Class Intro

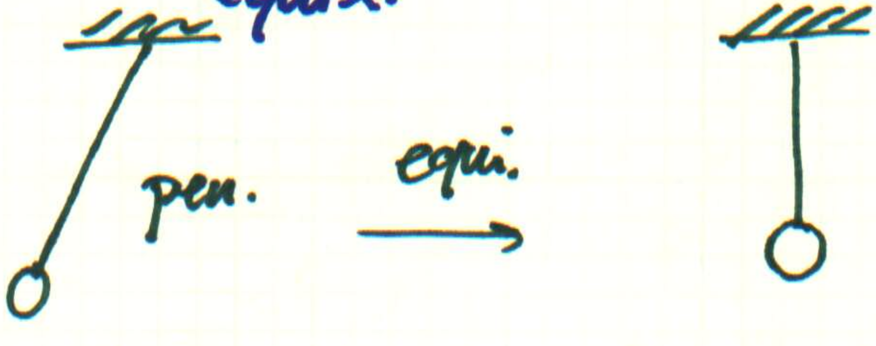
Osaillatory Motion

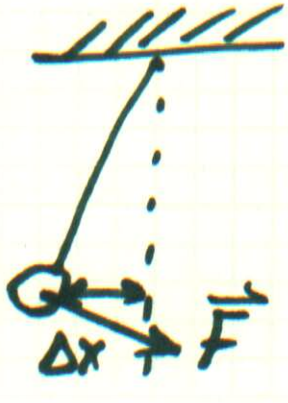
periodic motion: examples

↳ special kind in mech. sys.

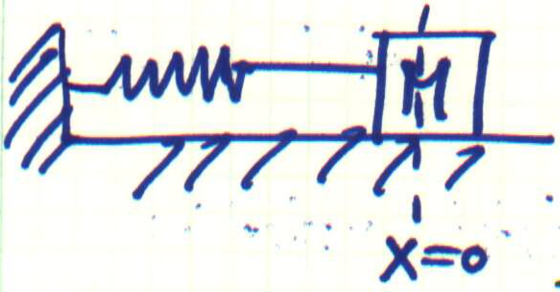
when <u>Force</u> is <u>proportional</u> to the <u>position</u> of obj.

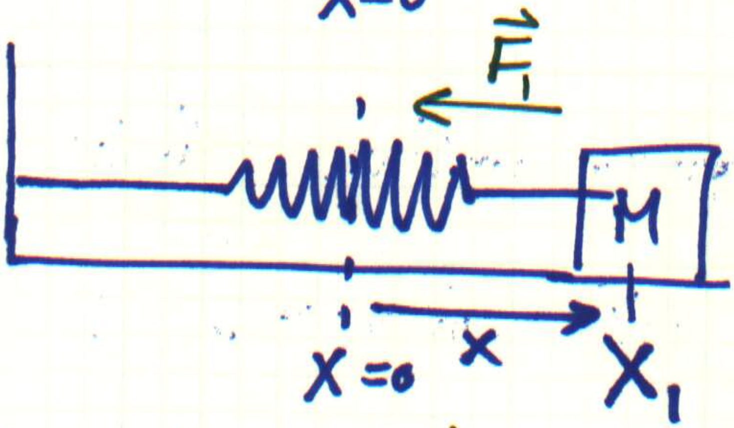
↳ <u>Force</u> always towards equil.





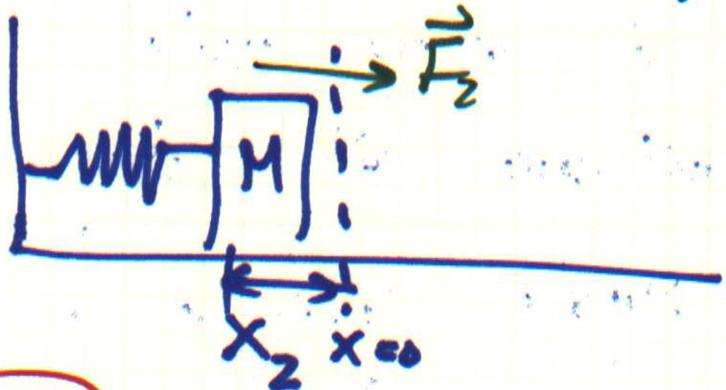
$$\vec{F} \propto \vec{\Delta x}$$





$\vec{F} \propto \vec{x}$

$\vec{F}_1 = -k\vec{x}_1$



$\vec{F}_2 = -k\vec{x}_2$

spring const.

$\boxed{\vec{F} = k\vec{x}}$ ← Hooke's law

Newton's 1st law ⟶ $\vec{F} = m\vec{a}$

$\Longrightarrow m\vec{a} = -k\vec{x}$

$$m\frac{d^2x}{dt^2} = -kx \xrightarrow{\text{ODE}} \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

$a = \frac{d^2x}{dt^2} = -\left(\frac{k}{m}\right) x \longrightarrow$ $\underline{a}$ propor. to negative of $\underline{\text{position}}$ (x)

$\hookrightarrow$ SHM whenever

Note:- $\vec{a}$ is not <u>const.</u> & changes ~~during~~ as $\vec{x}$ changes

(movie?)

\- no friction $\longrightarrow$ SHM will continue forever

ODE

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

** What should $\underline{x}$ be?

2nd deriv. of $\underline{x}$ is prop. to $\underline{x}$ w/ a (-) sign.

$\longrightarrow$ Sin & Cos

~~Try: $x(t) =$~~

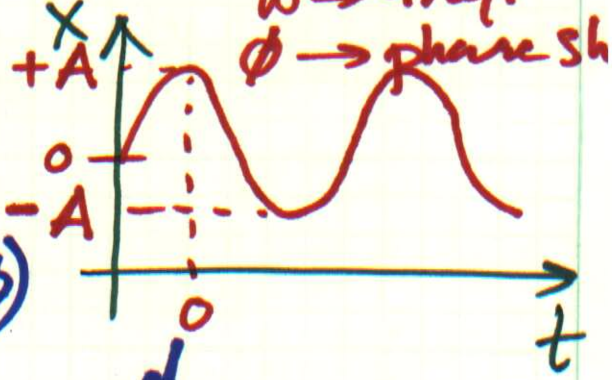
Reminder:

$$\cos(t) \longrightarrow A\cos(\omega t+\phi)$$

$\omega \rightarrow$ freq.

$\phi \rightarrow$ phase sh

try the gen. form:



$$x(t) = A\cos(\omega t+\phi)$$

$$\frac{d^2x(t)}{dt} = \frac{d}{dt}\left(\frac{dx(t)}{dt}\right) = \frac{d}{dt}\left(A\omega\sin(\omega t+\phi)\right)$$

$$= -\underbrace{A\,\omega^2\cos(\omega t+\phi)}_{x(t)}$$

$$\rightarrow \frac{d^2x(t)}{dt^2} = -\omega^2 x(t)$$

$$a = -\left(\frac{k}{m}\right)x(t) \longrightarrow \boxed{\omega = \sqrt{\frac{k}{m}}}$$

SHM def.

$$x(t) = A \cos(\omega t + \phi)$$

A: Amplitude (units: dist.)

ω: Ang. frequency (units: rad/s)

ϕ: phase const. (unit. phase) (rad)

<u>Amp.</u>

A is ~~eq~~ max dist. from equil.

(if $x = A$ @ $t = 0$ $\rightarrow$ $\phi = 0$)

$t = 0$

<u>ω</u>

reminder $\rightarrow$ ~~F~~ period T

time required to ~~go~~ complete 1 cycle

x, v, a are the same @

$t, t + T, t + 2T, \ldots$

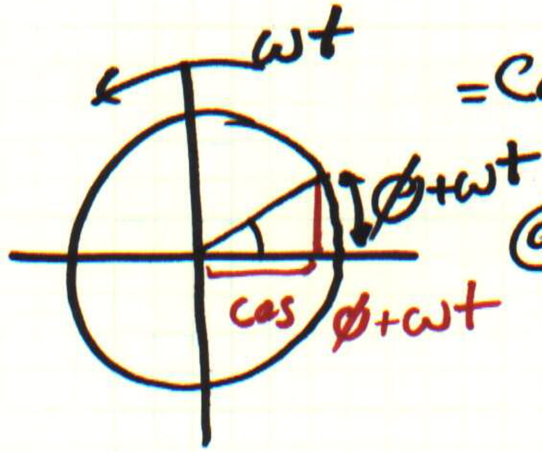
$$\boxed{T = 2\pi/\omega \;;\; f = 1/T = \omega/2\pi}$$

$[T] = s \qquad [f] = \frac{1}{s} = Hz$ ⑤

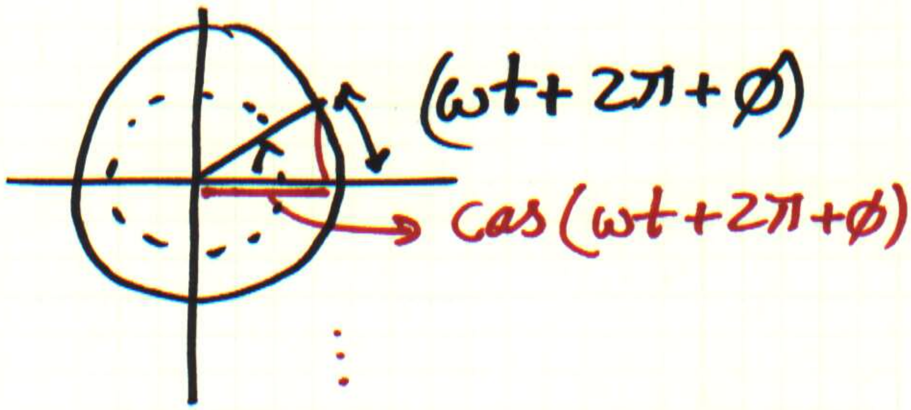
why $T = \frac{2\pi}{\omega}$?

$$\text{Cos}(\omega + \phi) = \text{Cos}(\underline{2\pi + \omega t} + \phi)$$

$$= \text{Cos}(\underline{2\pi \cdot n + \omega t} + \phi)$$



@ t



$$\boxed{T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \qquad f = \ldots}$$

only dep. on <u>mass</u>↑ & <u>k</u>↓ nothing else

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$$\vec{X}(t) \xrightarrow{dx/dt} \vec{V}(t) \xrightarrow{dv/dt} \vec{a}$$

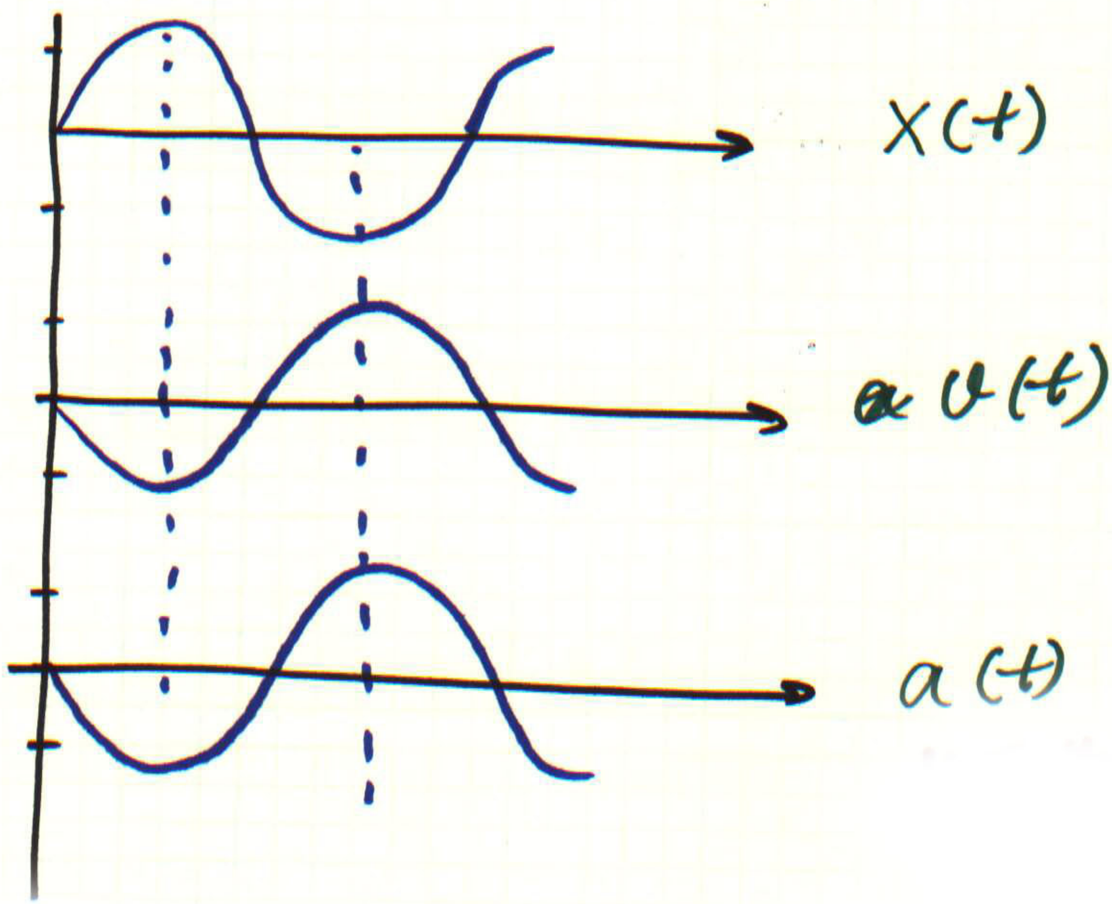
$$\vec{x}(t) = A\cos(\omega t + \phi)$$

$$\vec{v}(t) = -\omega A \sin(\omega t + \phi)$$

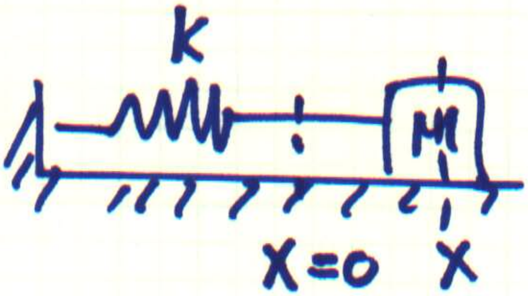
$$\vec{a}(t) = -\omega^2 A \cos(\underline{\omega t} + \phi)$$

time dep.

Note: not uniform $\underline{acc.}$



Energy in SHM



(no friction)
↓
E in cost.

E ? $\quad E = K + U$ ← pot. E

kinetic E

$$K = \frac{1}{2} m v^2 = \frac{1}{2} m \left(\frac{dx}{dt}\right)^2$$

$$= \frac{1}{2} m \left[-\omega A \sin(\omega t + \phi)\right]^2$$

$$= \frac{1}{2} m A^2 \omega^2 \sin^2(\omega t + \phi)$$

U = ? $\quad F = -\frac{dU}{dx} \longrightarrow U = \int F dx$

$$F = -kx \underset{dU/dt}{\overset{int.}{\rightleftarrows}} U = \frac{1}{2} k x^2$$

$$U = \frac{1}{2} k x^2 = \frac{1}{2} k A^2 \cos^2(\omega t + \phi)$$

$$K = \frac{1}{2} m A^2 \omega^2 \sin^2(\alpha)$$

$$\rightarrow m\omega^2 = m\left(\sqrt{\frac{k}{m}}\right)^2 = \cancel{m} \cdot \frac{k}{\cancel{m}} = k$$

$$K = \frac{1}{2} \cancel{A^2}\, k A^2 \sin^2(\alpha)$$

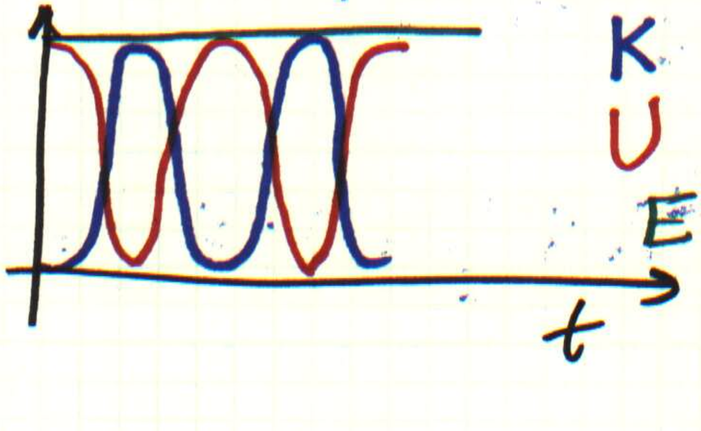
$$U = \frac{1}{2} k A^2 \cos^2(\alpha)$$

$$E = K + U = \frac{k}{2} A^2 \underbrace{\left(\sin^2\alpha + \cos^2\alpha\right)}_{1}$$

$$\boxed{E = \frac{1}{2} k A^2}$$

$\Longrightarrow$ total mech. E is const.

$\hookrightarrow$ cont. transferred between K. & U.



$$E = \frac{1}{2} k A^2 \xrightarrow[\text{solve for } A]{\cancel{A \text{ can be}}} \boxed{A = \pm\sqrt{\frac{2E}{k}}}$$

or

$$E = K + U = \cancel{\tfrac{1}{2}} m v^2 + \cancel{\tfrac{1}{2}} k x^2 = \cancel{\tfrac{1}{2}} k A^2$$

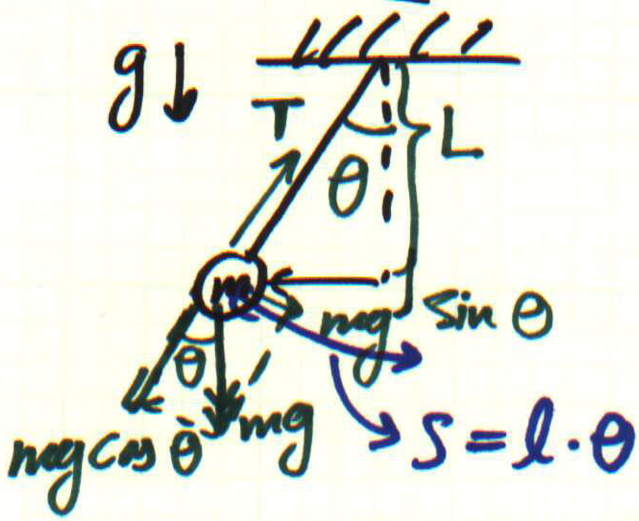
solve for v:

$$v^2 = k \, \frac{A^2 - x^2}{m} \longrightarrow \boxed{v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)}}$$

Simple Pendulum

~~consit~~

- consist of mass (m) & light string (l)
- 2D motion~~s~~



tangential component of the grav. force is the restoring force.

$$F_t = ma_t \longrightarrow -\cancel{m}g \sin\theta = \cancel{m}\frac{d^2s}{dt^2}$$

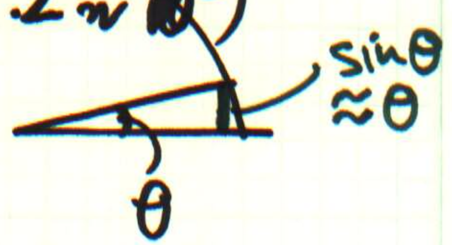
$$-g \sin\theta = \frac{d^2}{dt^2}(l\cdot\theta) \qquad l \rightarrow \text{const.}$$

$$= l\,\frac{d^2\theta}{dt^2}$$

$$\frac{d^2\theta}{dt^2} = -\left(g/l\right)\sin\theta$$

$\theta \ll 1$, small θ: $(\theta < 2 \sim 10^\circ)$

$$\sin\theta \approx \theta$$



[note: θ in Rad.]

$$\frac{d^2\theta}{dt^2} \approx -\left(g/l\right)\theta \xrightarrow[\text{as SHM}]{\text{same form}} \frac{d^2\theta}{dt^2} = -\omega^2\theta$$

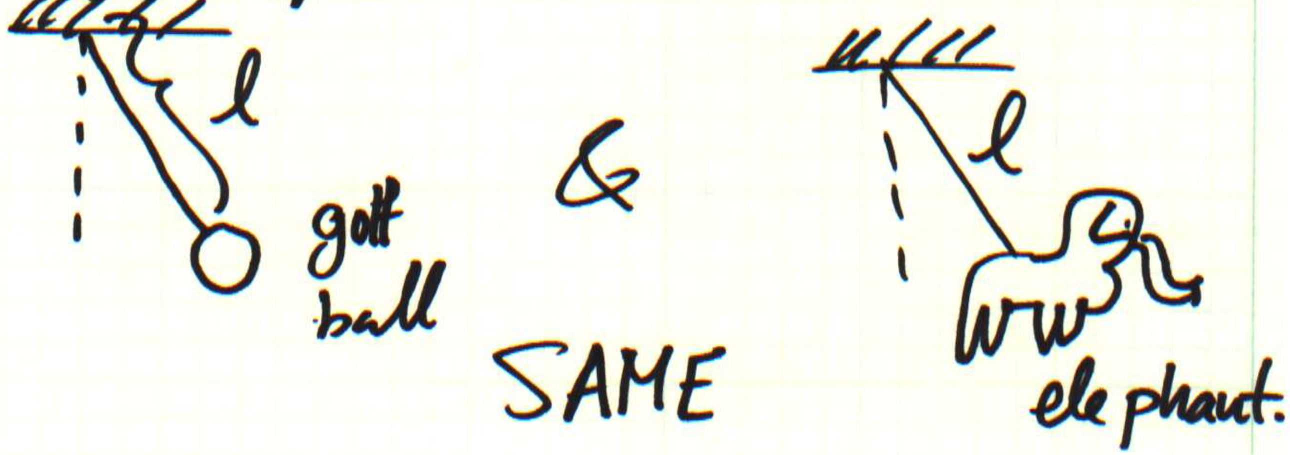
$$\frac{d^2\theta}{dt^2} = -\omega^2\theta \quad \text{w/} \quad \omega = \sqrt{g/l}$$

$$\theta(t) = \theta_{max}\cos(\omega t + \phi)$$

$$\rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}}$$

$\Longrightarrow$ * T only dep. on only l & g

* ind. of mass (m)



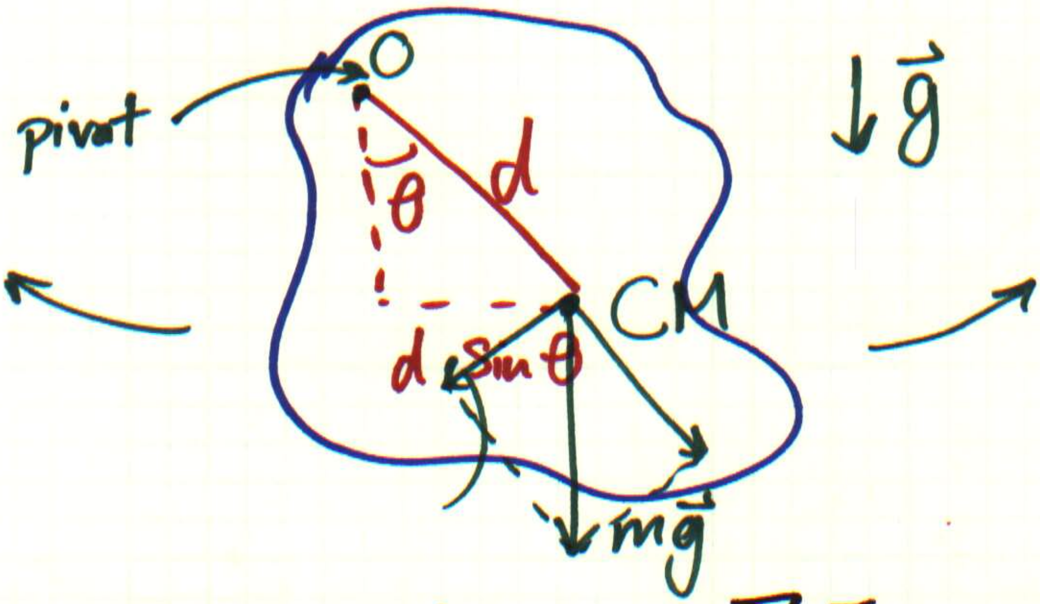
(qust. & exampls) AF 12.10

Physical Pendulum

- Obj. not approx. as a particle
- Obj. ~~hanging~~ not osaillating around a fixed axis that dos not pass through its center. of mass.

→ - not a simple pendulum!!

- rigid obj model



Newton's 2nd law: $\sum_i F_i = ma$

$$\sum_i \tau_i = I \cdot \alpha$$

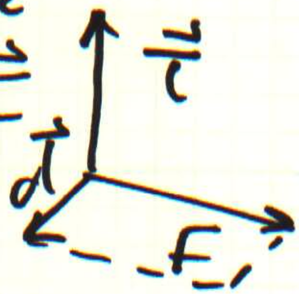
$\vec{\tau} = I \cdot \vec{\alpha}$

$\alpha = d^2\theta / dt^2$

$I \equiv$ moment of inertia

$\tau \equiv$ torque

$\tau = \vec{d} \times \vec{F}$



$\vec{\tau} = -m\vec{g} \times \vec{d} = -mgd \sin\theta$

$\longrightarrow -mgd \sin\theta = I \dfrac{d^2\theta}{dt^2}$

$$\frac{d^2\theta}{dt^2} = -\left(\frac{mgd}{I}\right) \sin\theta$$

same form SHM $\longrightarrow$ $\dfrac{d^2\theta}{dt^2} = -\omega^2 \sin\theta$

w/ $\omega = \sqrt{mgd/I}$

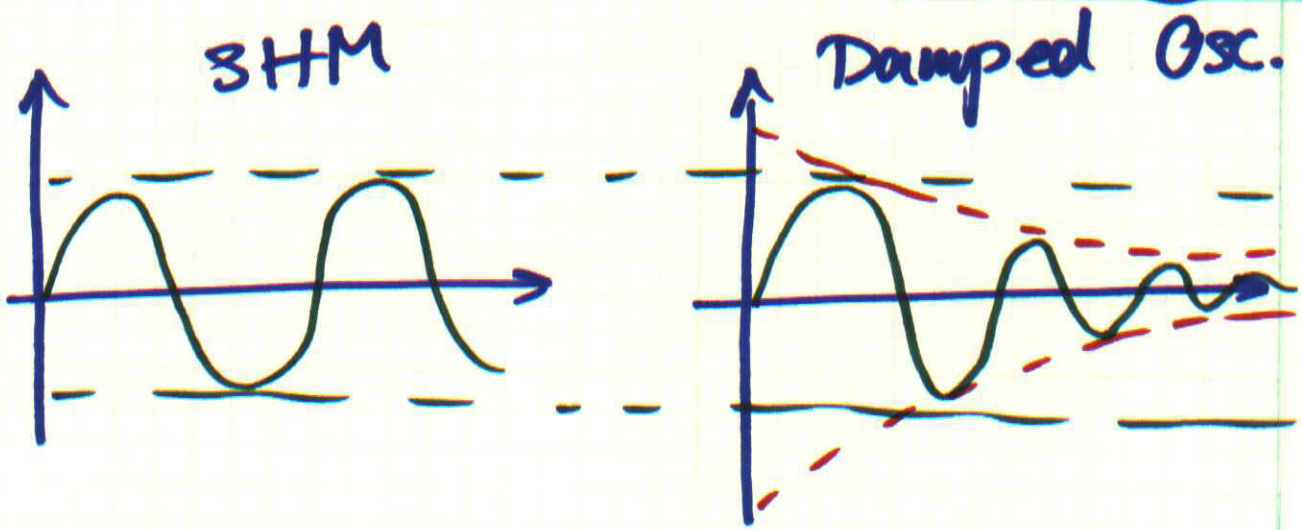
$$\Longrightarrow T = 2\pi \sqrt{\frac{I}{mgd}}$$

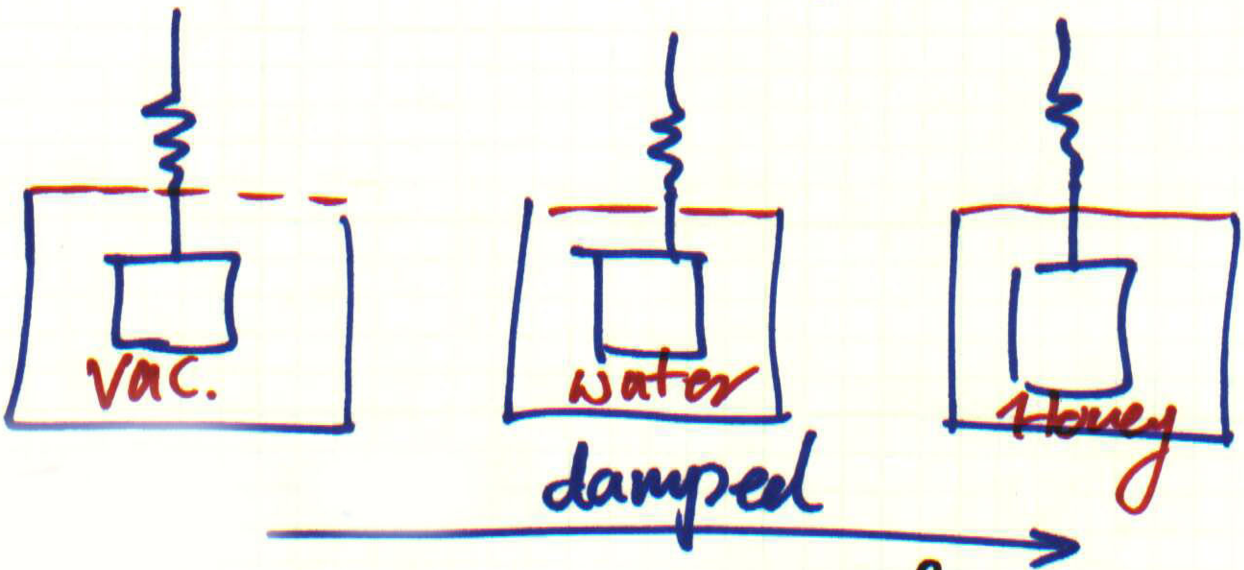
* Phys. Pend. ⟶ measure I
(knowing d)

* If $I = md$ then phys. pend.
is the same as ~~sy~~ simple pend.
(mass all at the center)

Damped Oscillation:

* In real sys. noncons. forces are present.
  → not an ideal sys.
  → Friction present
* E is not a const anymore
  ⟹ damped osc.





presence of resistive force:

$$\vec{R} = -\underbrace{b}_{\text{const.}}\vec{v}$$

$$\sum F_x = -\underbrace{kx}_{\text{restoring F}} - \underbrace{bv}_{\text{resistive force}} = m a_x$$

$$-kx - bv = m\,\frac{d^2x}{dt^2}$$

$$\Rightarrow m\,\frac{d^2x(t)}{dt^2} + b\,\frac{dx(t)}{dt} + k\,x(t) = 0$$

* the sol. of the above equation is beyond the scope of this class.

* we can solve this for the special case $b < \sqrt{4mv}$

$$\boxed{x = \underbrace{\left[A\,e^{-(b/2m)t}\right]}_{A(t)} \cos(\omega t + \phi) \quad \text{w/} \quad \omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}}$$

* $A(t)$ decays exp. w/ time

* motion ultimately stops

* $\omega = \sqrt{\omega_0 - \left(b/2m\right)^2}$

natural freq.

* If $R_{max} = b\,v_{max} < kA$

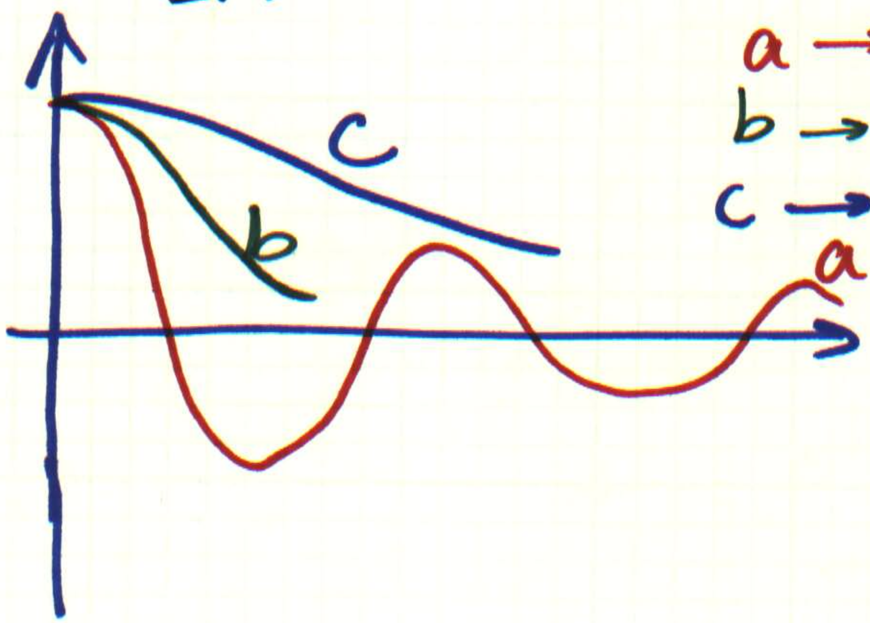
$\longrightarrow$ underdamped

* If $b = b_{critic.} \xRightarrow{s.t.} \frac{b_c}{2m} = \omega_0$

$\longrightarrow$ no osc. (critic. damped)

* If $b/_{2m} > \omega_0 \longrightarrow$ Over damped

a $\rightarrow$ und. dam.
b $\rightarrow$ crit. damp.
c $\rightarrow$ over damp.



(Q&E)

Forced Osc.

* Compensate loss of energy by applying ext. force

* A will remain const. if

energy input/cycle = energy loss/cycle

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stationary obj $\xrightarrow[\text{force}]{\text{driving}}$ osc. starts.

$\xrightarrow[\text{time}]{\text{long enough}}$ $E_{\text{driving}} = E_{\text{lost. to internal}}$

$\longrightarrow$ steady state

$\longrightarrow$ const A (ampl.)

$$m\,\frac{d^2x(t)}{dt} + b\,\frac{dx(t)}{dt} + k\,x(t) = F(t)$$
$$= F_0 \sin\omega t$$

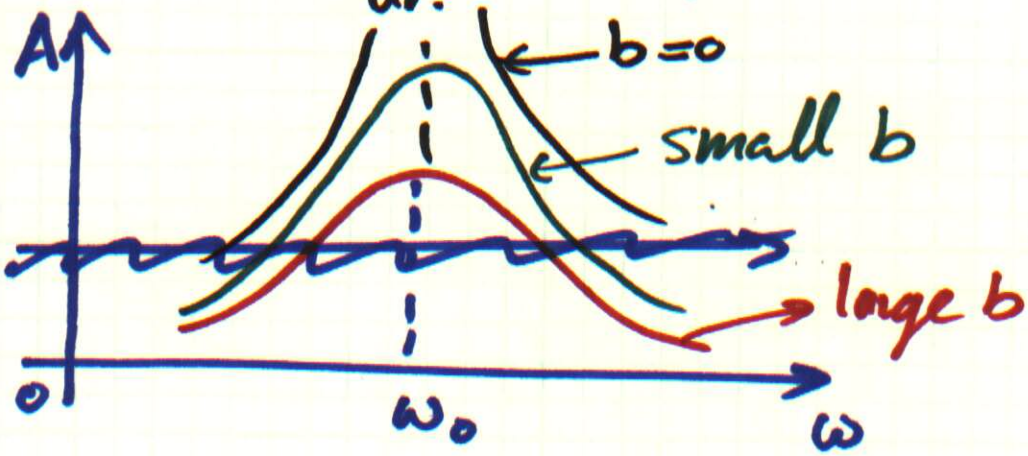
$$A = \frac{F_0/m}{\sqrt{(\omega^2-\omega_0^2)^2 + \left(\frac{b\omega}{m}\right)^2}} \quad \left(\begin{array}{l} x(t) = \ldots \\ \omega_0 = \ldots \end{array} \right.$$

As $\omega \to \omega_0$ $A \to A_{max}$

this dramatic increase in A is known as Resonance

<u>Resonance:</u>

- when $\omega_{dr.} = \omega_0$



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(Q & E)