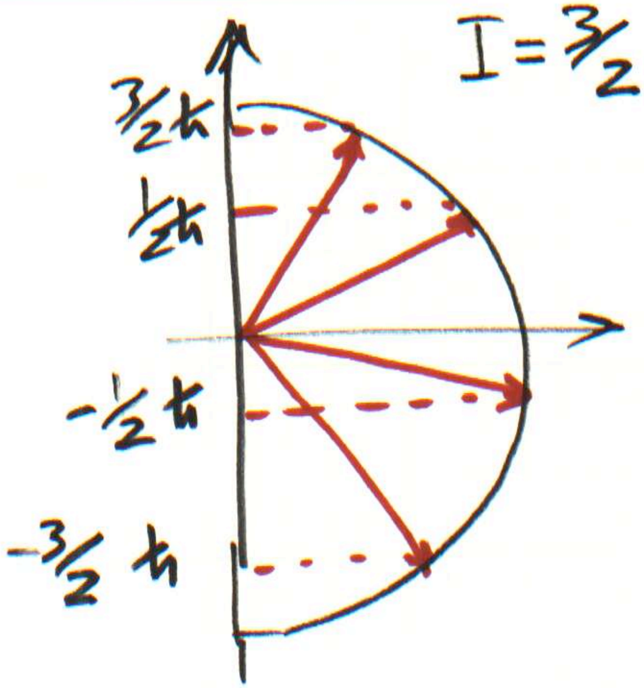
* P & n have intrinsic ang. mom. (spin)

* Nucleus → spin of n's + spin of p's

* Nucleus spin obeys the same quantum rules

* Mag. of spin:

$\sqrt{l(l+1)}\,\hbar$ Nuclear spin quantum # (int. or ½-int.)



Nuclear Magneton:

* Nuclear ang. mom. has a nuclear mag. mom. associated w/ it.

* $$\boxed{\mu_n \equiv \frac{e\hbar}{2m_p} = 5.05\times 10^{-27}\ J/T}$$

nuclear magneton

$$\mu_B \approx 2{,}000\,\mu_n$$

Bohr's magneton

* magnetic mom. of a free proton $= 2.7928\ \mu_n$

→ not explained by a gen. theory yet

* neutron has a magnetic mom.

⇒ $-1.9135\ \mu_n$

opp. to spin

→ means their sub-structure

for neutrons → quarks

<u>Nuclear Magnetic Resonance</u>

mag mom. quantized

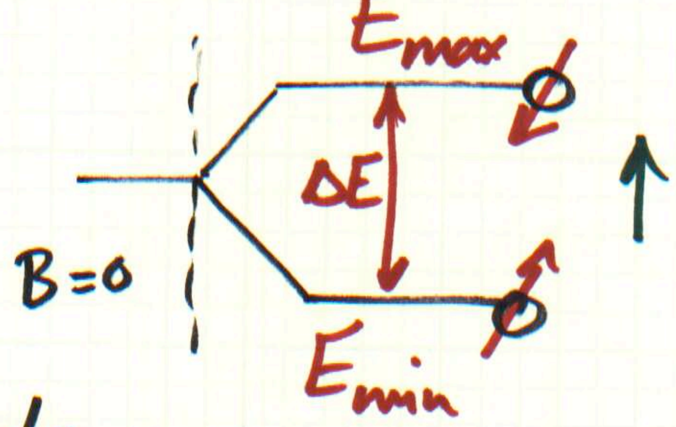
⇒ Es are also quantized

Spin vec. cannot align exactly

w/ ~~the~~ $\vec{B}$

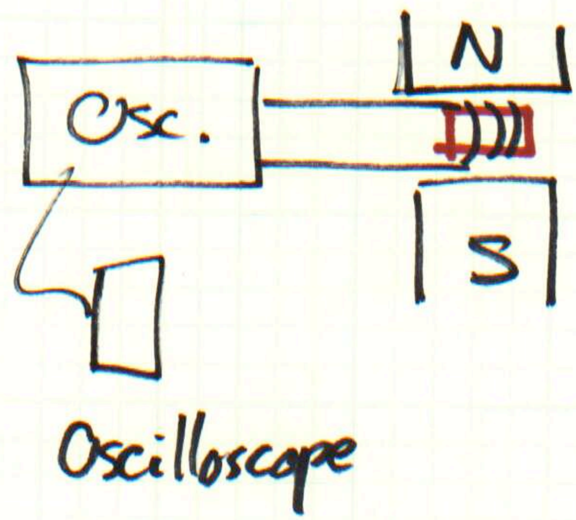
⇒ E values → $\pm \mu_z B$

These are called spin states:



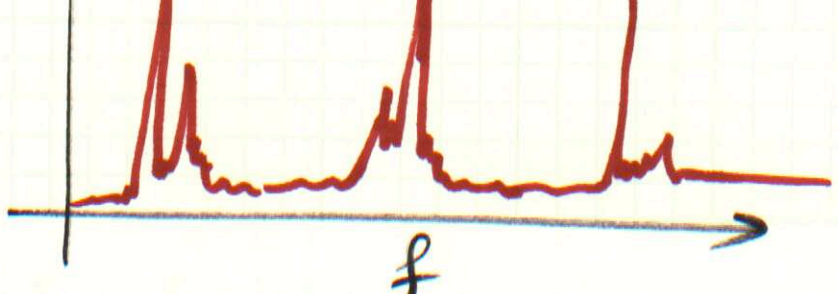
* B splits states
* transition can be observed using NMR

* Sample irradited w/ EM waves



* The freq. of EM waves is matched w/ sep. of spin states

*



~~MRI~~

MRI:

* Uses NMR
* protons in diff. parts of body have diff. ΔE ← splitting E
* Resonance signal provides info about the position of protons.

Binding E

total E of the bound state < Combined $\overset{\text{rest}}{E}$ of nucleons

⇒ diff in E ≡ binding E

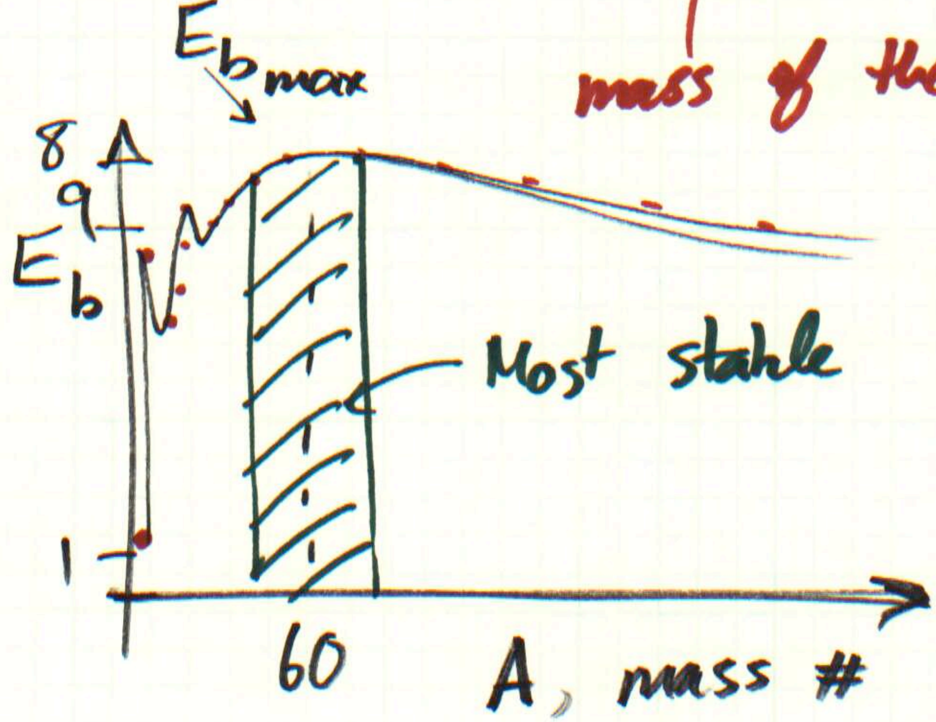
→ the amount of E to sep. nucleons

$$E_b(\text{MeV}) = \left[Z\,\underline{M(H)} + N\,\underline{m_n} - M(^{A}_{Z}X)\right] \times 931.494\ \text{MeV}/u$$

atomic mass of H-atom

mass of neutron

mass of the isotope



For $A > 56 \rightarrow E_b$ saturates

$\rightarrow$ A nuclear can interact only w/ certain # of nuclears

Radioactivity

Spontaneous emission of radiation

1896 → Bacquerel

experiments → Curies + Bacquerel

$\xrightarrow{\text{exp.s}}$ radioactivity

the result of the decay or disintegration of unstable nuclei

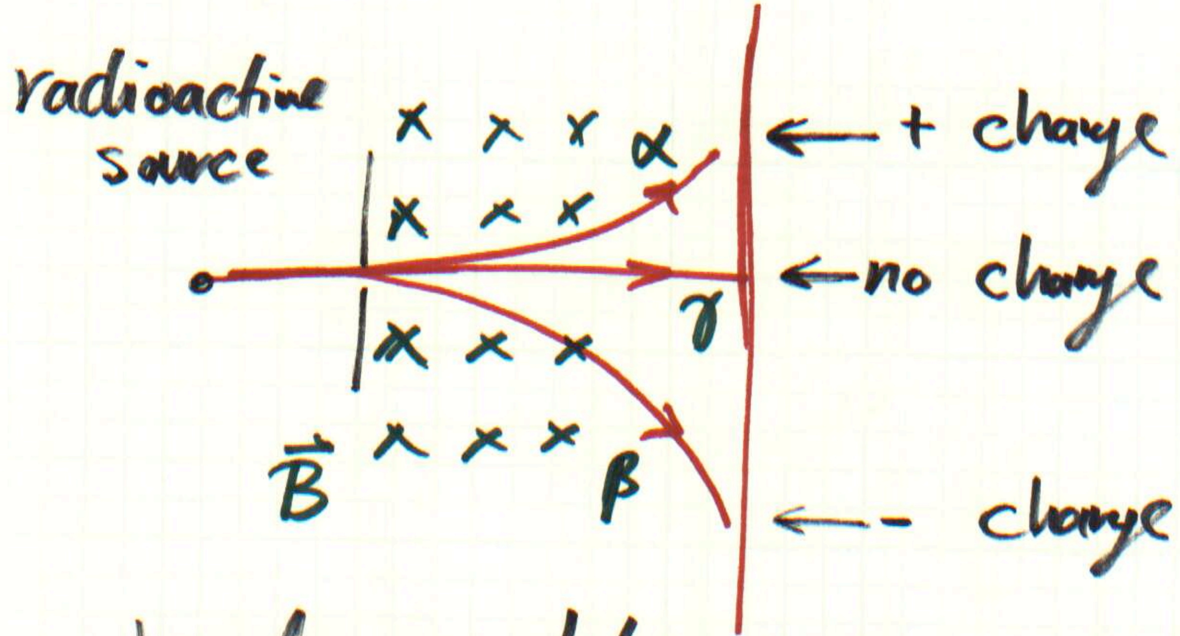
Types:

α particle : ^{4}He nuclei

β " : e^- or ~~p~~ e^+

antimatter of e^-

γ ray : high E photon



α: barely penetrates a piece of paper

β: penetrates a few mm of al.

γ: penetrates several cm. of lead

<u>Decay Const.</u> decay const.

undecayed nuc. at t

$$\boxed{\frac{dN}{dt} = -\lambda N} \Longrightarrow \boxed{N(t) = N_0 e^{-\lambda t}}$$

undecayed nuclei at $t=0$

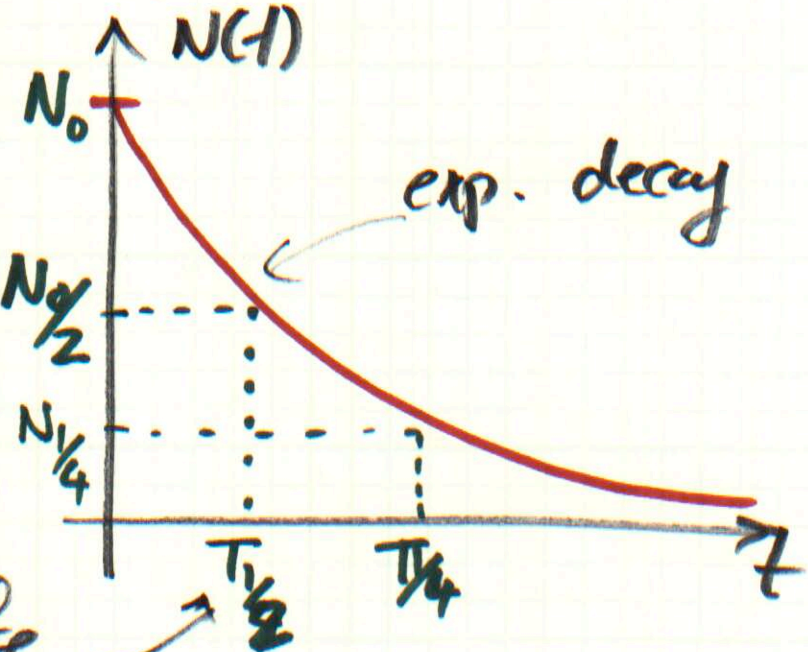
rate of decay proportional to radioactive nuclei present

$$\boxed{R = \left|\frac{dN}{dt}\right| = \underbrace{N_0 \lambda}_{R_0} e^{-\lambda t} = R_0 e^{-\lambda t}}$$

Decay rate

$\equiv$

activity of a sample

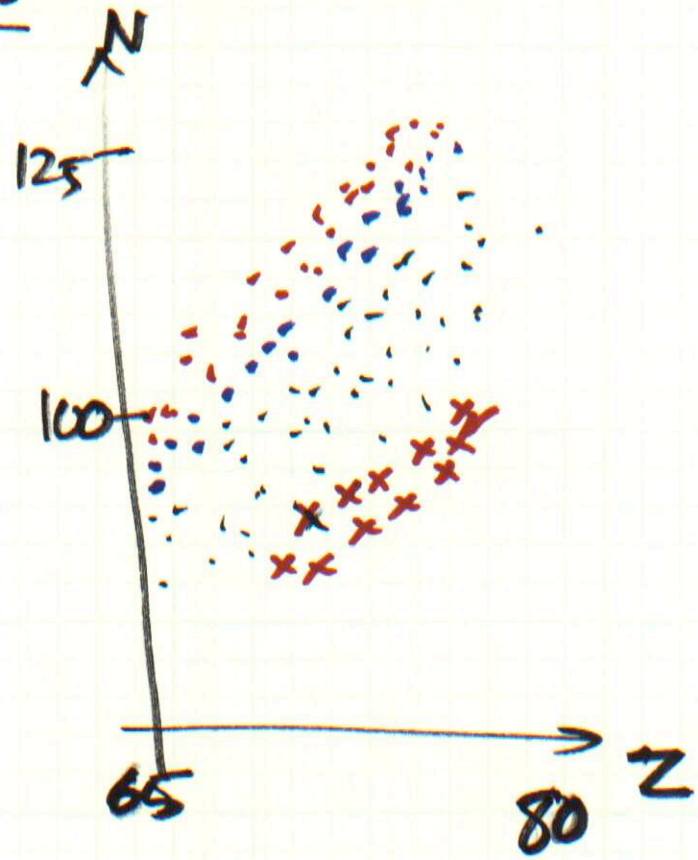


half-life

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

Decay Processes

- Stable
- $\beta\ (e^-)$
- $\beta\ (e^+)$
- × α



α-Decay

N dec. by 2

Z " " 2

$\Rightarrow$ A " " 4

$${}^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$$

$\rightarrow$ β- particle

Parent nucleus

daughter nucleus

* One element changes into another element $^{A}_{Z}X \longrightarrow ^{A-4}_{Z-2}Y + ^{4}_{2}He$

* ~~A~~ Conserved #s:
  A , N, Z

* E is also conserved

Disintegration E:

$$\boxed{Q = (M_X - M_Y - M_\alpha)c^2}$$

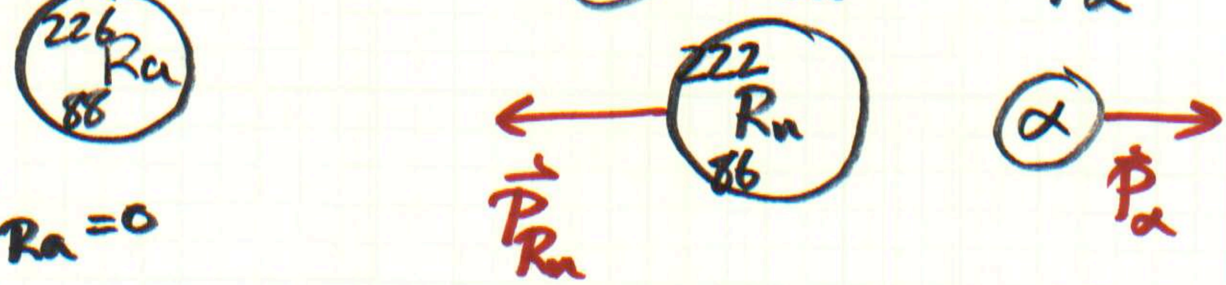
→ Q appears in form of K.E. of daughter nucleus & ~~the~~ α-particle

Also referred to as Q-value of nuclear decay

Example:

$${}^{226}_{88}Ra \longrightarrow {}^{222}_{86}Rn + {}^{4}_{2}He$$

① ② K_{Rn} K_α

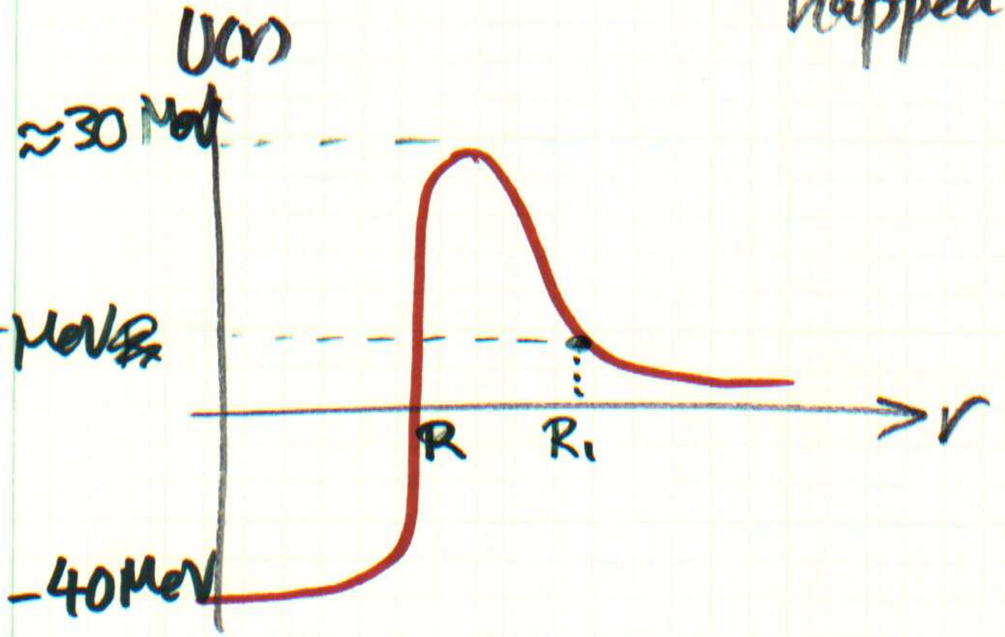


$K_{Ra} = 0$

$\vec{P}_{Ra} = 0$

- ${}^{226}Ra$ at rest $\longrightarrow$ $K_{Rn} \sim 4.87$ MeV
- In general less massive particles carry most of the K.E.
- Observations $\longrightarrow$ A number of discrete energies
  - × daughter may be in excited state
  - × Not all E goes to K.E.

Neg. Q means → decay does not happen spontaneously



- α-particle → tunnels

- For higher E particles
  → barrier is narrower
  ⇒ shorter ½-life

β-Decay

$${}^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}X + e^{-}$$

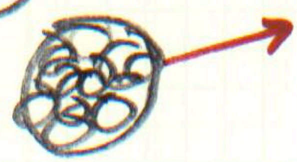
$${}^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}X + e^{+}$$

parent & mother nucleus have the same A.

This is not the complete picture

①

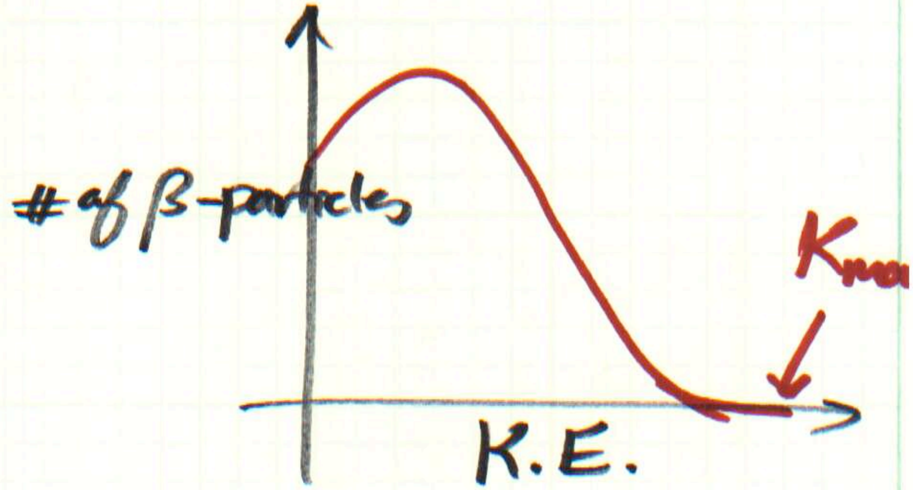
②



$\bar{e}/e^{+}$

- Emission of $e^{\pm}$ is from nucleus
- $\begin{cases} n \longrightarrow p + e^{-} \\ p \longrightarrow n + e^{+} \end{cases}$
- E conserved

- E released → almost all in K_β
- Q should be the same for all decays



missing E & p

⇒ 1930 Pauli proposed the existance of another particle

neutrino

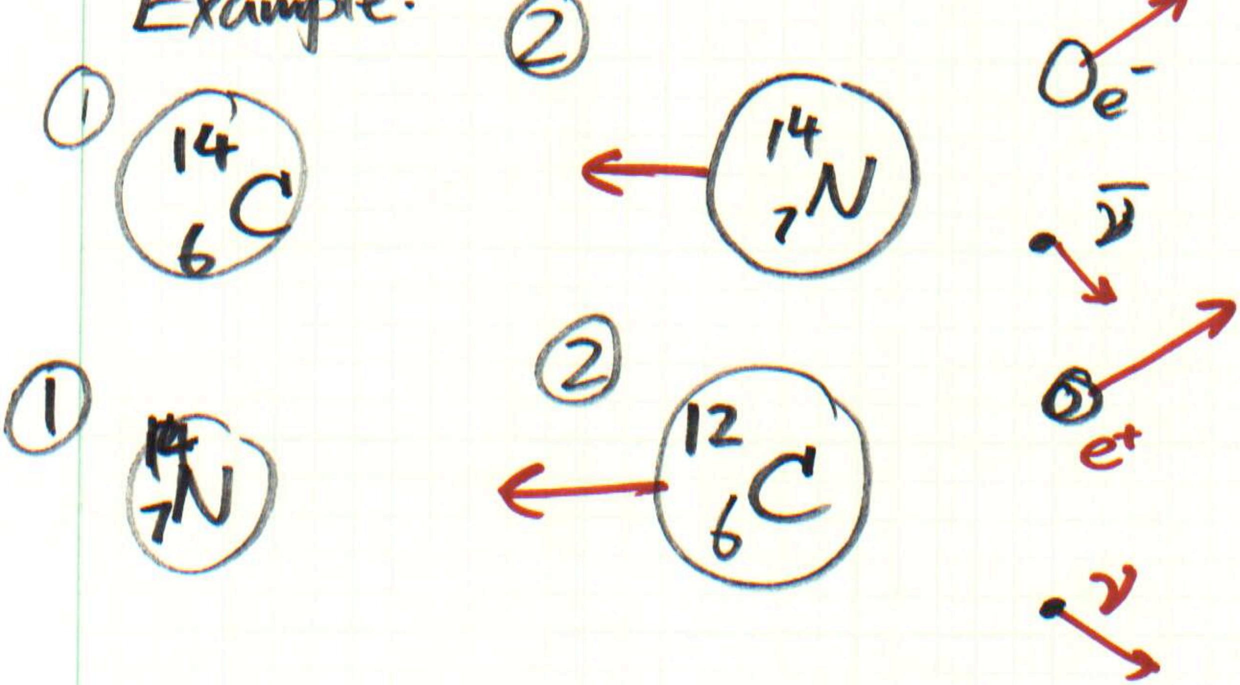
Neutrino: - 0 e.charge
- tiny mass $2.8\ eV/c^2$

spin ½
- Weak interaction w/ matter
→ hard to detect

$$ {}^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + \underbrace{e^{-} + \bar{\nu}}_{\text{Pair}} $$ ← antineutrino

$$ {}^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}Y + \underbrace{e^{+} + \nu}_{\text{Pair}} $$ ← neutrino

Example:



Fundamental processes:

$$\begin{cases} n \longrightarrow p + e^- + \bar{\nu} \\ p \longrightarrow n + e^+ + \nu \end{cases}$$

This can only occur inside a nucleus

This cannot happen for an isolated proton $\longrightarrow$ b/c $m_p < m_n$

e^- Capture

$$^{A}_{Z}X + {}^{0}_{-1}e \longrightarrow {}^{A}_{Z-1}Y + \nu$$

- Competes w/ e^+ decay
- Most of the times:

  K-shell e^- is captured

  $\rightarrow$ K-capture

Carbon Dating

$$^{14}_{6}C \longrightarrow {}^{14}_{7}N + e^- + \bar{\nu}$$

- process dep. on ratio of ^{14}C to ^{12}C in the atmosphere $\rightarrow$ fairly const.
- ratio decrease when an organism dies $\longrightarrow$ b/c of β-decay

Gamma Decay

Excital nucleus decays to a lower E state

$$^{A}_{Z}X^{*} \longrightarrow {}^{A}_{Z}X + \gamma$$

Excited

Example:

$$^{12}_{5}B \longrightarrow {}^{12}_{6}C^{*} + e^{-} + \bar{\nu}$$ β emission

$$^{12}_{6}C^{*} \longrightarrow {}^{12}_{6}C + \gamma$$ γ emission

- γ emission does not change:

  A, Z, N

- $E_\gamma = hf = \Delta E_{nucleus}$

Nuclear Reactions

- Structure of nuclei can change by bombarding them w/ energetic particles
  → the changes are called nuclear reactions
- Z & A should balance on both sides of the eq.

× X bombarded by a

$$a + X \rightarrow Y + b$$

reaction E:

$$\boxed{Q = (M_a + M_X - M_Y - M_b)c^2}$$

Value of Q determines reaction type

- Exothermic Reaction:
  - * mass 'loss'
  - * E release
  - * $Q >$
- Endothermic Reaction:
  - * mass 'gain'
  - * E is need (as K.E. of incoming
  - * particle)
  - * $Q < 0$
  - * E_{min} neccessary for the reaction to occur $\longrightarrow$ threshold E

If a & b and therefore X & Y are identical

→ scattering event

$K_i = K_f$ → elastic scattering

$K_i \neq K_f$ → inelastic 〃

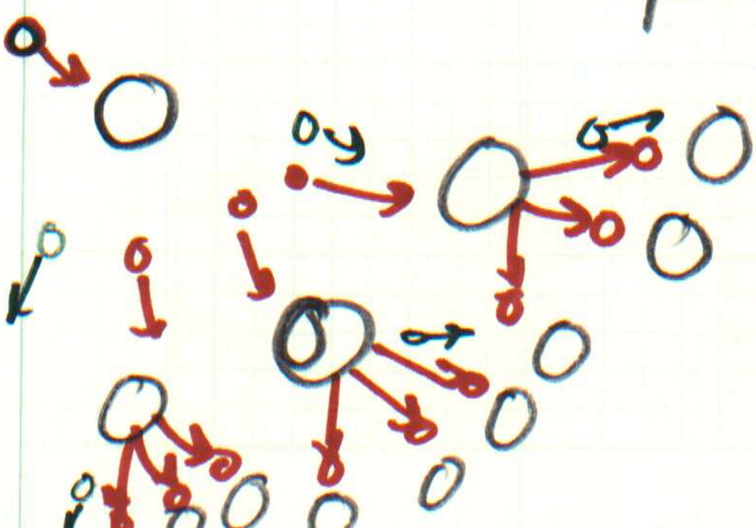
Conserved quantities:

E, P, charge, # of nucleons

- Nuclear ~~E~~ reaction releases much more E than chemical reacti
- 2 Types:
  - Fission
  - Fussion

Nuclear Fission

- Heavy nucleus splits to two smaller nuclei
- A Fissionable nucleus (X) absorbs a neutron (a) and splits into (Y_1) & (Y_2) and multiple nuetrons (several b)
- No control → Chain reaction
- w/ Control → Nuclear power plants



Nuclear Fision:

- 2 light nuclei → heavier nucle
- More difficult → b/c nuclei shoul

    overcome Coulomb repulsion

    * raise temp → high K.E.
    * high density

This is what happens in the sun

$${}^{1}_{1}H + {}^{1}_{1}H \rightarrow {}^{2}_{1}H + e^{+} + \nu$$

$${}^{1}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{2}He + \gamma$$

$${}^{1}_{1}H + {}^{3}_{2}He \rightarrow {}^{4}_{2}He + e^{+} + \gamma$$

or

$${}^{3}_{2}He + {}^{3}_{2}He \rightarrow {}^{4}_{2}He + {}^{1}_{1}H + {}^{1}_{1}H$$