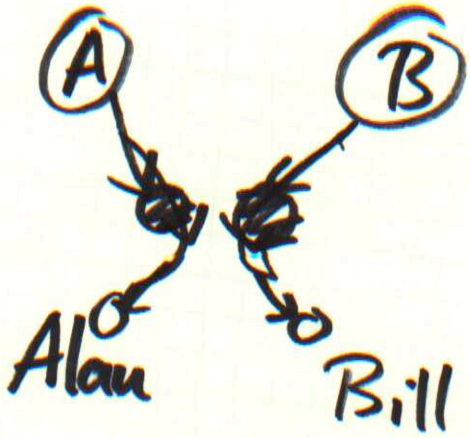
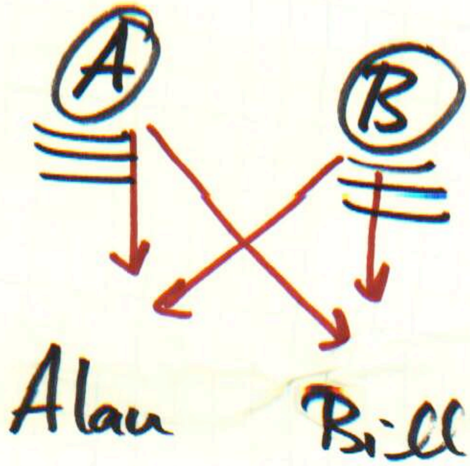
Ch 14

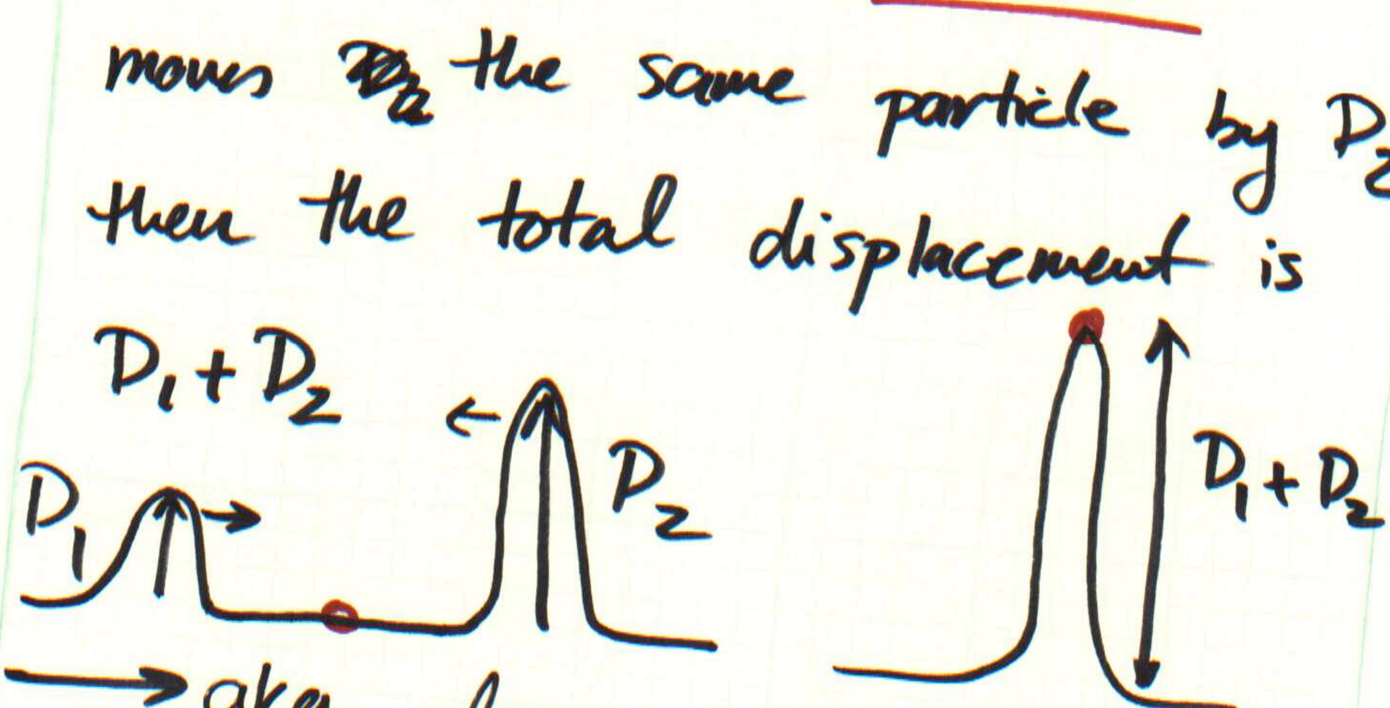
1

Wave Vs. Particle





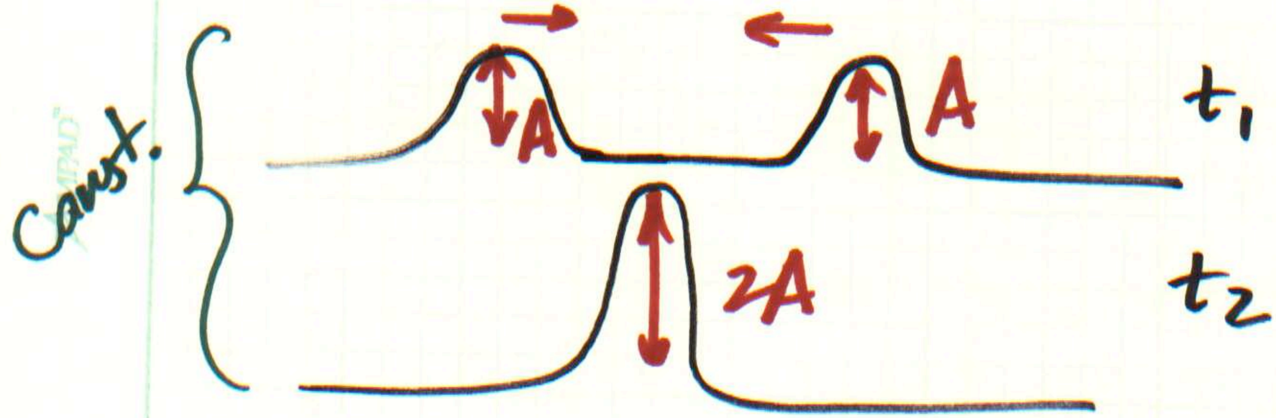
Principle of superposition:
wave 1 moves a particle in medium by D_1 & wave 2 moves ~~D_2~~ the same particle by D_2 then the total displacement is $D_1 + D_2$

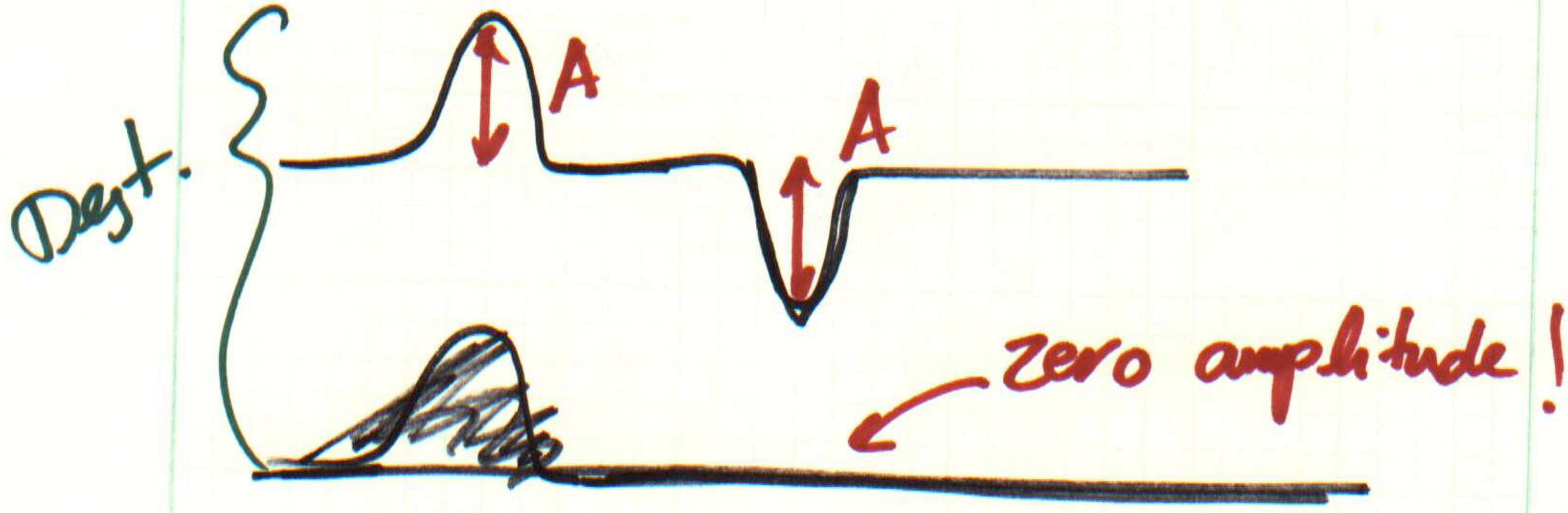


→ a.k.a. linear waves

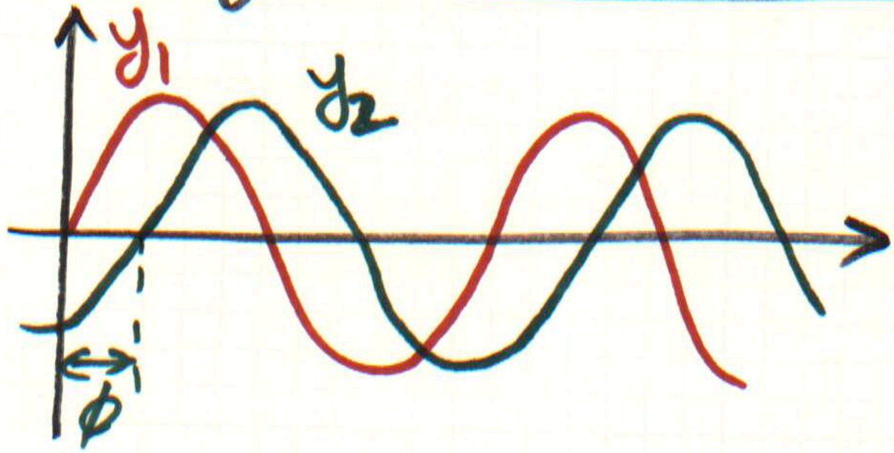
Types of Interference

Const. Vs. Dist.





S.P. of Sinusoidal Waves



$Y_1 = A \sin(kx - \omega t)$

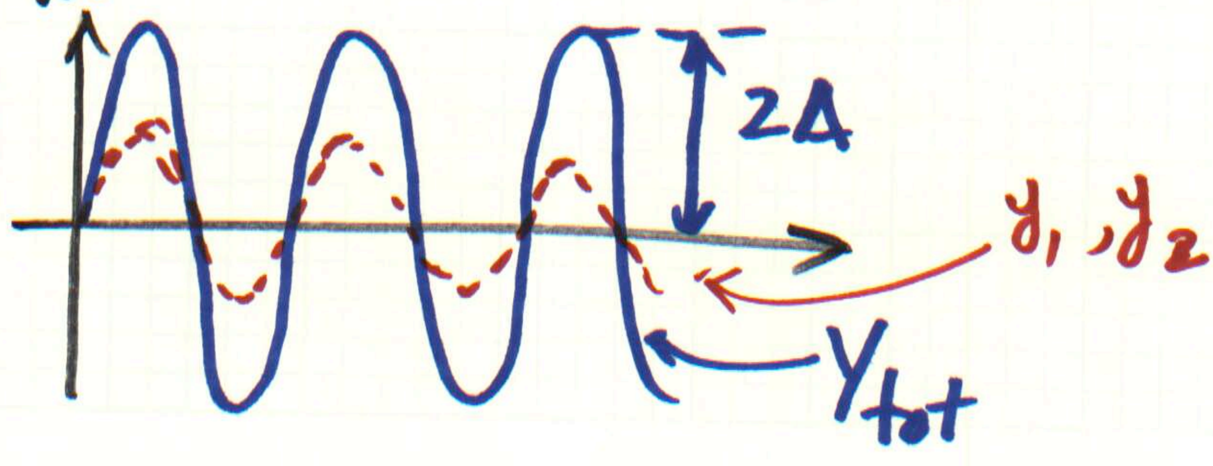
$Y_2 = A \sin(kx - \omega t + \phi)$

If $\phi = 0$:

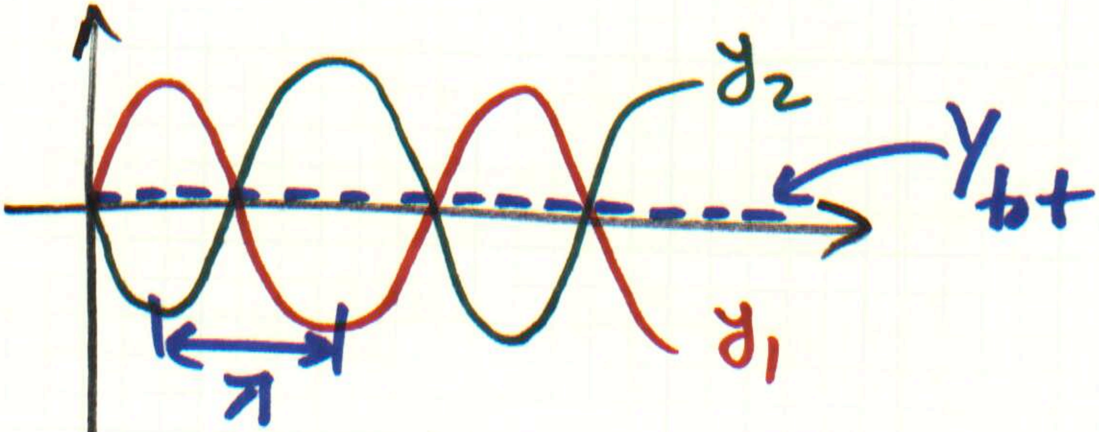
Y_1 & Y_2 identical. Completely

const.

$Y_{tot} = 2A \sin(kx - \omega t)$



If $\phi = \pi$:



Completely dest.

In general $\phi = (2n+1)\pi \longrightarrow$ dest.

$\phi = 2n\pi \longrightarrow$ const.

$n \in \mathbb{Z}$

$n = \{0, 1, 2, \cdots\}$

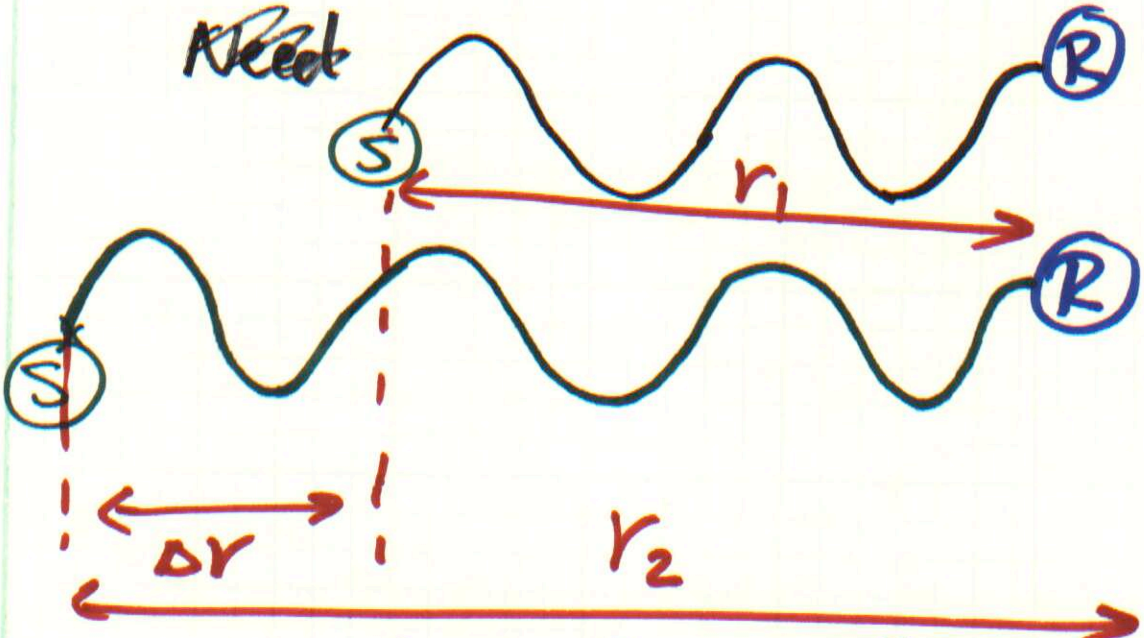
ϕ other than special cases:
general interference

$$Y = Y_1 + Y_2$$
$$= 2A \cos(\phi/2) \sin(kx - \omega t + \phi/2)$$

Interference in Sound Waves

Const. Interference:

~~Need~~



$$\Delta r = |r_2 - r_1| \Longrightarrow \text{if } \Delta r = n\lambda \rightarrow \text{const}$$

$$\text{if } \Delta r = \left(\frac{2n+1}{2}\right)\lambda \rightarrow \text{dest.}$$

$n = 0, 1, 2, \ldots$

Δr can be represented in terms of phase angle ϕ:

$$\Delta r = \frac{\phi}{2\pi}\lambda$$

Const. $\Delta r = m\lambda \longrightarrow \phi = 2m$

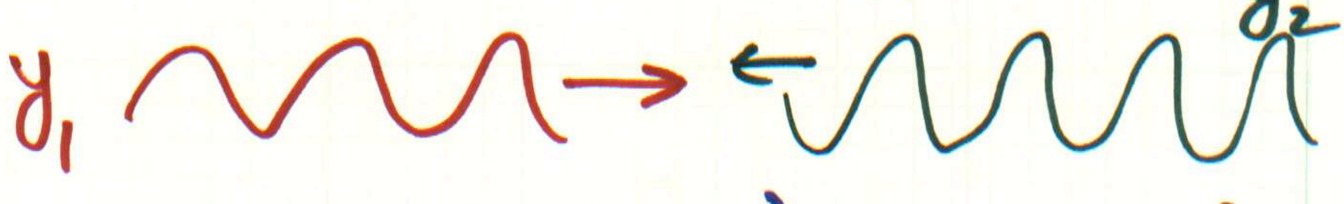
$m = 0, 1, 2, \ldots$

Dest. $\Delta r = \frac{(2m+1)}{2}\lambda \longrightarrow \phi = (2m+1)\pi$

$m = 0, 1, 2, \ldots$

<u>Standing Waves</u>

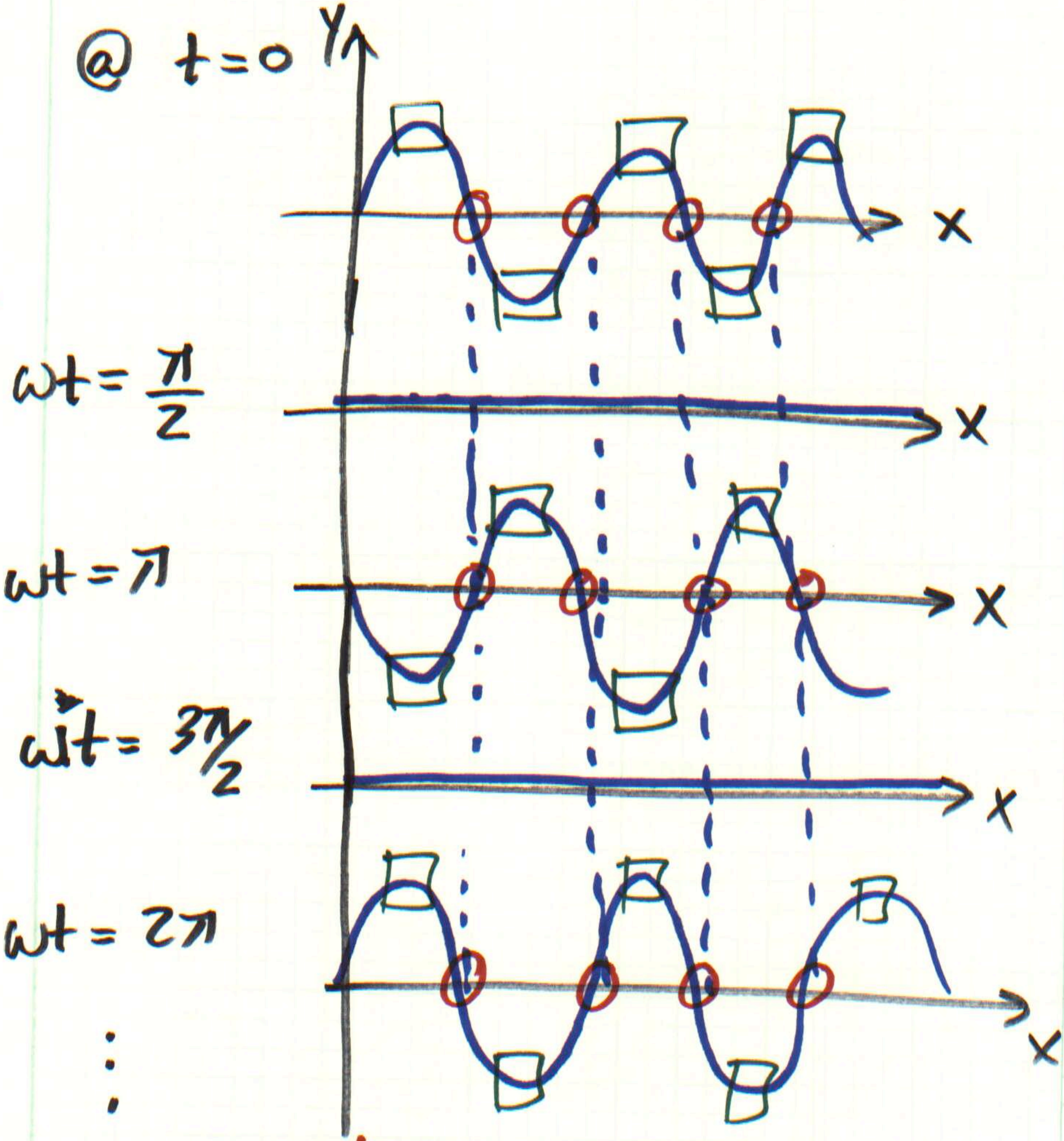
2 identical waves y_1, y_2



$y_1 = A \sin(kx - \omega t)$ → going left

$y_2 = A \sin(kx + \omega t)$ → going right

$$Y = Y_1 + Y_2 = 2A \sin(kx)\cos(\omega t)$$



○ ⟶ nodes

□ ⟶ anti-nodes

$y = \left[2A \sin \overset{kx}{\cancel{\omega t}}\right] \cos(\omega t) \leftarrow$ standing wave

$\rightarrow$ no $(kx - \omega t)$ argument

$\Rightarrow$ does not travel in space

Amplitudes:

~~Amplitude of S.W. vs time~~

$A \rightarrow$ amp. of each wave y_1, y_2

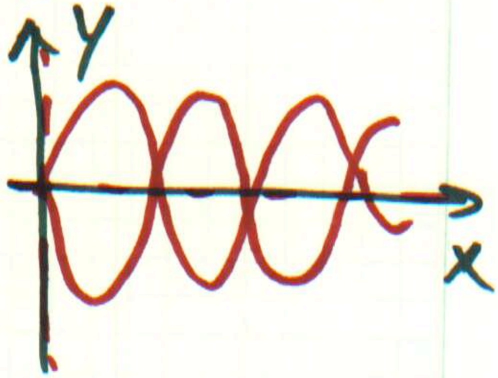
$2A \sin kx \rightarrow$ amp. of SHM of each particle in medium

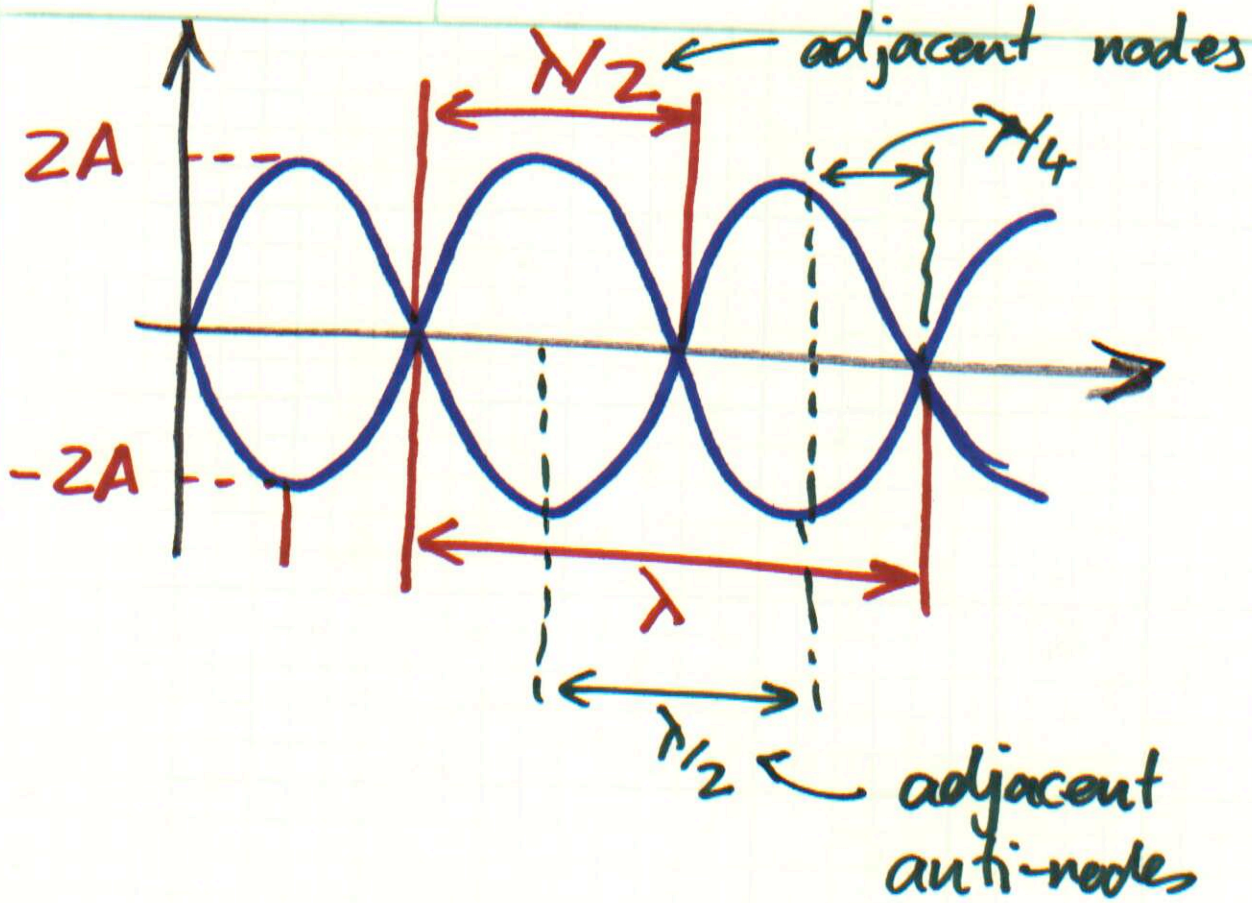
$2A \rightarrow$ amp. of S.W.

SHM of each particle:

freq. $\rightarrow \omega$

amp $\rightarrow 2A \sin kx$



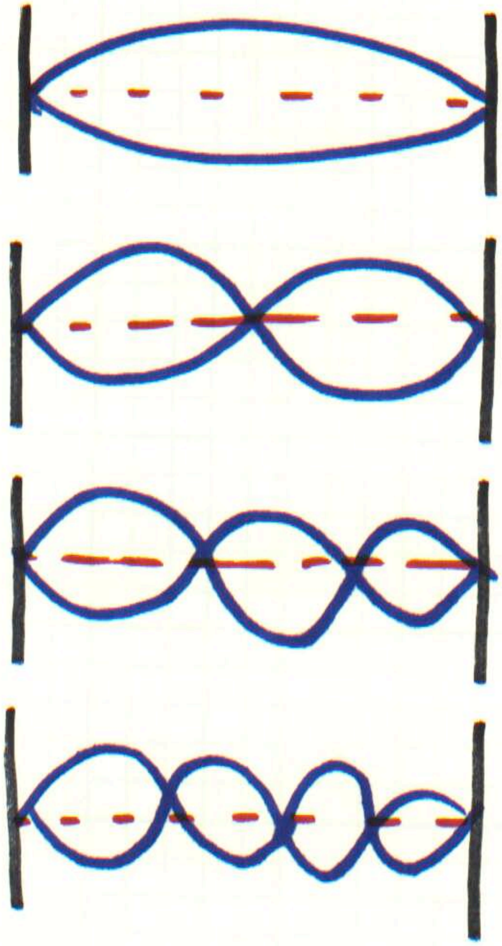


Standing Wave in a string:

have to be nods!

This boundary condition results in <u>normal modes</u>

long λ



λ_1 $\quad L = \frac{1}{2}\lambda_1$

λ_2 $\quad L = \lambda_2$

λ_3 $\quad L = \frac{3}{2}\lambda_3$

λ_4 $\quad L = 2\lambda_4$

short λ

$\vdots$

$$L = \frac{n}{2}\lambda_n \qquad n = 1, 2, \ldots$$

$$\Longrightarrow \boxed{\lambda_n = \frac{2}{n} L \qquad n = 1, 2, 3, \ldots}$$

Wave length of nth normal mode

$$v = \lambda/T = \lambda \cdot f$$

$$\Rightarrow v = \lambda_n \cdot f_n$$

$$\left.\begin{array}{l} f_n = \dfrac{v}{\lambda_n} = n\,\dfrac{v}{2L} \\ v = \sqrt{T/\mu} \end{array}\right\} \Rightarrow \boxed{f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}}}$$

On a string w/ tension T, linear mass density μ, and length L only specific freq. are allowed

$\Longrightarrow$ a.k.a quatization of freq.

e.g. guitar strings.

$$\left.\begin{array}{l} L \rightarrow \text{same} \\ T, \mu \rightarrow \text{diff} \end{array}\right\} \Rightarrow \text{diff freq.}$$

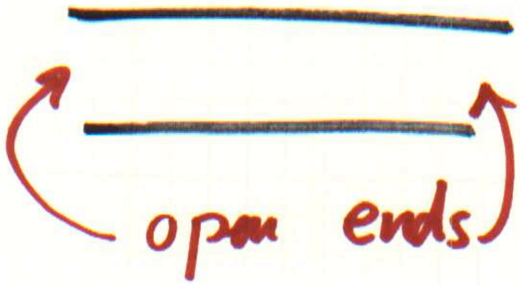
$n = 1 \longrightarrow$ Fundamental Freq.

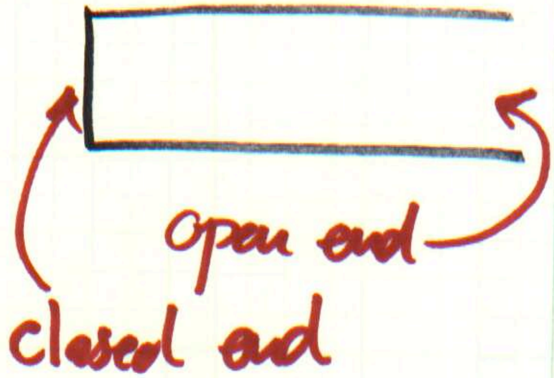
$f_n = n \cdot f_1$ ← fundamental freq.

↑ harmonics

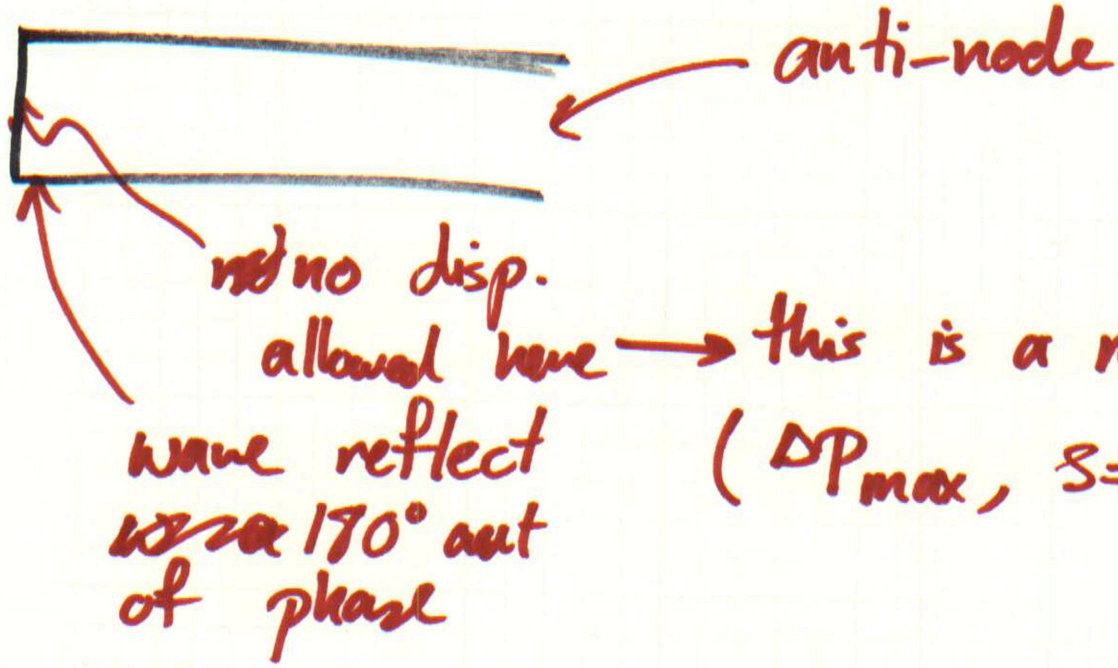
Standing Waves in Air Columns

Open end pipe Vs. Closed end pipe

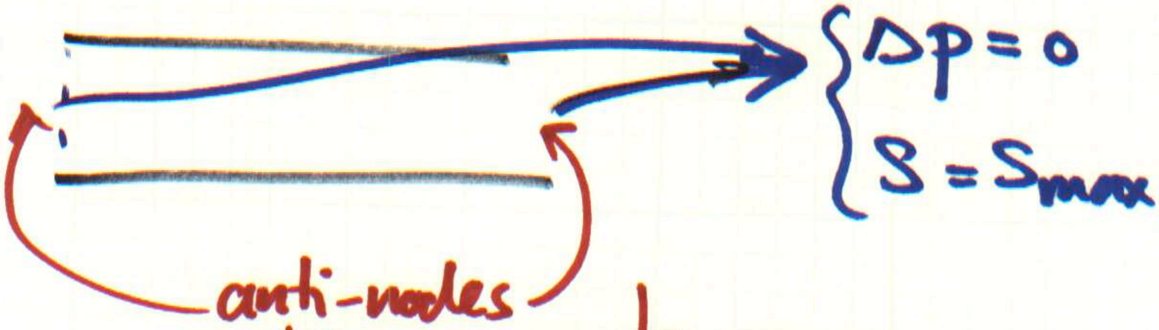


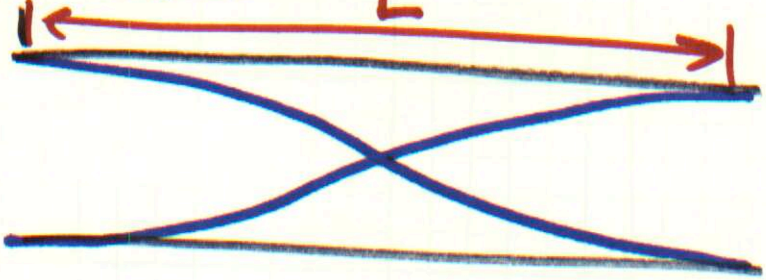


Closed End Pipe:

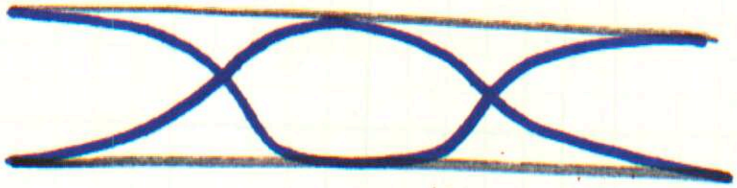


Open End Pipe:

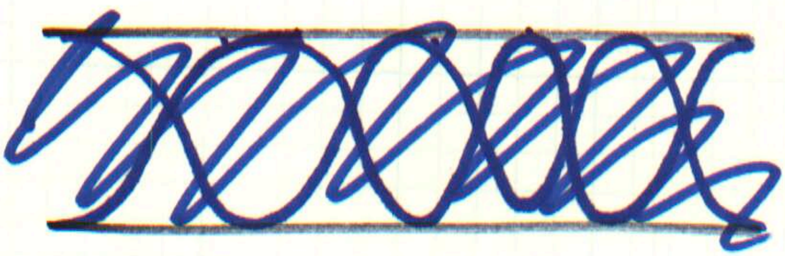


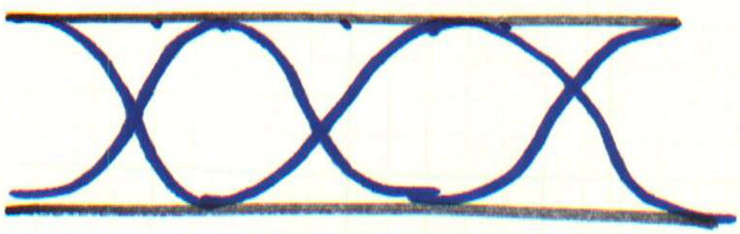


$L = \lambda_1/2$ λ_1



$L = \lambda_2$ λ_2





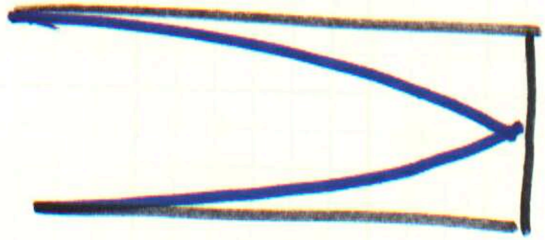
$L = \frac{3}{2}\lambda_3$ λ_3

$\vdots$ $\vdots$

$L = \frac{n}{2}\lambda_n \longrightarrow \boxed{\lambda_n = \frac{2}{n} L \quad n = 1, 2, \ldots}$

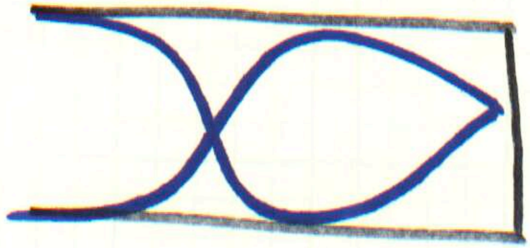
$\boxed{f_n = \frac{n}{2L} \cdot v \quad n = 1, 2, \ldots}$

$L = \lambda_1/4$



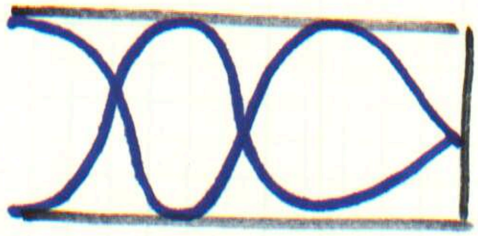
λ_1

$L = \frac{3}{4}\lambda_2$



λ_2

$L = \frac{5}{4}\lambda_3$



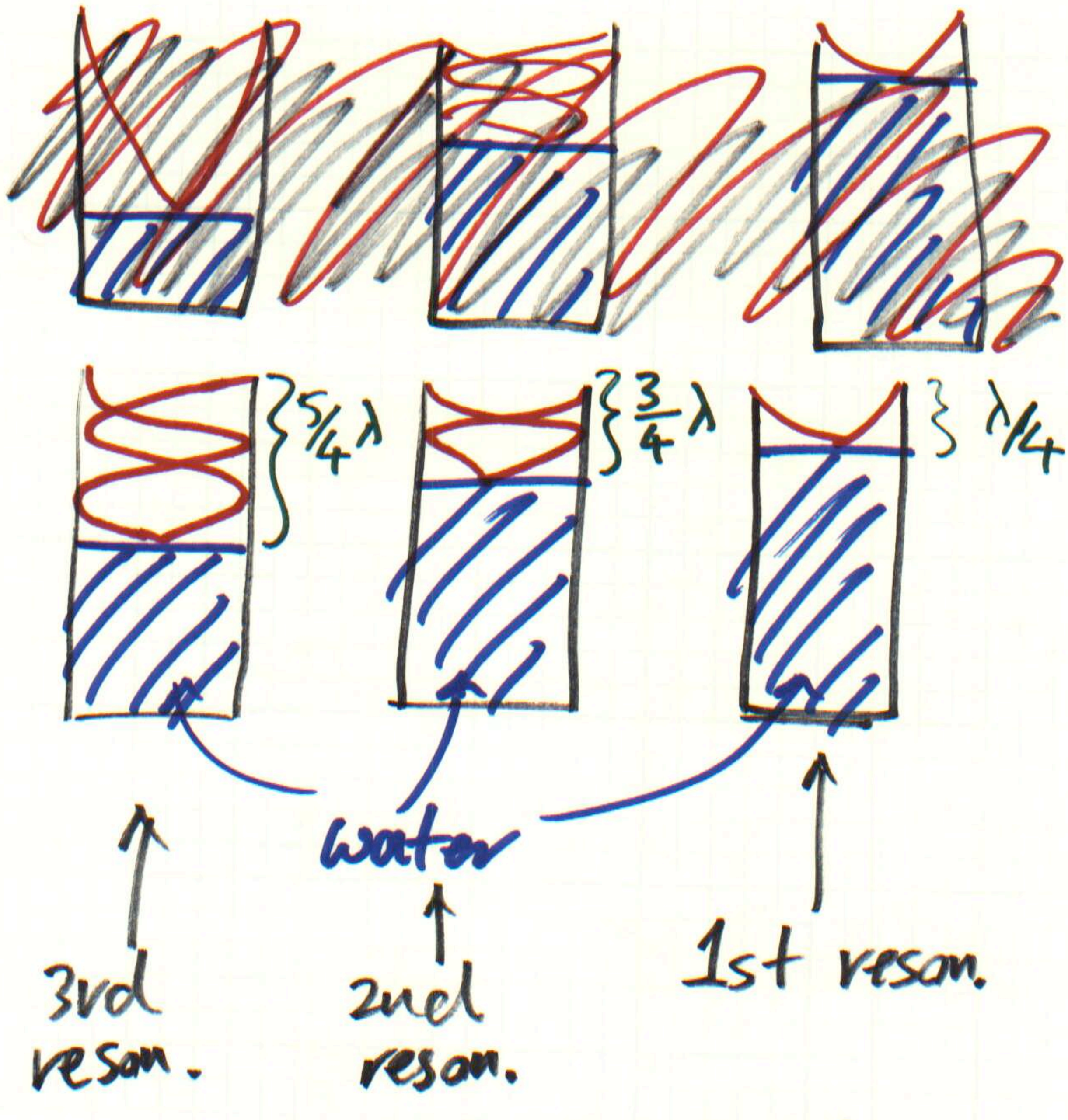
λ_3

$\vdots$

$L = \frac{(2n-1)}{4}\lambda_n \longrightarrow \boxed{\lambda_n = \left(\frac{4L}{2n-1}\right) ;\, n=1,2,\ldots}$

$$\boxed{f_n = \frac{2n-1}{4L}\cdot v \qquad n = 1,2,\ldots}$$

Find freq. of a tuning fork using air columns:

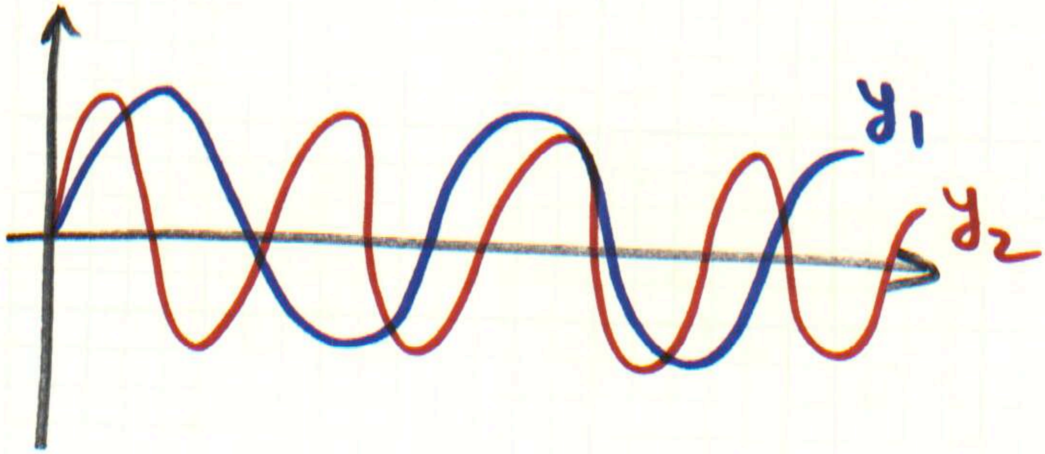


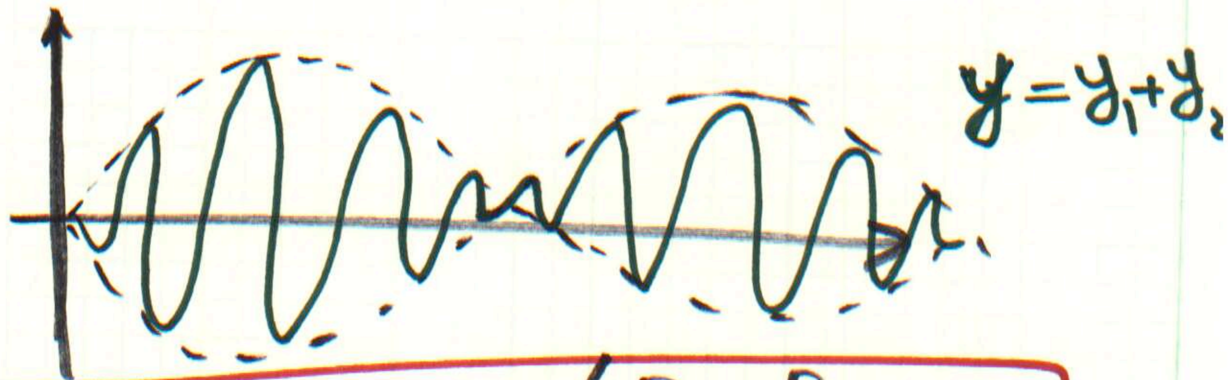
λ can be found $\Longrightarrow$ freq.

Beats

y_1 & y_2 have diff. frequencies

$f_1 \neq f_2$





$$\boxed{A_{tot.} = 2A \cos 2\pi \left(\frac{f_1 - f_2}{2}\right) t}$$

$\Longrightarrow$ Intensity varies in time

$$\boxed{f_{beat} = |f_1 - f_2}$$