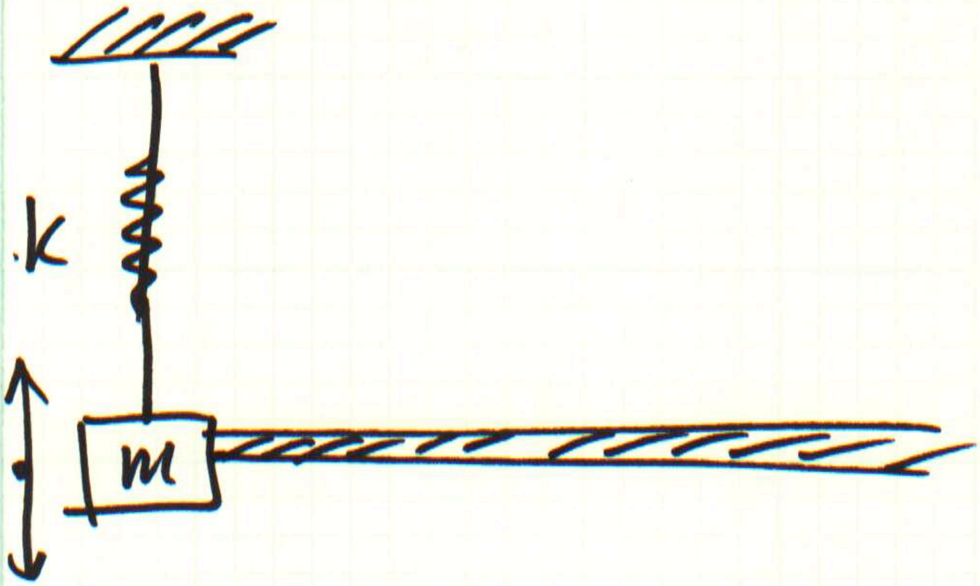
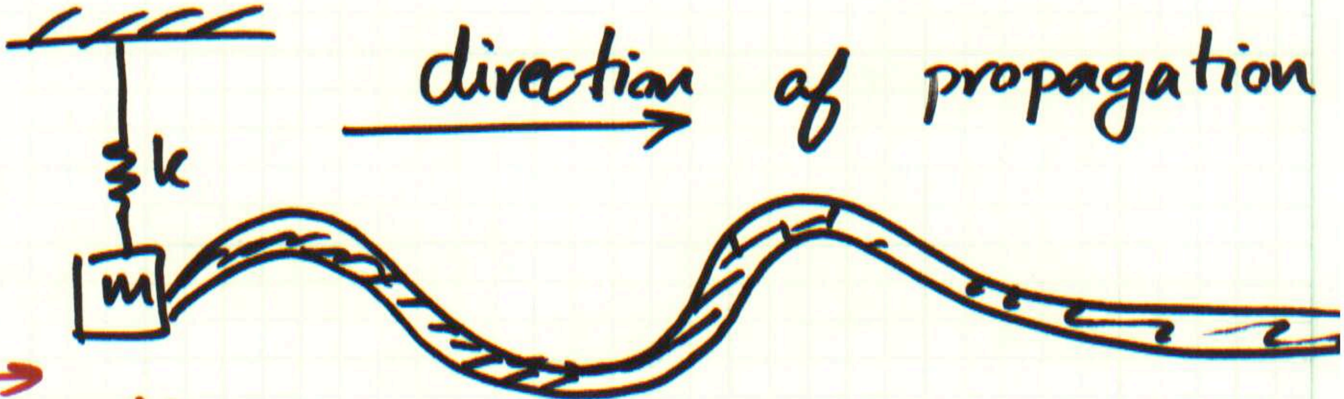
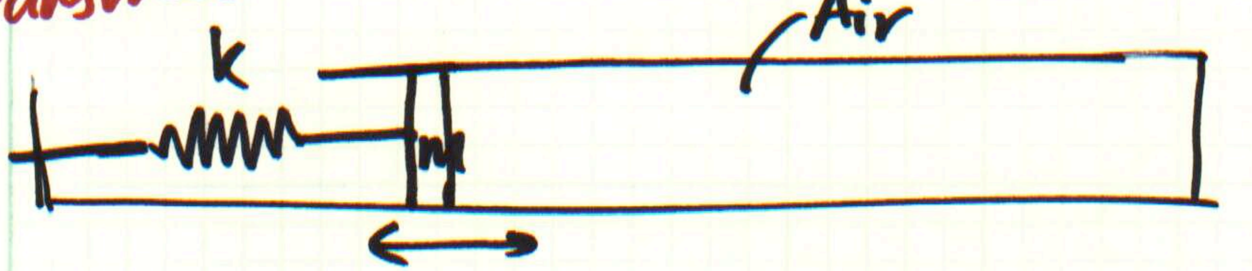
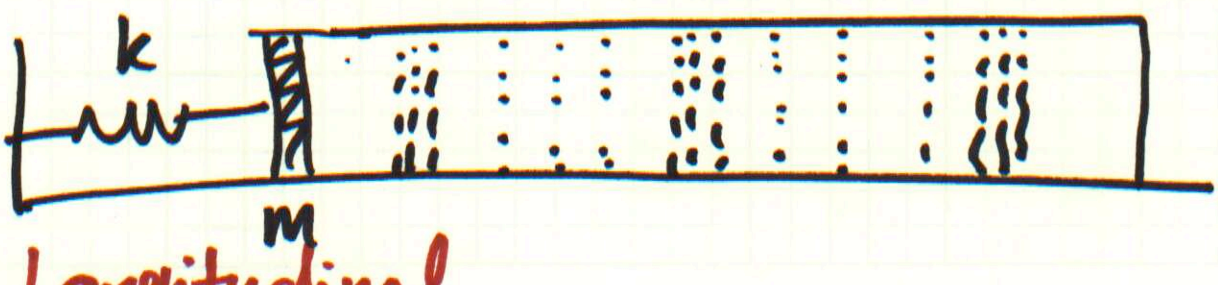
Traveling Waves





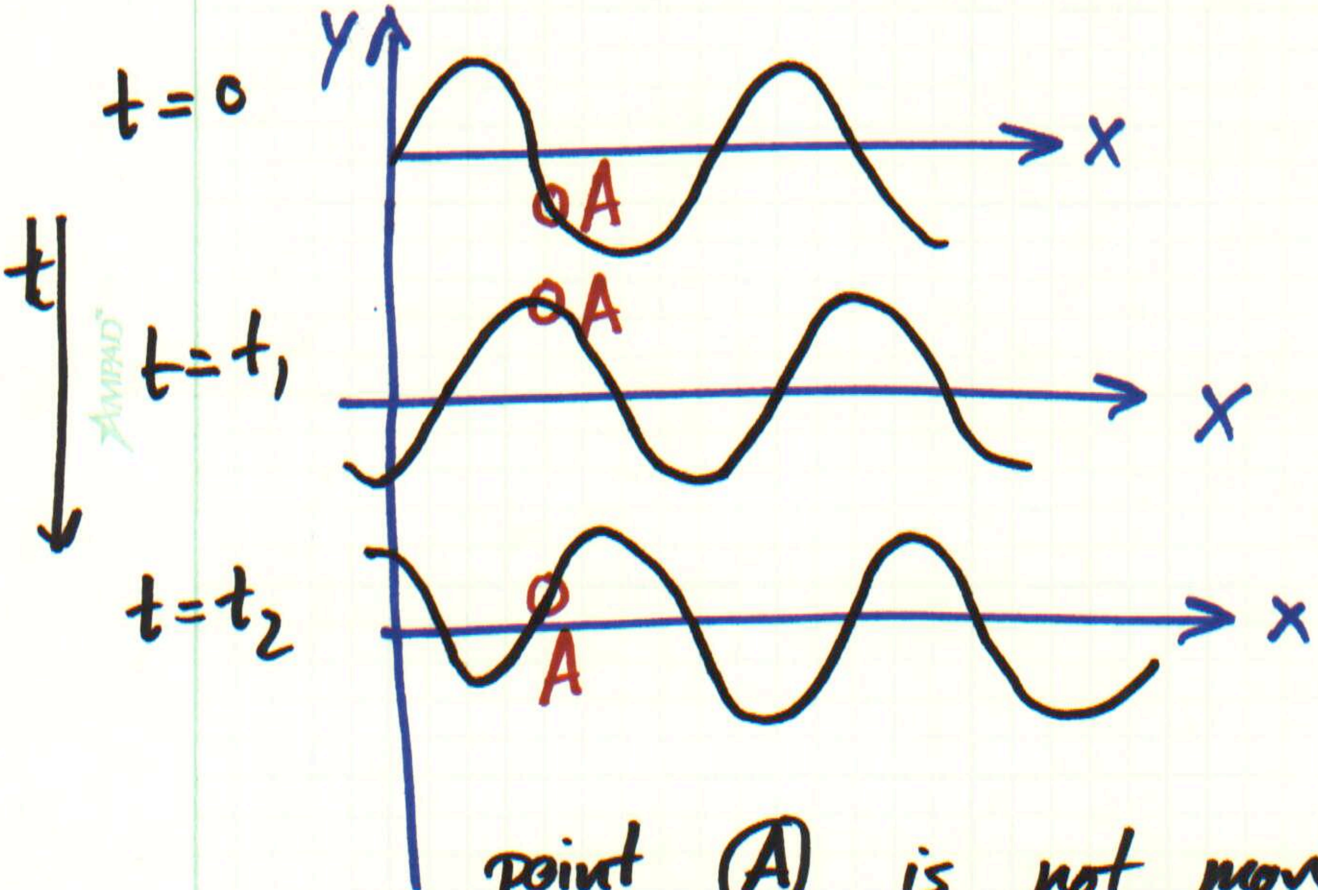
Transverse





Longitudinal

note: Matter is not transferred



point A is not moving w/ the wave.

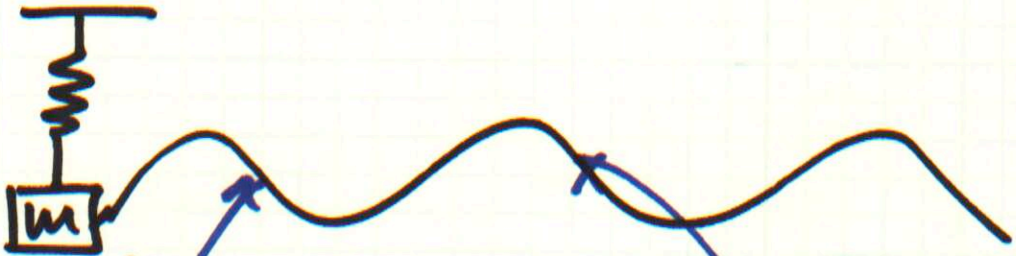
Trans. ⟶ Osc. $\perp$ ~~to~~ Prop.

Long. ⟶ Osc. $\parallel$ Prop.

Waves

- Source of disturbance
- medium
- Some physical mechanism for elements of medium to interact w/ each other

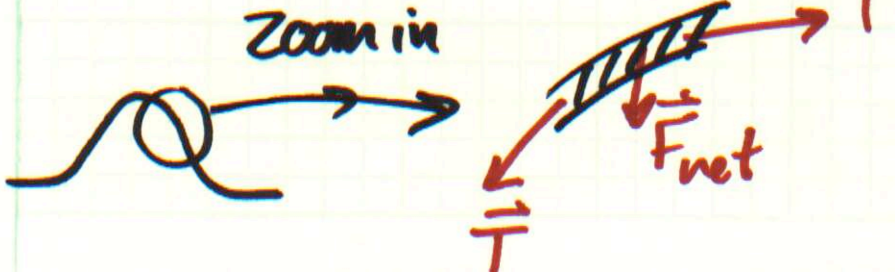
Ex. Rope



spring Osc. → Source of disturb.

rope → medium

tension of rope → phys. mech.

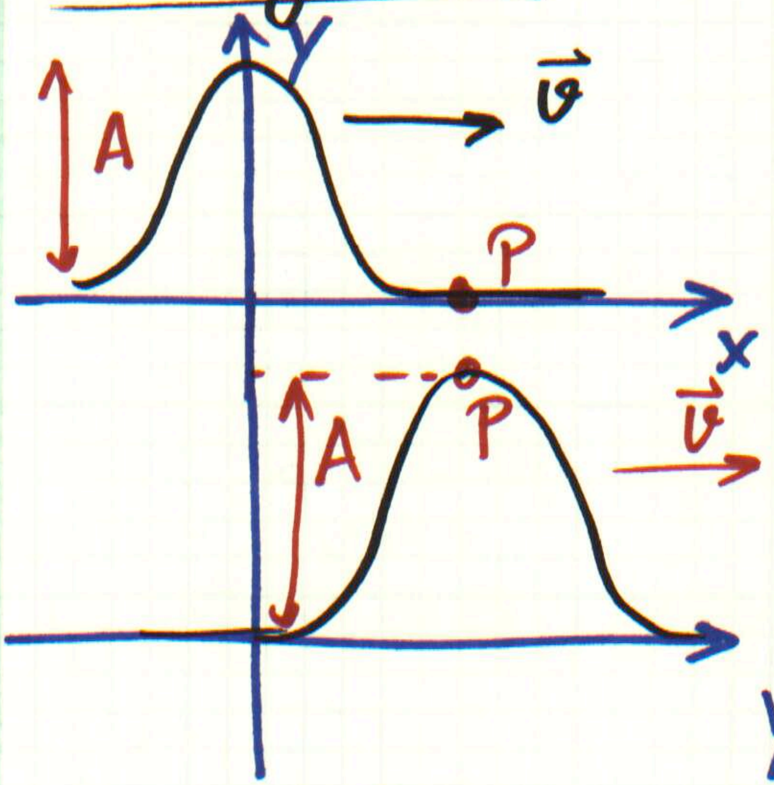


Other ex.

Sound waves, EM waves, ...

Demo

Travelling Wave



$t=0$

$y = f(x)$ (for p

$t>0$

$$y = f(x - vt)$$

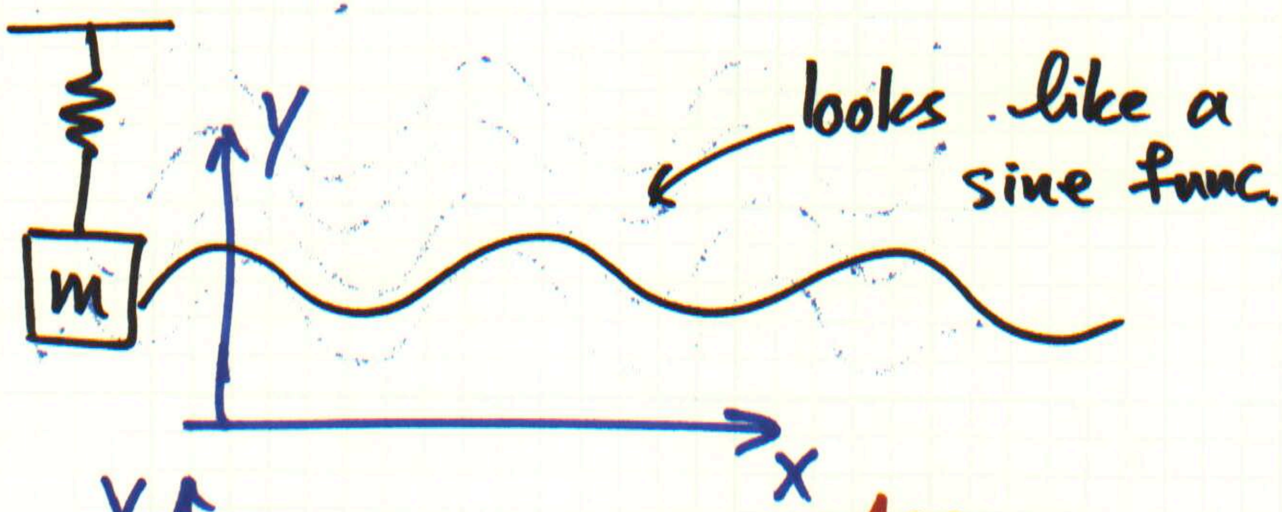
$x \longrightarrow$ $\quad \xrightarrow{\vec{v}}$ $\quad y = f(x - vt)$

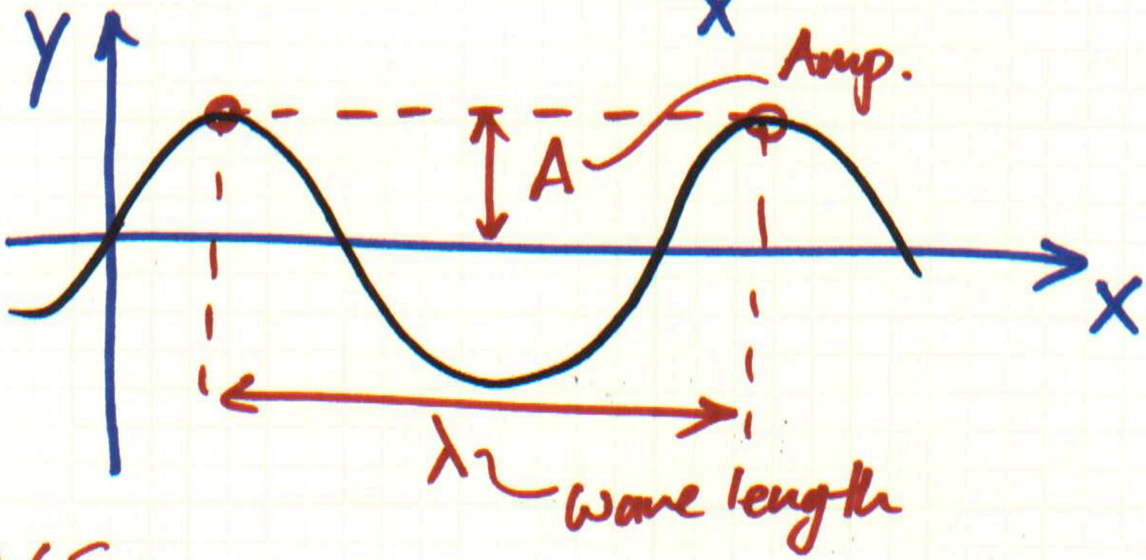
$\xleftarrow{\vec{v}}$ $\quad y = f(x + vt)$

Fixed t $\longrightarrow$ wave form

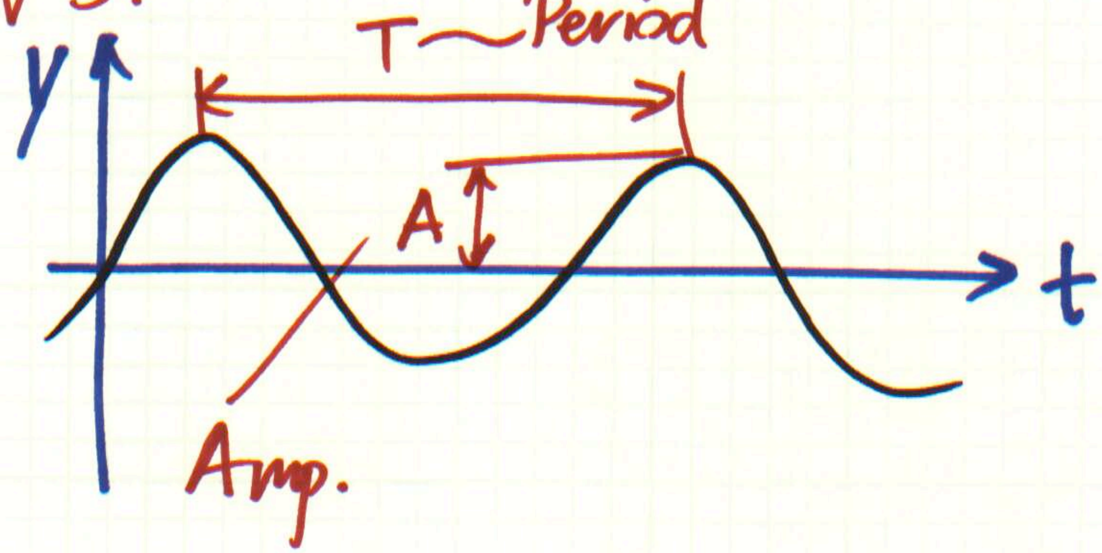
Like a snapshot of wav

Sinusoidal Wave





VS.



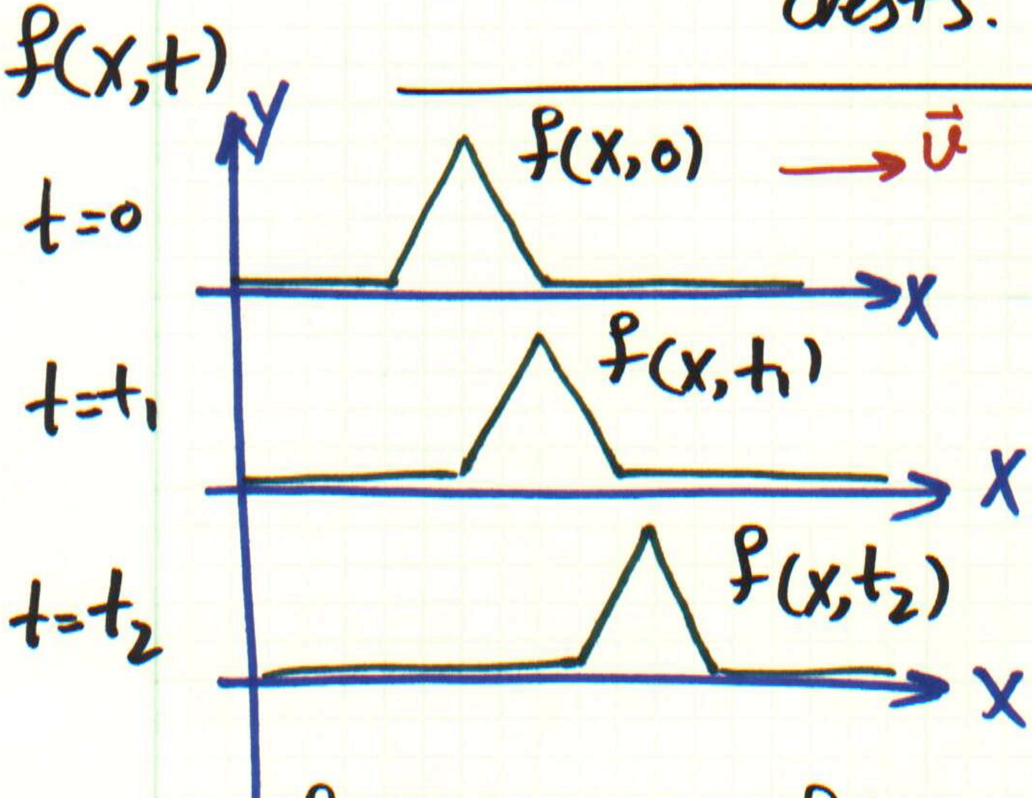
$T \equiv$ Period $\equiv$ ~~dist~~ Δt between two crests

unit $\rightarrow$ sec.

$\frac{1}{T} = f \equiv$ freq. $\equiv$ # of crests that pass a given point in a unit of time

unit $\rightarrow$ Hz

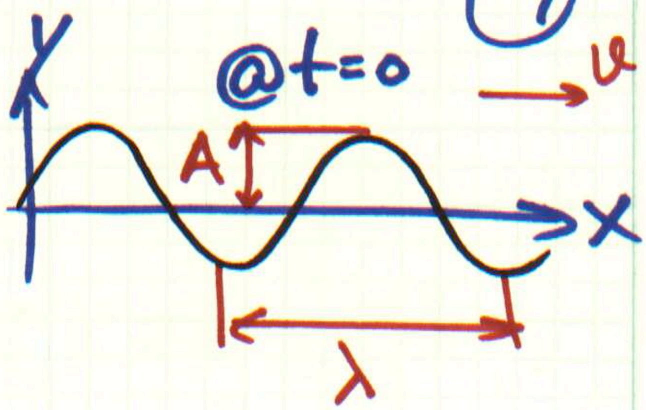
$\lambda \equiv$ wave length $\equiv$ Dist. between two crests.



[wave form doesn't change]

$f(x,t) \longrightarrow f(x-vt)$

Sinusoidal Wave



$f(x, t=0)$

$= A \sin\left(\frac{2\pi}{\lambda} \cdot x\right)$

why?

$x=0 \rightarrow f(x,t) = A \cdot \sin(0) = 0$

$x=\frac{\lambda}{4} \rightarrow f(x,t) = A \sin\left(\frac{2\pi}{\cancel{\lambda}} \cdot \frac{\cancel{\lambda}}{4}\right) = A$

$x=\lambda/2 \rightarrow f(x,t) = A \sin\left(\frac{2\pi}{\lambda} \cdot \frac{\lambda}{2}\right) = 0$

$x=\frac{3\lambda}{4} \rightarrow f(x,t) = A \sin\left(\frac{2\pi}{\lambda} \cdot \frac{3\lambda}{2}\right) = -A$

$x=\lambda \rightarrow f(x,t) = A \sin\left(\frac{2\pi}{\lambda} \cdot \lambda\right) = 0$

$t > 0$:

Amp. Pos.

$$\boxed{y(x,t) = A \sin\left[\frac{2\pi}{\lambda} \cdot (x \mp v t)\right]}$$

w.l. vel.

$$v = \frac{\Delta x}{\Delta t}$$

in how much time does a wave travel λ? $\longrightarrow T$

$$\rightarrow v = \frac{\lambda}{T} = \lambda \cdot f$$

$$\cancel{y(x,t) = A \sin\left[2\pi \cdot T\,(x - v t)\right]}$$

$$\boxed{y(x,t) = A \sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]}$$

define $k = 2\pi/\lambda$ (wave number)

$\omega = \dfrac{2\pi}{T}$ (Ang. freq.)

m

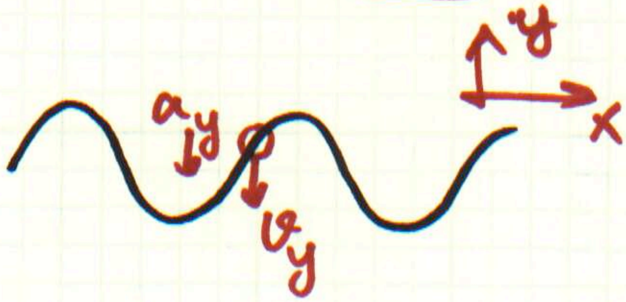
$$y(x,t) = A \sin(kx - \omega t)$$

or in general ($\phi \neq 0$):

$$\boxed{y(x,t) = A \sin(kx - \omega t + \phi)}$$

Q.E

v & a



$$v_y = \left.\frac{dy}{dt}\right|_{x=\text{const.}}$$

$$= \frac{\partial y}{\partial t} = -\underbrace{\omega A}_{v_{y_{max}}} \cos(kx - \omega t)$$

$$a_y = \left.\frac{dv_y}{dt}\right|_{x=\text{const.}}$$

$$= \frac{\partial v_y}{\partial t} = -\underbrace{\omega^2 A}_{a_{y_{max}}} \sin(kx - \omega t)$$

What does this mean physically?

Each point has a SHM

x der.:

$$\left.\frac{dy}{dx}\right|_{t=\text{const.}} = \frac{\partial y}{\partial x}$$

$$= -kA \cos(kx-\omega t)$$

$$\left.\frac{\partial^2 y}{\partial x^2}\right|_{t=\text{const.}} = \frac{\partial^2 y}{\partial x^2}$$

$$= -k^2 A \sin(kx-\omega t)$$

$$\Longrightarrow A \sin(kx-\omega t) = -\frac{1}{k^2}\frac{\partial^2 y}{\partial x^2}$$

$$A \sin(kx-\omega t) = -\frac{1}{\omega^2}\frac{\partial^2 y}{\partial t^2}$$

$$\longrightarrow \frac{\partial^2 y}{\partial x^2} = \frac{k^2}{\omega^2}\cdot\frac{\partial^2 y}{\partial t^2}$$

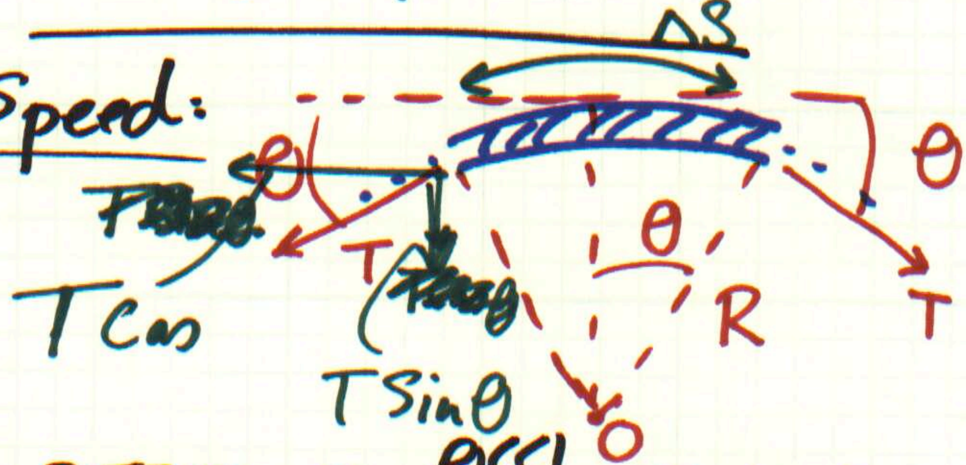
$$\frac{k}{\omega} = \frac{2\pi/\lambda}{2\pi/T} = \frac{T}{\lambda} = \frac{1}{v}$$

$$\Longrightarrow \boxed{\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \cdot \frac{\partial^2 y}{\partial t^2}}$$

General wave equ. for many types of waves.

e.g. spring, rope, EM, ...

Wave Speed:



$$\sum F = 2T \sin\theta \overset{\theta \ll 1}{\approx} 2T\theta$$

$$\text{mass} \longrightarrow m = \mu \cdot \Delta S = \mu \cdot \underbrace{2R\theta}_{\Delta S}$$

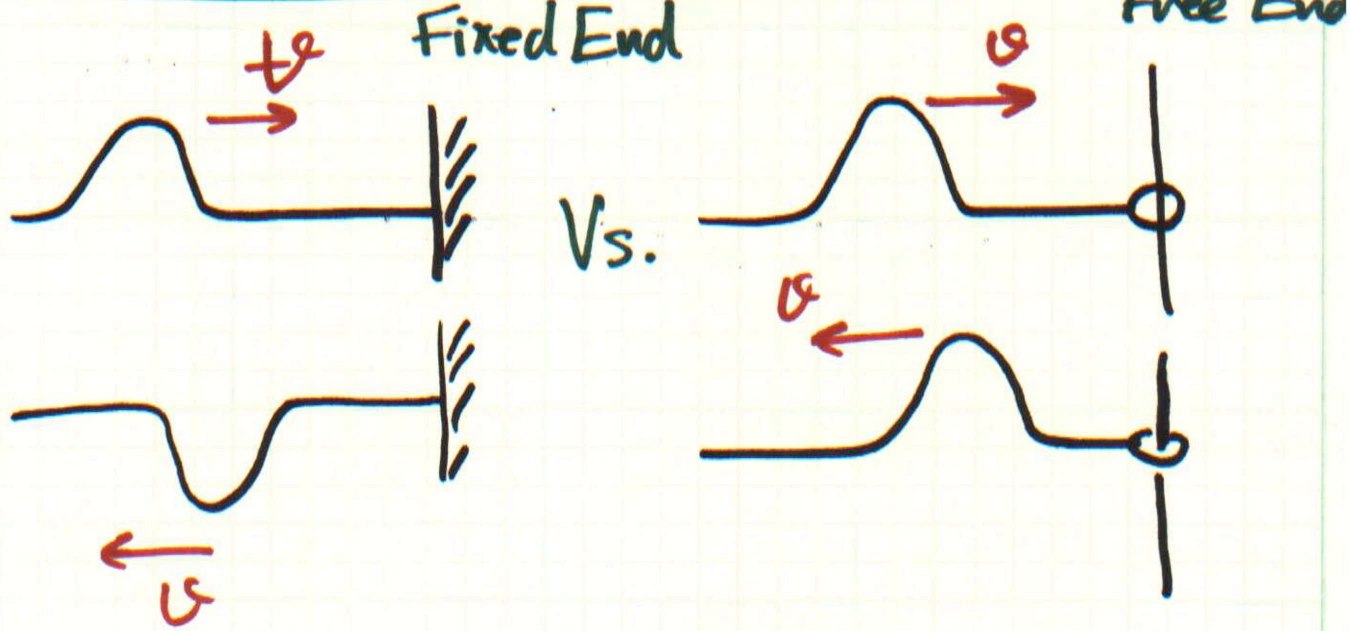
$$F = \frac{mv^2}{R} = \frac{\mu \cdot 2\theta \cancel{R} v^2}{\cancel{R}}$$

$$\cancel{2}T\cancel{\phi} = \cancel{2}\mu\cancel{\phi}v^2 \implies \boxed{v=\sqrt{\frac{T}{\mu}}}$$

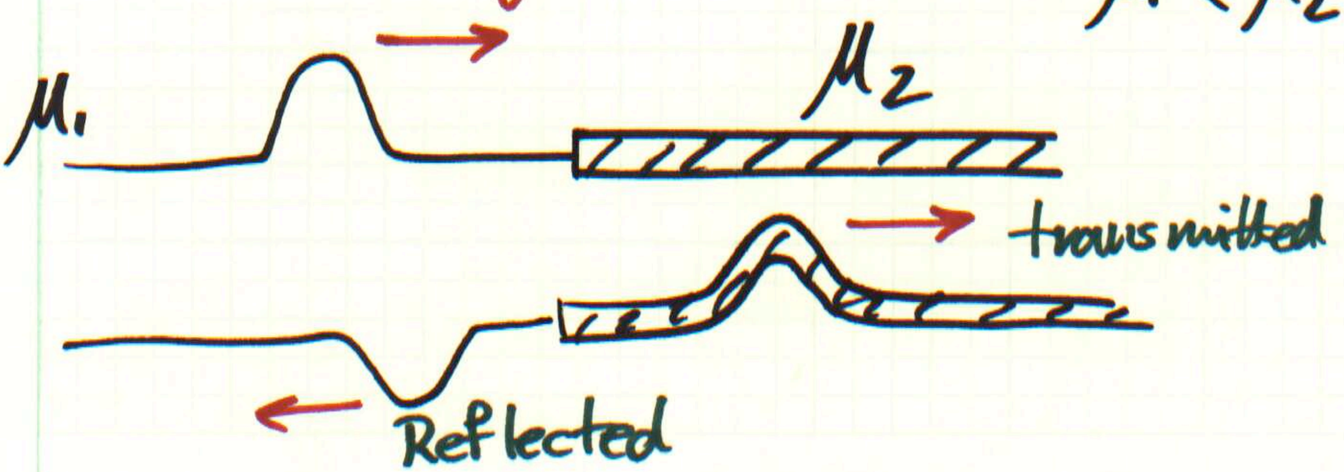
vel. Tension linear mass density

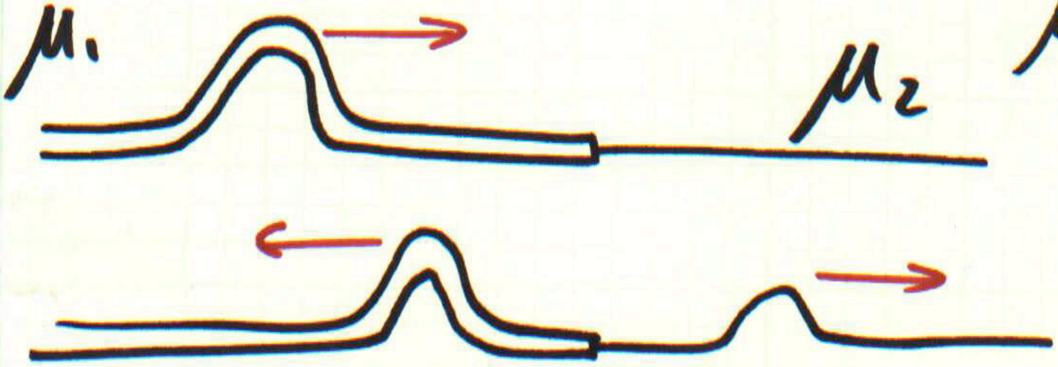
QE

Reflection & Transmission



$\mu_1 < \mu_2$



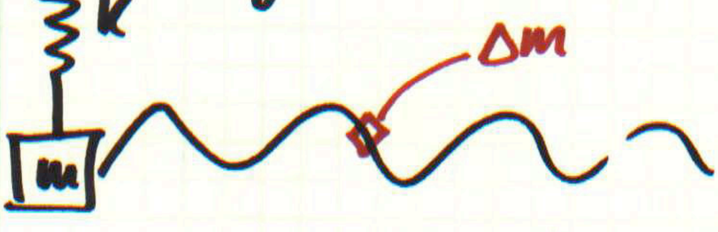


$\mu_1 > \mu_2$

Energy transfer

- Waves carry E. through a medium.
- Source of E $\longrightarrow$ source of disturb.

e.g.



Each element of the rope has a SHM $\xrightarrow[\text{element}]{\text{for each}}$ $\Delta K = \frac{1}{2} \Delta m \, v_y^2$

$$\Delta K = \frac{1}{2} \cdot \Delta m \cdot v_y^2$$

$$= \frac{1}{2} \cdot \mu \cdot \Delta x \cdot v_y^2$$

as $\quad \Delta K \longrightarrow dK$

$$dK = \frac{1}{2} \cdot \mu \cdot dx \cdot v_y^2$$

$$= \frac{1}{2} \mu \omega^2 A^2 \cos(kx - \omega t)\, dx$$

$$\xrightarrow{\underline{\text{Int.}}\, \Big|_0^{\lambda}} K_\lambda = \frac{1}{4} \mu \omega^2 A^2 \lambda$$

$U = ?$

$$\longrightarrow U_\lambda = \frac{1}{4} \mu \omega^2 A^2 \lambda$$

$$\Longrightarrow \boxed{E_\lambda = \frac{1}{2} \mu \omega^2 A^2 \lambda}$$

<u>Power</u>

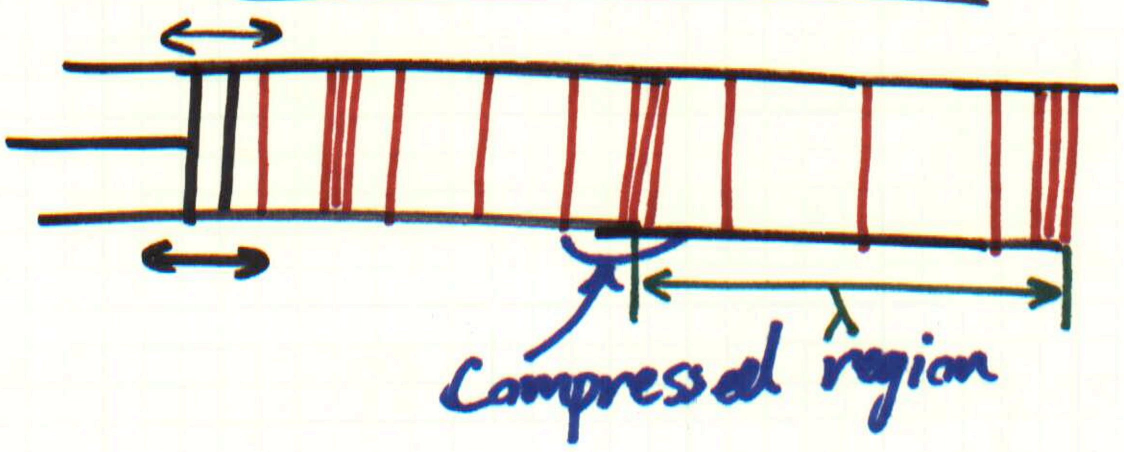
v

$$\mathcal{P} = \frac{\Delta E}{\Delta t} = \frac{\frac{1}{2} \mu \omega^2 A^2 \lambda}{T} \qquad \boxed{\mathcal{P} = \frac{1}{2} \mu \omega^2 A^2 v}$$

Sound Waves

- longitudinal waves
- travel through any medium (gas, liquid, solid)
- ~~v depends on~~
- Wave speed depends on properties of medium

in gen. $v \propto \sqrt{\dfrac{\text{elastic prop.}}{\text{inertial prop.}}}$



Wave Eqn.

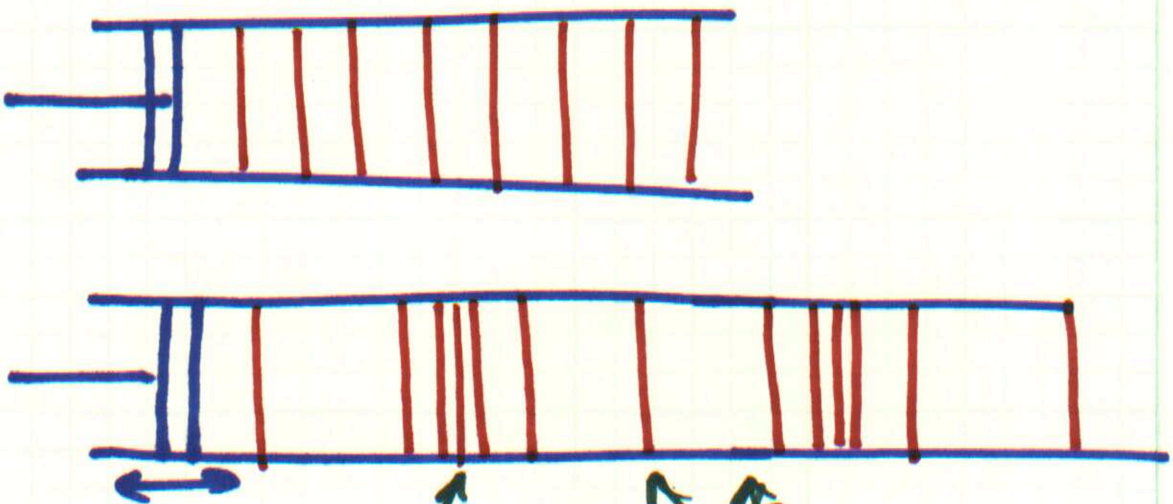
$$s(x,t) = \underbrace{S_{max}}_{A} \sin(kx - \omega t)$$

wave number

Ang. freq.

S represents displacement

@ rest



high pressure

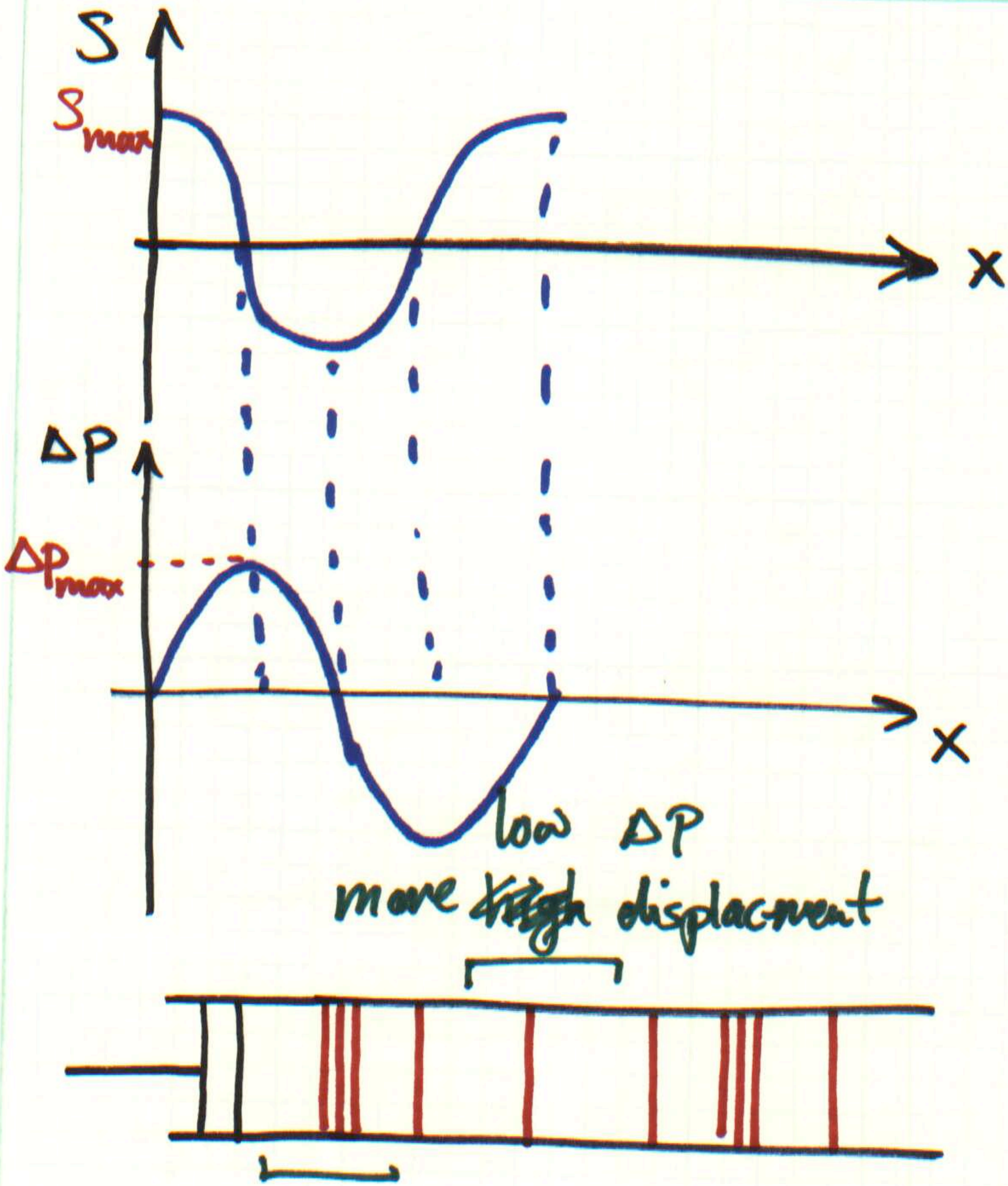
low pressure

pressure wave

$$\Delta P = \Delta P_{max} \cos(kx - \omega t)$$

Compare to sin in displacement eqn.

$$\Delta P_{max} = \rho v \omega s_{max}$$



high ΔP
less displacement

q: At what speed do sound waves travel? ~~at~~

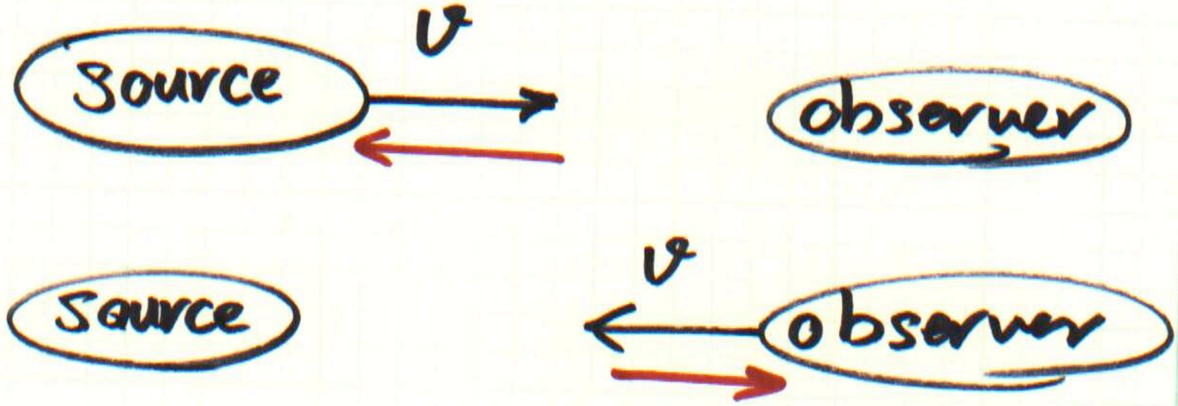
v_{sound} depends on temperature of the air

$$v = 331\ m/s + (0.6\ m/s \cdot {}^\circ C)\,T_C$$

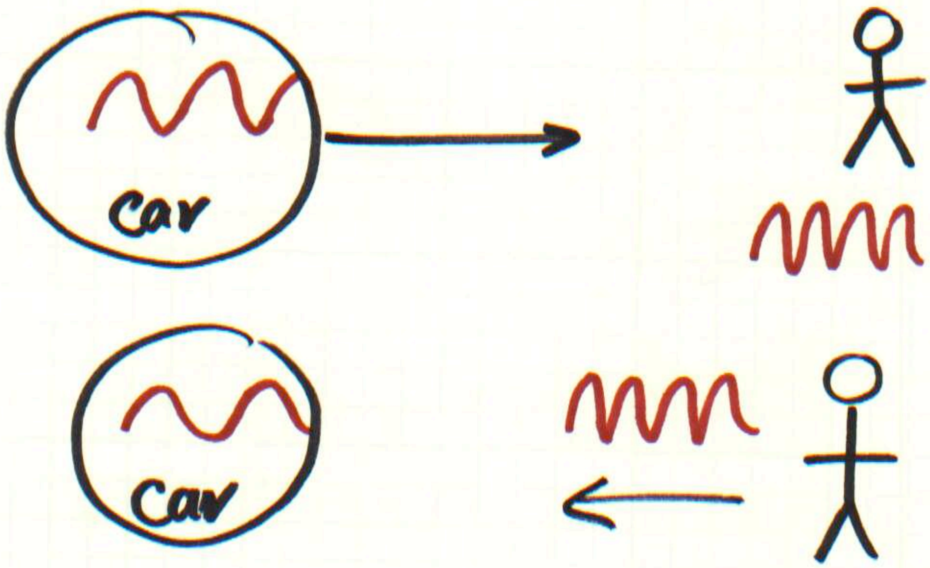
temp. in celcius

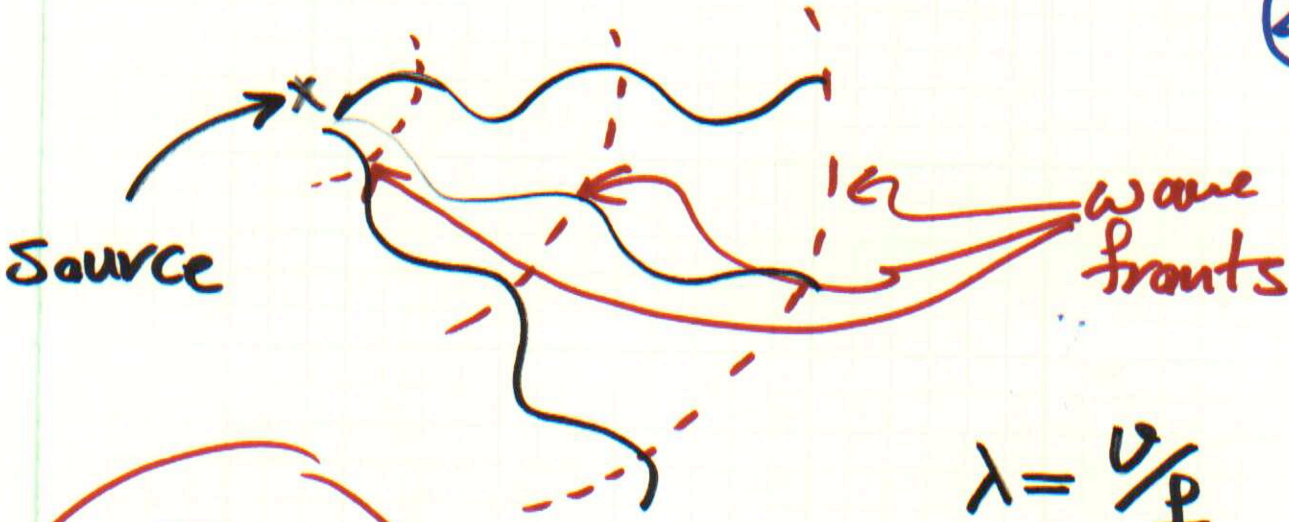
In general speed of sound in different media depends on many factors: gas/liquid/solid, temp, ...

Doppler Effect

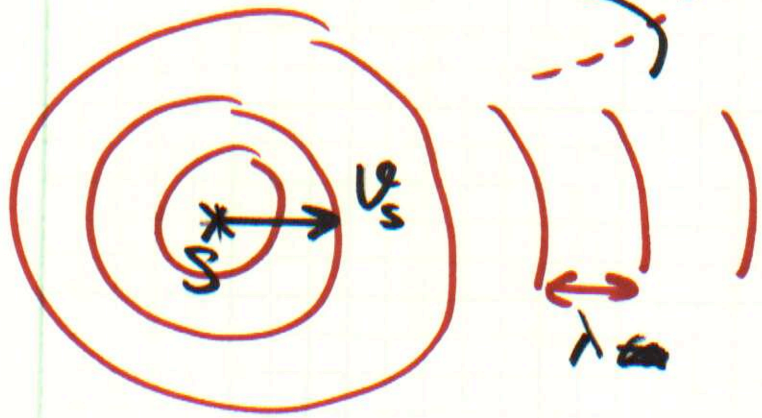


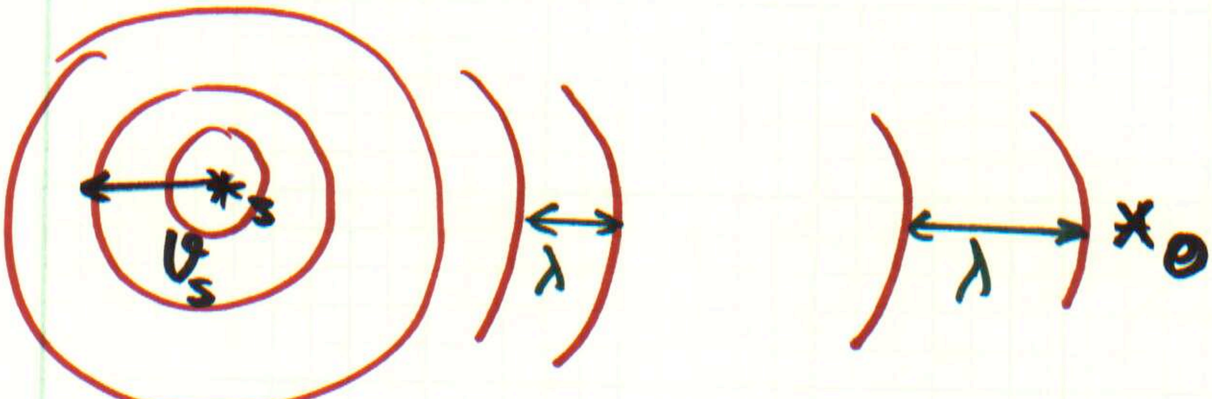
freq. of sound that source is making (f_s) is different for the observer. (f_o)





$\lambda = v/f$





Similar when the observer moves.

$\cancel{f' = \left(\frac{v+v_o}{v}\right) f}$ $f = v/\lambda$ → freq. also change in DE

$$f' = \left(\frac{v \pm v_o}{v}\right) f$$

← → Source obs.

$$f' = \left(\frac{v}{v \mp v_o}\right) f$$

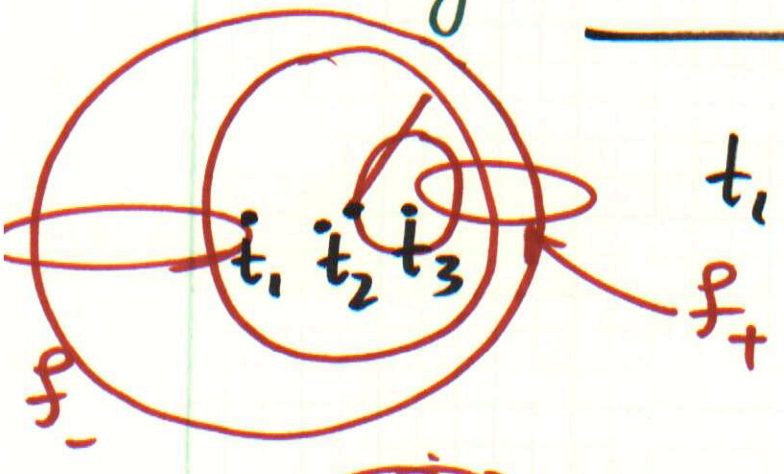
Source ← → obs.

$$f' = \left(\frac{v + v_o}{v - v_s}\right) f$$

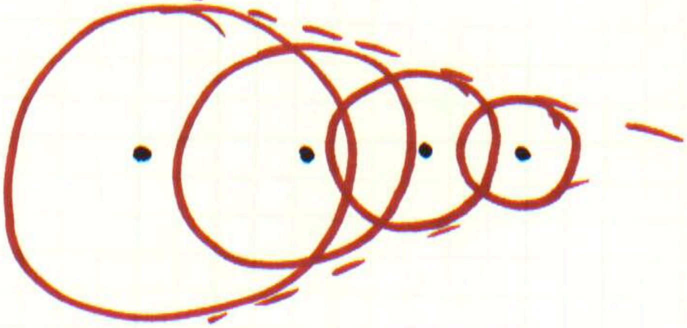
Source → ← obs.

towards $\longrightarrow f_+$

away from $\longrightarrow f_-$



$t_1 < t_2 < t_3$



Shock wave

$v_o > v$