Chapter 27

Wave Optics

- Interference
- Diffraction

→ Cannot be explained by geometric optics

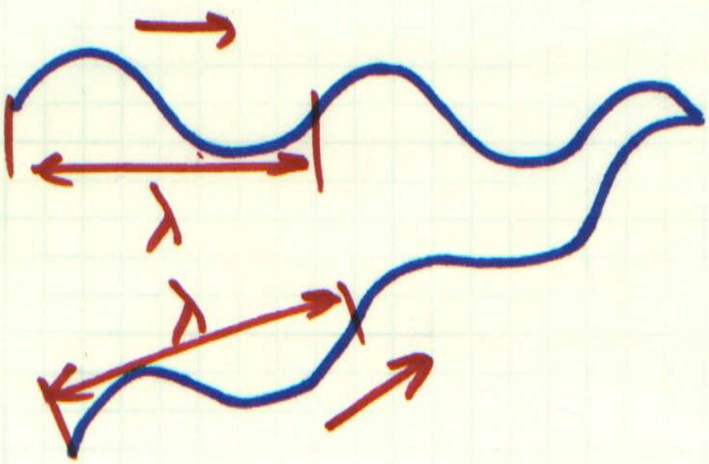
Interference:

Superposition of 2 EM waves

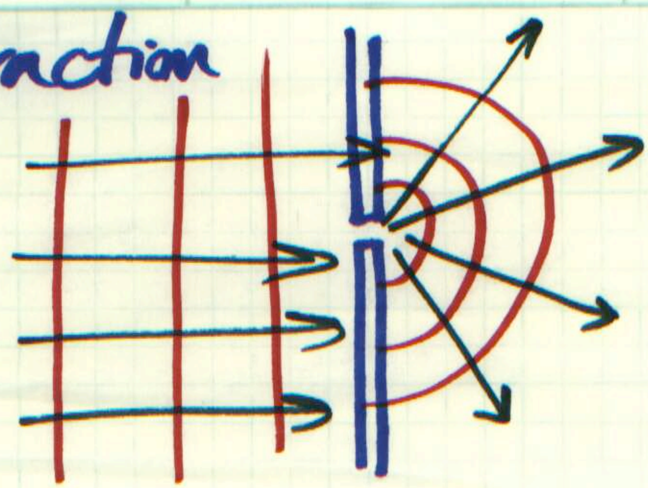
Const. vs. dest.

In order to observe interference:
~~Interferen~~

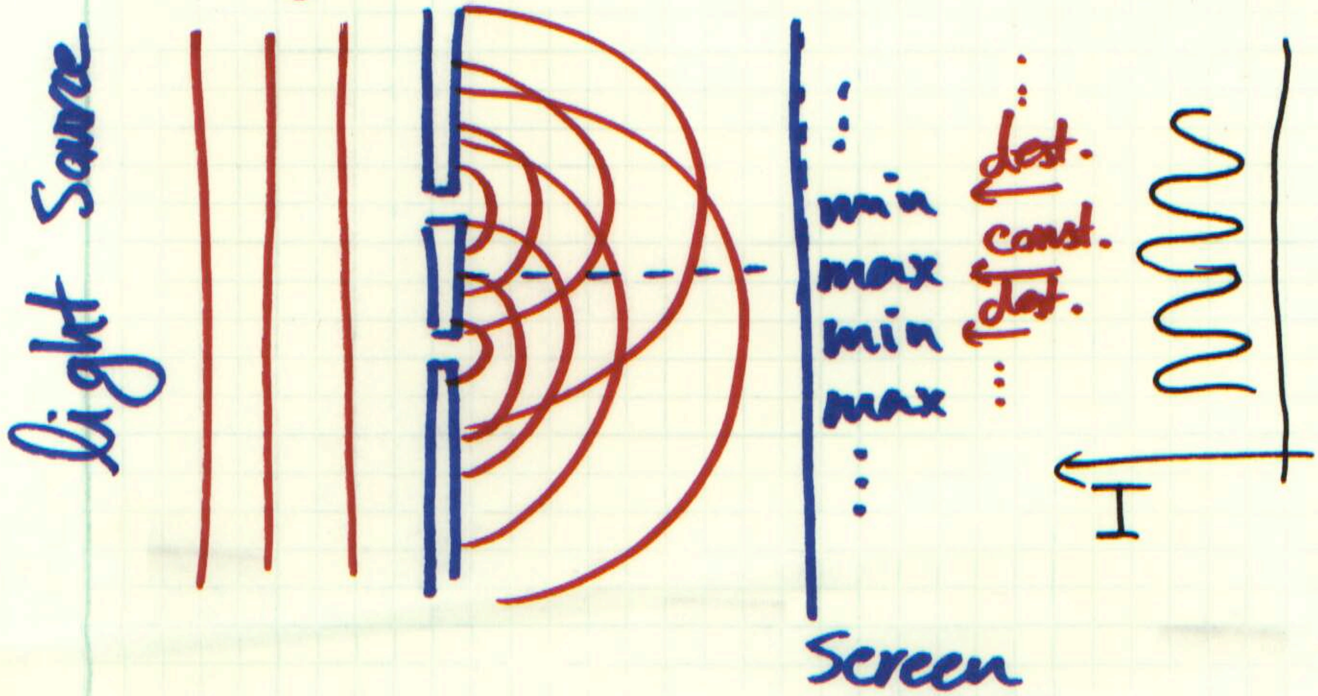
* Monochromatic (same λ)
* Coherent (Const phase)



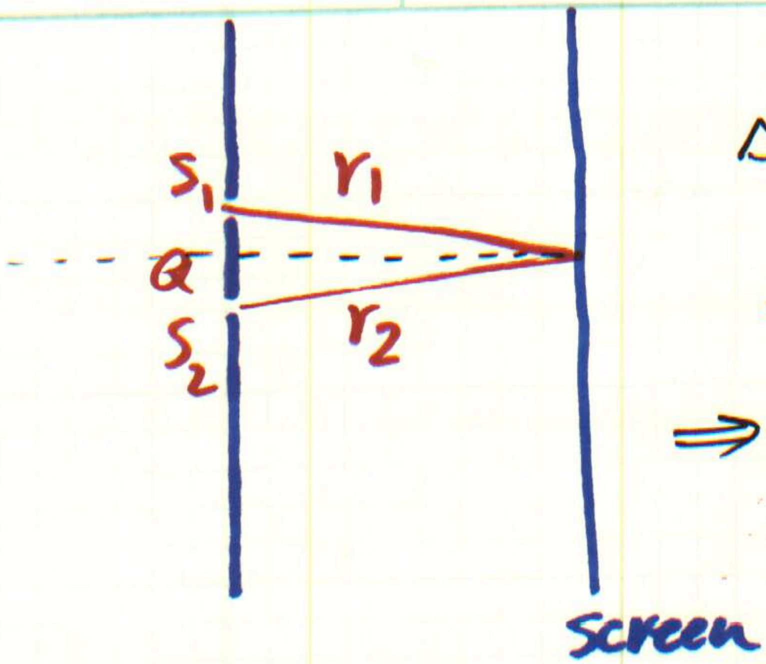
Diffraction



Young's Double Slit Experiment:



* Each slit acts as a point source of light
* Coherent & Monochromatic (same source)

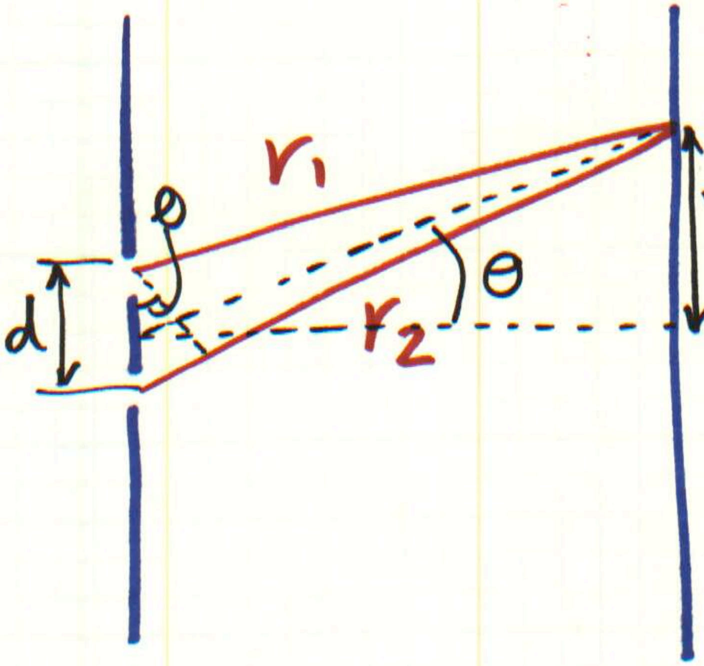


$\Delta r = |r_2 - r_1| = 0$

$\Rightarrow \Delta\phi = 0$

$\Rightarrow$ Const. interferance

Brigh Spot



$\Delta r = |r_2 - r_1|$

$= d \sin\theta$

$\Delta r = m\lambda$

$m = 0, \pm 1, \pm 2, \cdots$

const.

$\Rightarrow$ $m = 0$ zeroth order max

$m = \pm 1$ first-order max

$m = \pm 2$ second-order max

$\vdots$

$\Delta r = d \sin\theta = (m+\frac{1}{2})\lambda \leftarrow$ Desk [Dark Spots]

$m = 0, \pm 1, \pm 2$

When $\theta \ll 1$ rad.

$\tan\theta \approx \sin\theta$

$\tan\theta = \frac{Y}{L}$

$\Rightarrow Y_{max} \approx L \cdot \sin\theta_{max} = L \cdot \left(\frac{m\lambda}{d}\right)$

$Y_{min} \approx L \cdot \sin\theta_{min} = L \cdot \frac{(m+\frac{1}{2})\lambda}{d}$

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* λ could be measured using the double slit experiment
* Double slit exp. provides an evidence for the wave behaviour of light

Intensity Distribution

no sharp edges

Assumptions:

- coherent sources
- same ω
- const ϕ

path diff. (δ) vs. phase diff (ϕ):

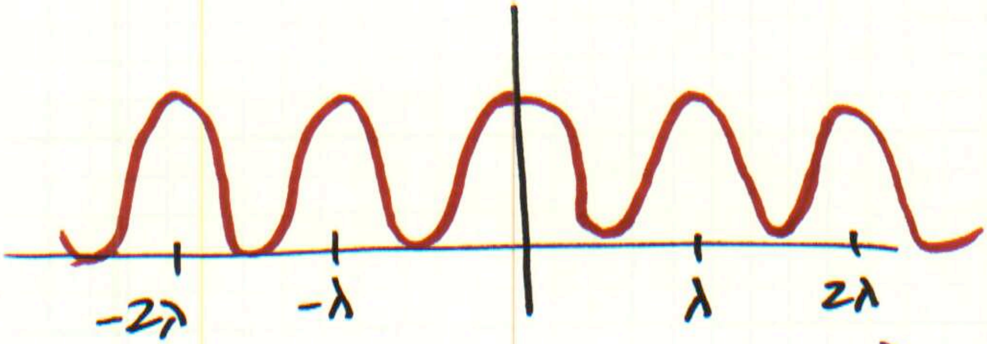
$$\frac{\phi}{2\pi} = \frac{\delta}{\lambda}$$

$$\phi = 2\pi \frac{\delta}{\lambda} = \frac{2\pi}{\lambda} \underbrace{d \sin\theta}_{\delta}$$

phase diff. as a func. of θ

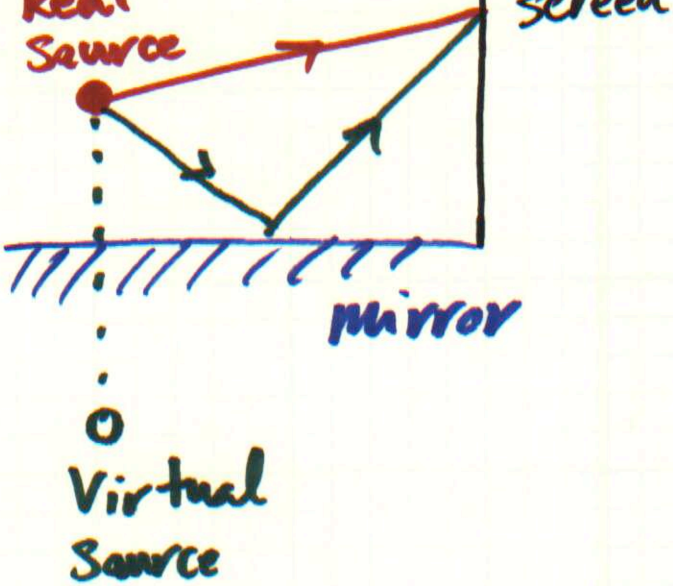
Analysis of intensity:

$$\boxed{I = I_{max} \cos^2\left[\frac{\pi d \cos\theta}{\lambda}\right]}$$



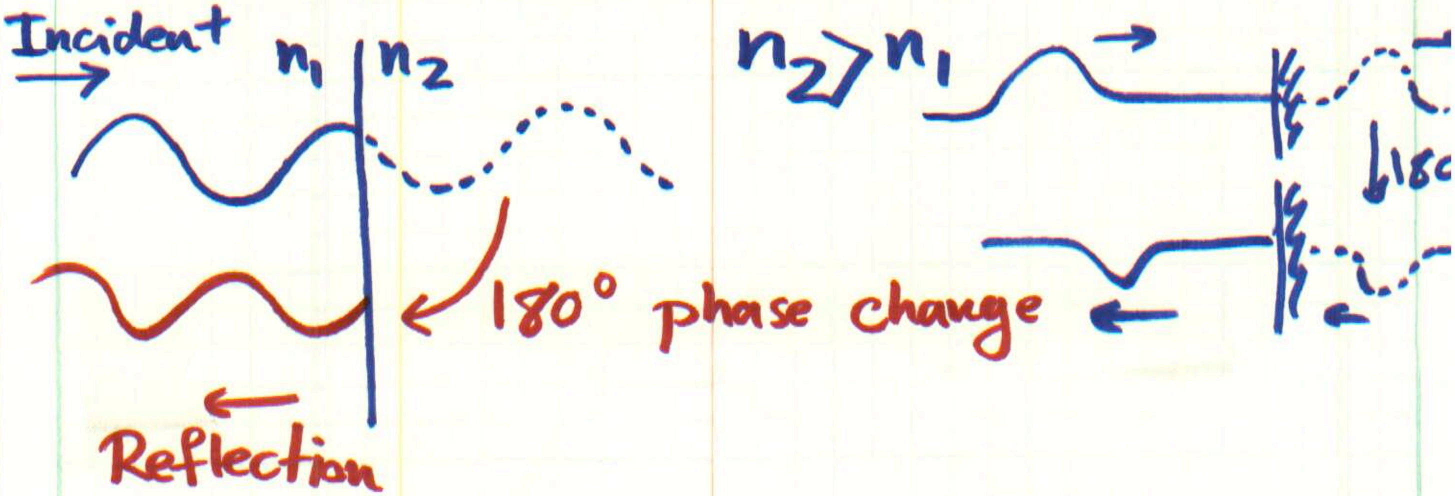
Only valid for $\begin{cases} L \gg d \\ \theta \ll 1 \text{ rad.} \end{cases}$

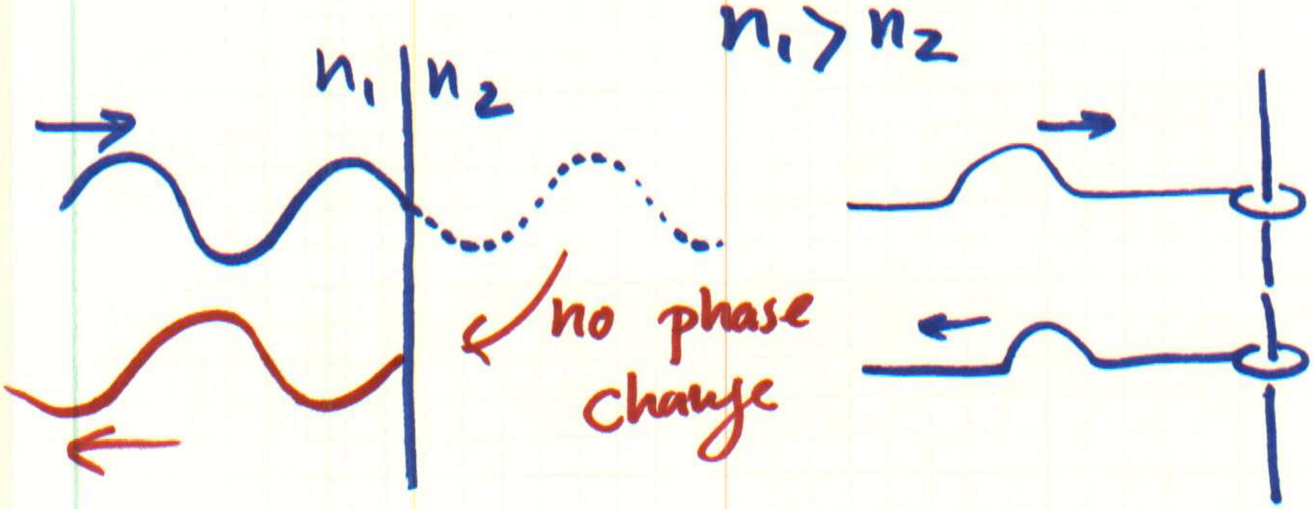
Lloyd's Mirror:



- Similar to d.s.e
- dark & bright fringes
- the dark and bright bands are switched because of 180° phase change due to reflection

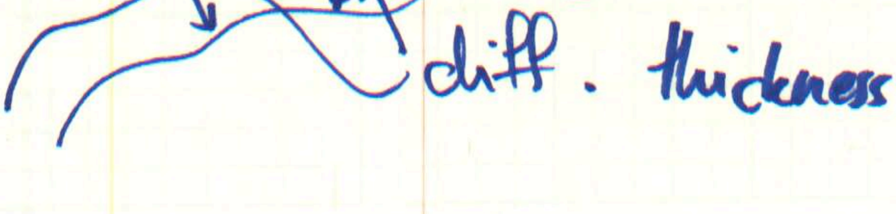
Thin Film Interference

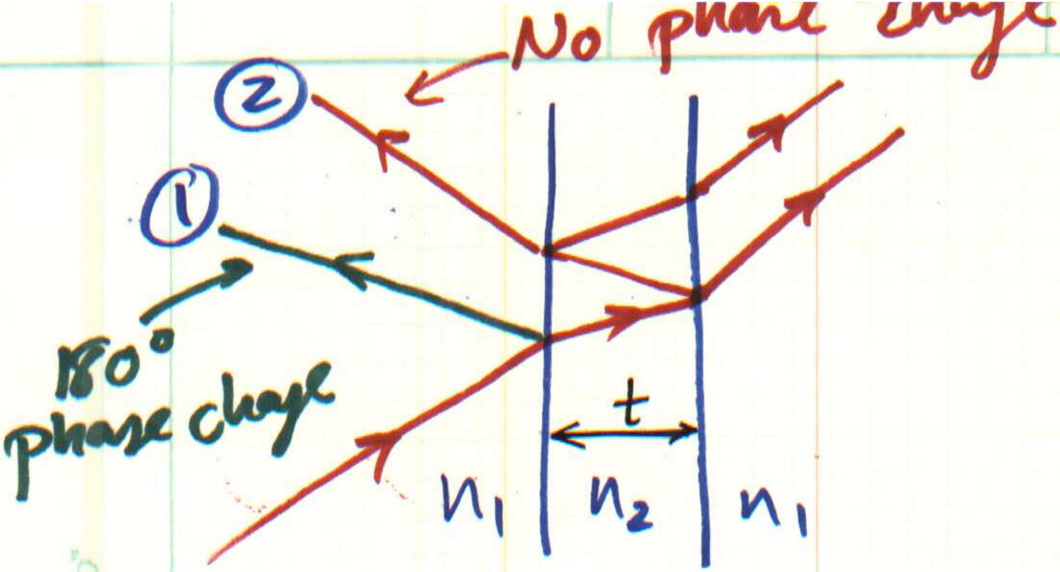




Examples of interferance in thin films:

Soap bubbles, Oil in water, ...





1st refl. $\Rightarrow$ $n_2 > n_1 \Longrightarrow$ 180° phase change

2nd refl $\rightarrow n_1 < n_2 \Longrightarrow$ No phase change

$$\lambda_{n_2} = \frac{\lambda_1 \cdot n_1}{n_2} \quad \text{if } n_1 \approx 1 \Longrightarrow \boxed{\lambda_n = \frac{\lambda}{n}}$$

Wave 2 travels an additional distance $2t$ befores waves interfere.

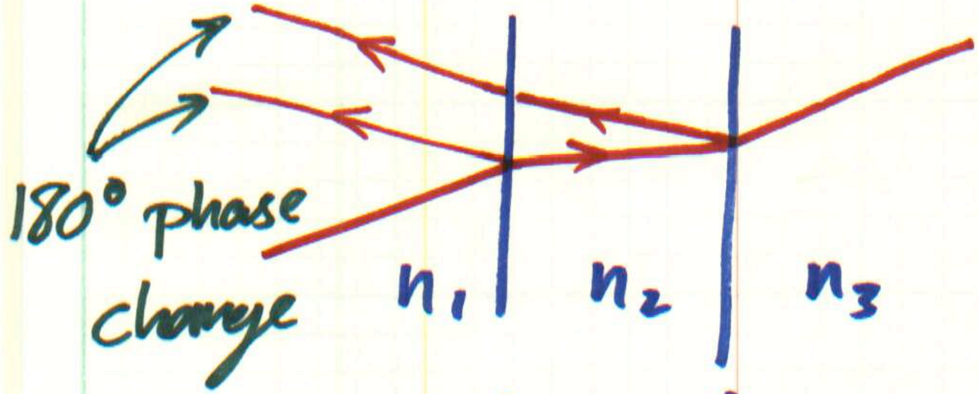
Const. Int.:

$$\Delta r = 2t = m \cdot \lambda_n = \boxed{m\left(\frac{\lambda}{n}\right) = 2t \quad n = 0, 1, 2 \cdots}$$

Dest. Int.:

$$\Delta r = \boxed{2t = \left(m+\tfrac{1}{2}\right)\frac{\lambda}{m} \qquad m = 0, 1, 2, \ldots}$$

This would be diff. if we have 3 media w/ 3 diff. n's.



$$n_1 < n_2 < n_3$$

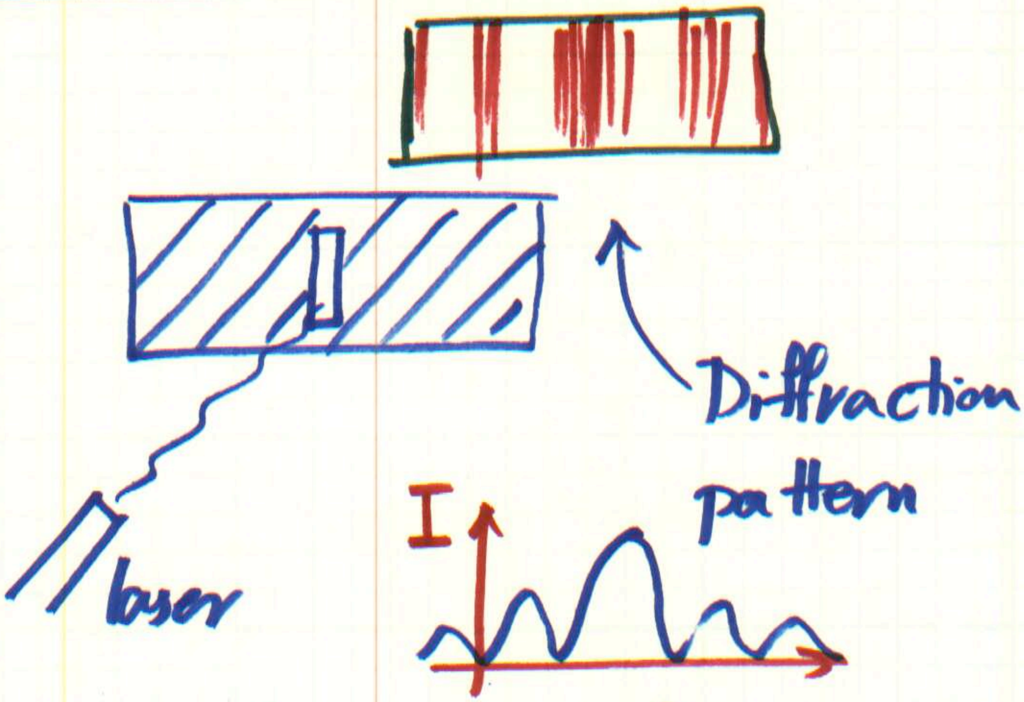
→ The conditions are reversed:

Const.

$$\boxed{\begin{aligned} 2t &= \left(m+\tfrac{1}{2}\right)\frac{\lambda}{n_2} \qquad m = 0, 1, 2, \ldots \\ 2t &= (m)\frac{\lambda}{n_2} \qquad m = 0, 1, 2, \ldots \end{aligned}}$$

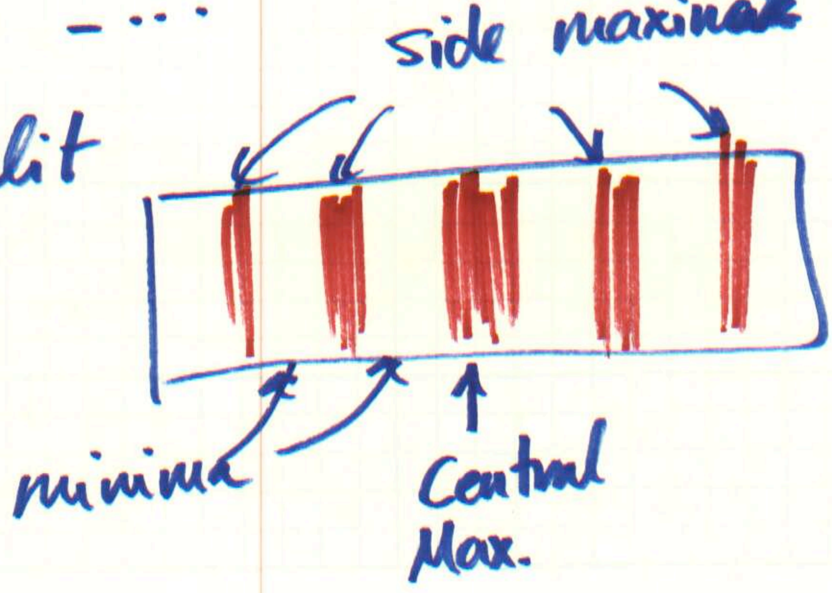
This is used ~~in~~ for making anti-reflecting coating.

Single-Slit Reflection

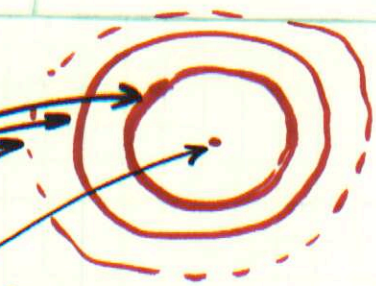


Diffraction: - light goes through a small opening
- On the edges
- Obstacles
- ...

Single-Slit

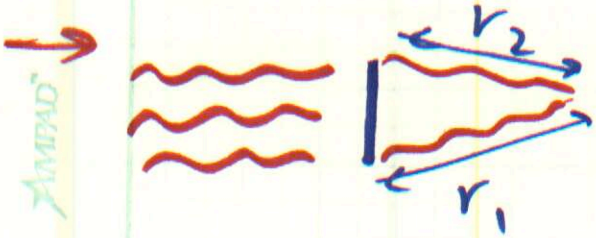


Circular Obstacle:



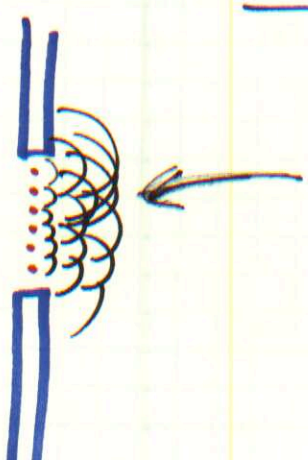
bright rings

central maximum
aka Arago bright spot



Arago bright point

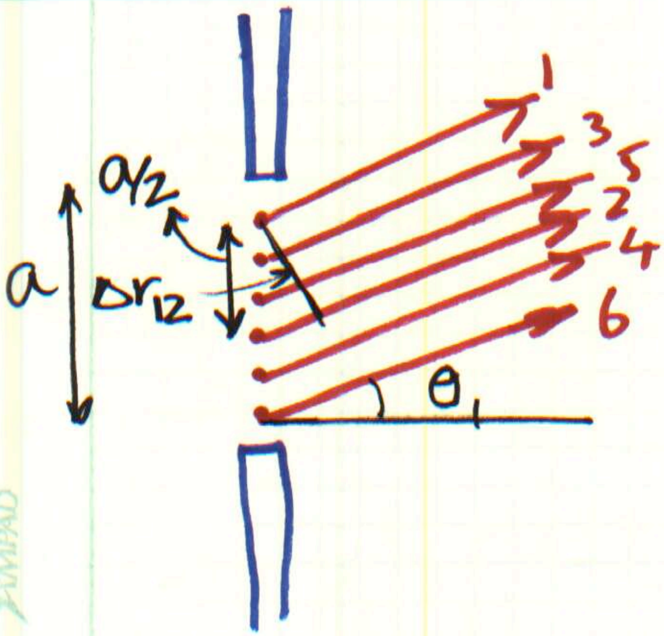
$r_1 = r_2 \Rightarrow$ waves travel the same dist. $\Rightarrow$ const.



Imagin many point sources inside the single-slit

$\theta = 0$

when $\theta = 0$ [screen far] all the waves travel eq. dist. $\Rightarrow$ cent. max.



If $\Delta r_{12} = \frac{\lambda}{2}$

$\Rightarrow$ dest. interference

same true fo (3,4) & (5,6)

$$\Delta r_{12} = \Delta r_{34} = \Delta r_{56}$$

$$\Delta r_{12} = \frac{a}{2} \sin\theta_1 = \lambda/2 \quad \text{or} \quad a \sin\theta_1 = \lambda$$

If $\Delta r_{15} = \frac{\lambda}{2} = \frac{a}{4}$

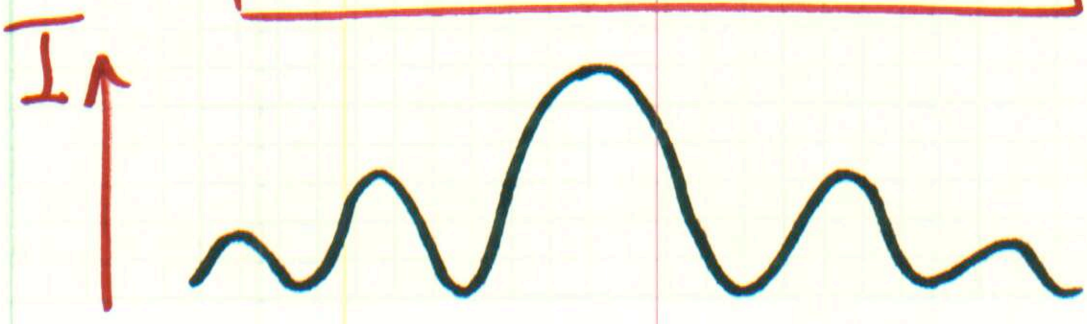
$\Rightarrow \Delta r_{15} = a \sin\theta_2 = 2\lambda$

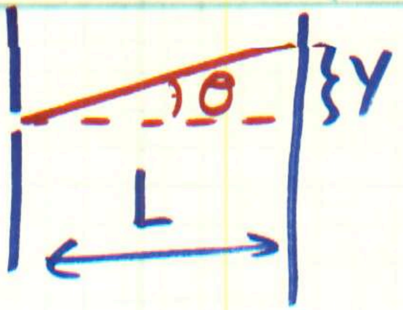
$\vdots$

$$a \sin\theta_p = p\lambda \qquad p = 1, 2, 3, \dots$$

for $\theta_p \ll 1$ rad. $\Rightarrow \sin\theta \approx \theta$

$$\Rightarrow \boxed{\theta_p \approx p\,\lambda/a \qquad p = 1, 2, 3, \dots}$$





$$\tan\theta_p = \frac{y_p}{L}$$

$$\sin\theta_p = p\frac{\lambda}{a}$$

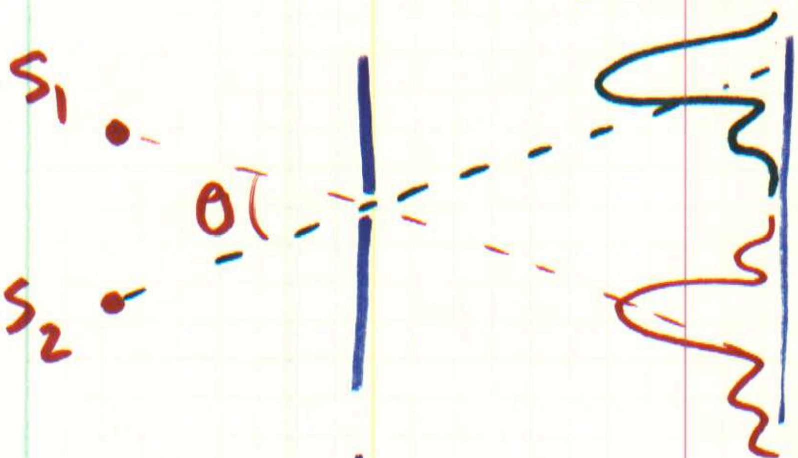
if $\theta \ll 1$ rad. $\Rightarrow$ $\sin\theta \approx \tan\theta$

$$\Rightarrow \boxed{y_p = \frac{p\lambda L}{a} \quad p = 1, 2, 3, \ldots}$$

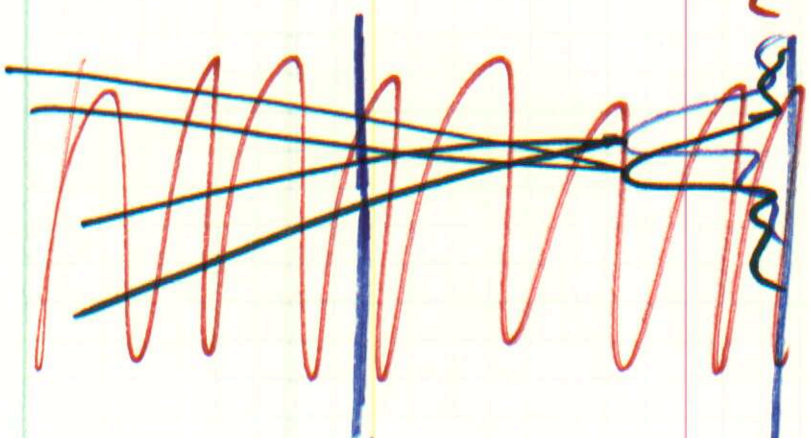
position of dark fringes

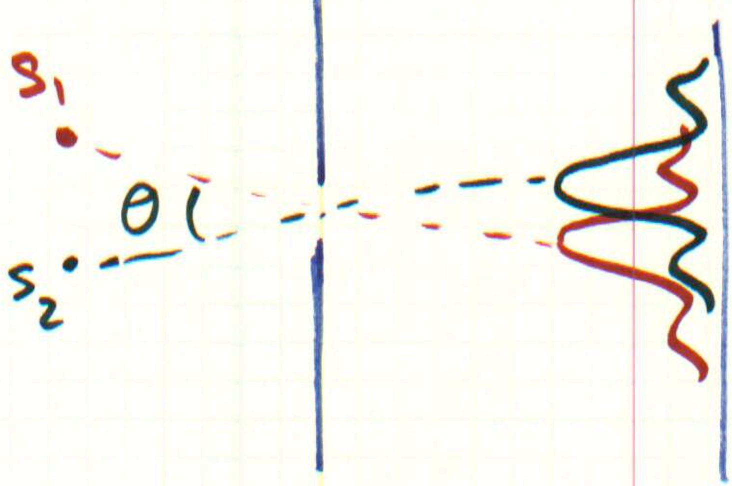
Resolution

resolved | just resolved | unresolved



Resolved

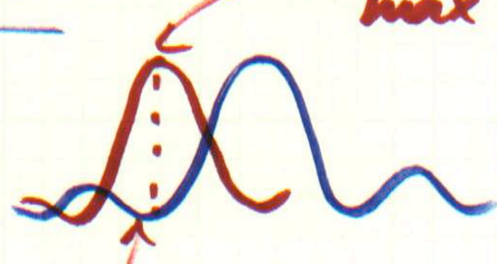




As θ gets smaller the two central maxima overlap and are ~~indistinguis~~ indistinguishable

Rayleigh's Criterion

Just Resolved $\longrightarrow$



The angle θ between S_1 & S_2 is called $\underline{\theta_{min}}$ in this case.

$\lambda \ll a \Rightarrow \theta \ll 1 \text{ rad} \Rightarrow \sin\theta \approx \theta$

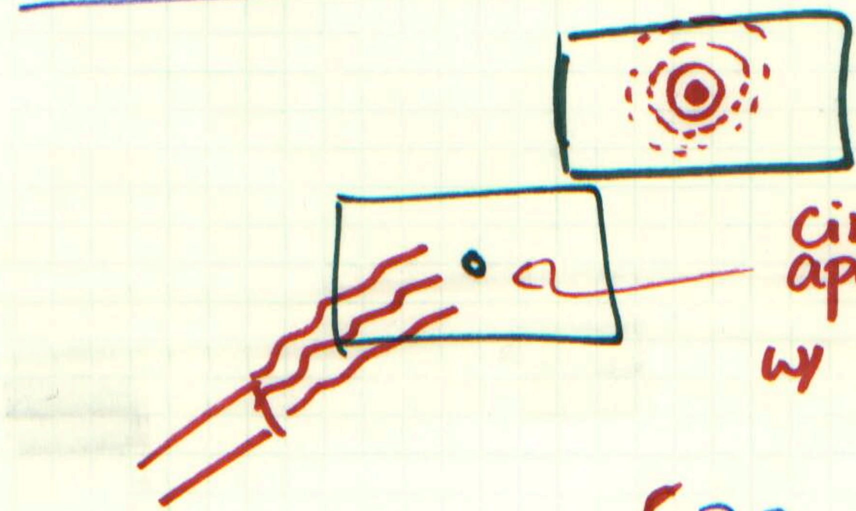
$\Rightarrow$ As we had for single slit:

$$\sin_{ss} \theta = \lambda/a$$

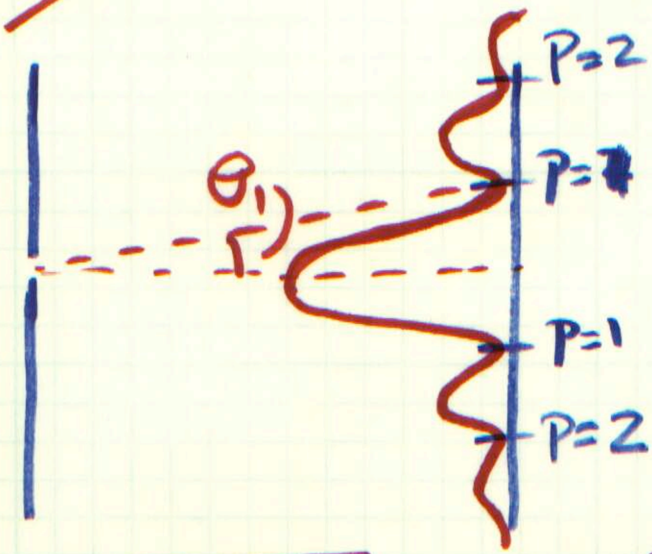
$$\Rightarrow \boxed{\theta_{min} = \lambda/a}$$

$\leftarrow$ θ_{min} represent the angle at which the two central maxima are just resolved.

Circular Aperture



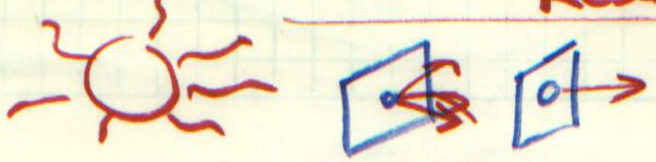
circular aperture w/ diameter D



$$\boxed{\theta_1 = \frac{1.22\ \lambda}{D}}$$

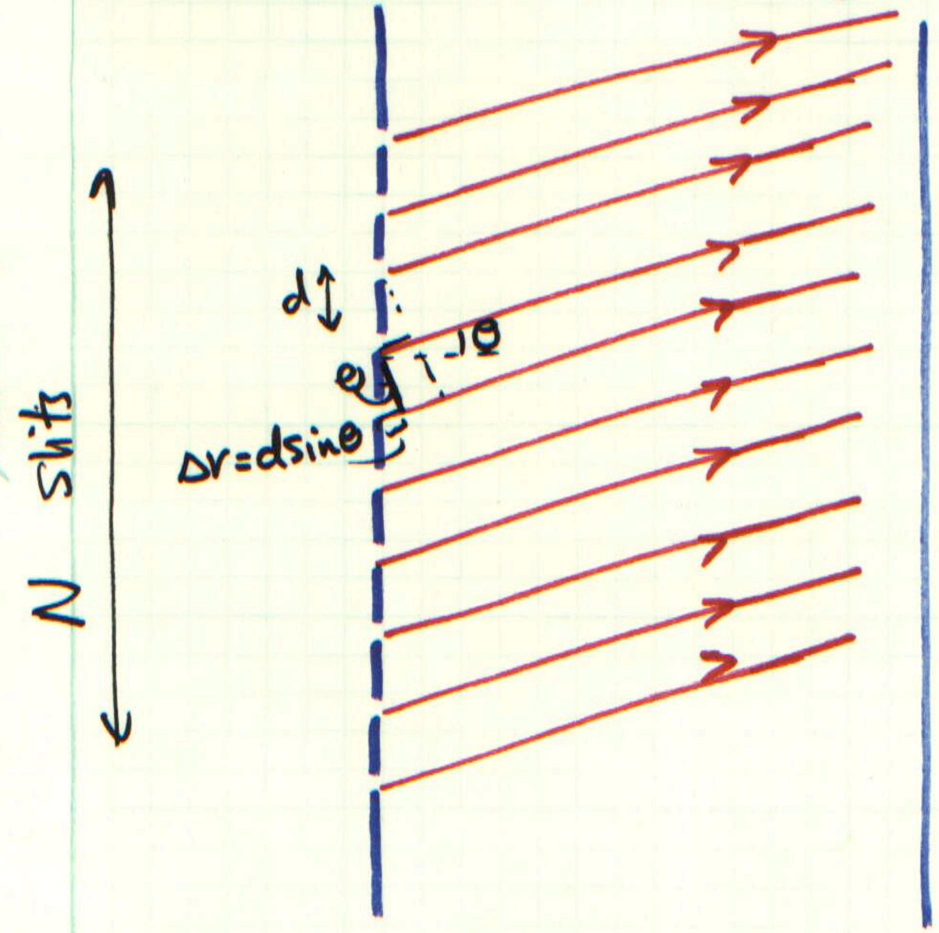
Angle for first dark ring

Resolved Just Resolved Not resolved



Circular aperture

Diffraction Grating



~~Each 2 sli~~

The ray from each slit travels a distance $\Delta r = d \cdot \sin\theta$ more than the one above and Δr distance less than the one below.

$\Rightarrow$ True for all slits.

if $\Delta r = d\sin\theta = m \cdot \lambda \longrightarrow$ const.

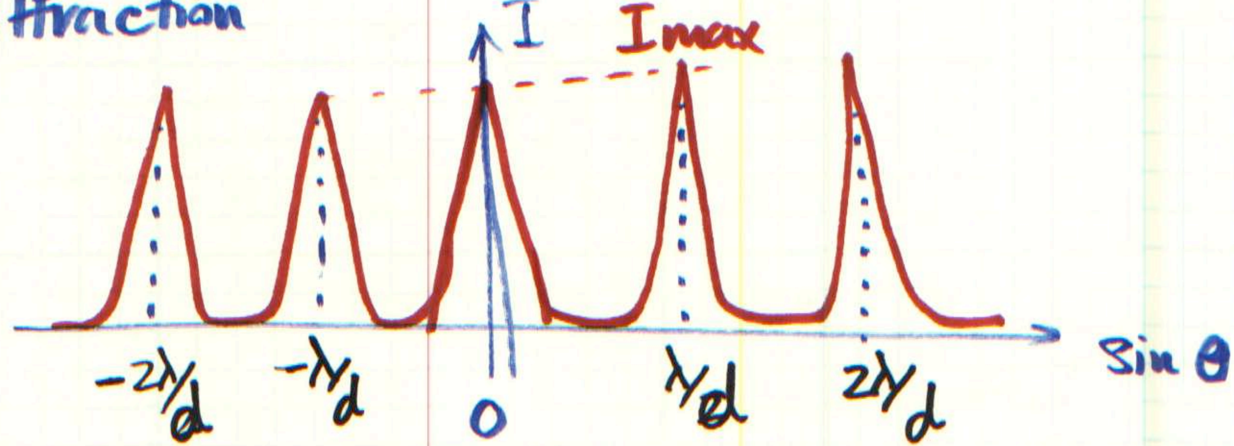
$m = 0, 1, 2, \ldots$

$$\boxed{d \sin\theta_m = m\lambda}$$ bright fringes

$m = 0, 1, 2, \dots$

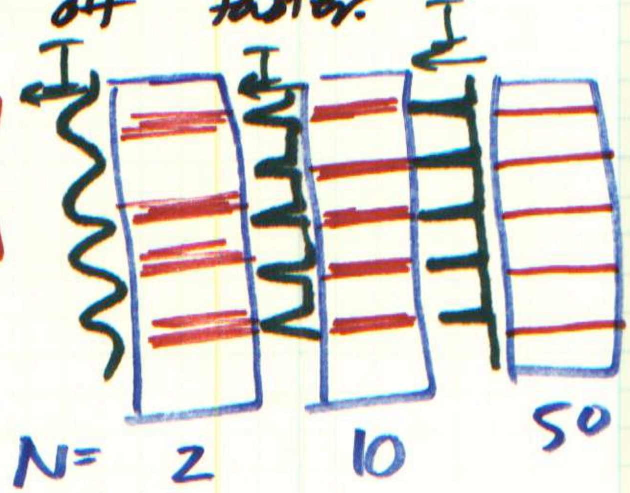
$$\boxed{y_m = L \tan\theta_m}$$

m in θ_m defined as order of diffraction



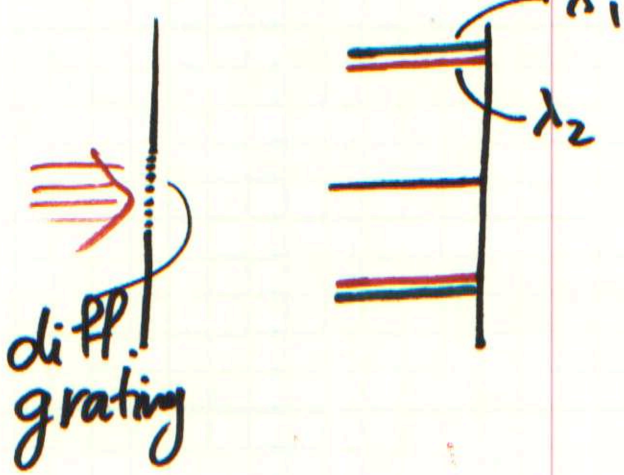
The bright fringes are sharper and the brightness fall off faster.

$$\boxed{I_{max} = N^2 I_1}$$



Spectroscopy

Measuring the wavelength of atomic & molecular emission



$\lambda_1 > \lambda_2$

diff. wavelengths can be distinguished

Diffraction of X-Ray by Crystals:

X-Ray: EM wave w/ a short λ