Physics 1C **Midterm 1**
Summer Session II, 2011 **Solutions**

1. Light of wavelength 550 nm in vacuum enters a substance with an index of refraction of 1.47. What is the wavelength in nm in the medium?
   (a) 293
   (b) 357
   (c) 374
   (d) 388
   (e) 401

   Solution:
   $$\lambda_n = \frac{\lambda}{n} = \frac{550nm}{1.47} \approx 374nm$$

2. Two mirrors are at right angles to one another. A light ray is incident on the first at an angle of 30° with respect to the normal to the surface. What is the angle of reflection from the second surface?
   (a) 30°
   (b) 60°
   (c) 45°
   (d) 53°
   (e) 75°

   Solution: The light ray will reflect off of the first mirror with an equal angle of 30°, and then arrives at the second mirror. Using the geometry of the two mirrors and the fact that they the angle between the two of them is 90°, the incident angle for the second mirror is 60° which is equal to the final angle of reflection.

3. A person in a boat sees a fish in the water (n=1.33) at an angle of 40° relative to the waters surface. What is the true angle in degrees relative to the waters surface?
   (a) 40
   (b) 35
   (c) 50
   (d) 55
   (e) 61

Solution: If the angle is 40° with respect to the surface of water the angle of refraction is $90° - 40° = 50°$. Now we can use Snell's law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Index of refraction of air is very close to unity, so one can rewrite the above equantion and solve for θ_1:

$$\sin\theta_1 = \frac{\sin\theta_2}{n_1} = \frac{\sin 50°}{1.33} \approx 0.576$$

Which gives a value of around 35° for θ_1. However, this is the angle between the ray and the normal to the surface, so the original angle is $90° - 35° = 55°$

4. A plano-convex lens is made of glass (n=1.5) with one surface flat and the other having a radius of 20 *cm*. What is the focal length in *cm* of the lens?

   (a) 20
   (b) 30
   (c) 40
   (d) 10
   (e) 50

   Solution: We can use the Lens maker's equation for this problem. The radius of the flat side can be considered to be infinity. ($R_2 = \infty$)

$$\begin{aligned} \frac{1}{f} &= (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = (n-1)\left[\frac{1}{R_1}\right] = \frac{n-1}{R_1} \\ f &= \frac{R_1}{n-1} = \frac{0.20m}{1.5-1} = \frac{0.2m}{0.5} = 0.40m = 40cm \end{aligned}$$

5. What is the focal length in *cm* of a convex mirror in which a virtual image is located 10.0 *cm* from the mirror and the object is 30.0 *cm* from the mirror. Both object and image are located on the principle axis of the mirror.

   (a) -5
   (b) -10
   (c) -15
   (d) -20
   (e) -25

Solution: Since this is a convex mirror and is making a virtual image, the sign on q is negative:

$$\begin{aligned}\frac{1}{f} &= \frac{1}{p} + \frac{1}{-q} = \frac{-q+p}{-pq} \\ f &= \frac{pq}{q-p} = \frac{10cm \times 30cm}{(10-30)cm} = \frac{300}{-20}cm = -15cm\end{aligned}$$

6. The image distance, q_A, of object A is twice as far from a converging lens as the image distance, q_B, of object B. Both images are real images. Which statement regarding the object distances is correct?

   (a) $p_B < p_A$

   (b) $p_B = p_A$

   (c) $p_B > p_A$

   (d) $p_B < -p_A$

   (e) $p_B = -p_A$

   Solution:

$$\begin{aligned}q_A &= 2q_B \\ \frac{1}{f} &= \frac{1}{p_A} + \frac{1}{q_A} = \frac{1}{p_B} + \frac{1}{q_B} \\ \frac{p_A + q_A}{p_A q_A} &= \frac{p_B + q_B}{p_B q_B} \\ \frac{p_A + 2q_B}{2p_A q_B} &= \frac{p_B + q_B}{p_B q_B} \\ \frac{p_A + 2q_B}{2p_A} &= \frac{p_B + q_B}{p_B} \\ p_B(p_A + 2q_B) &= 2p_A(p_B + q_B) \\ p_B p_A + 2p_B q_B &= 2p_A p_B + 2p_A q_B \\ 2q_B(p_B - P_A) &= p_A p_B\end{aligned}$$

   The right hand side of this equation is always positive which mean

$$p_B - p_A > 0$$

   Hence:

$$p_B > p_A$$

7. A convex lens and a concave mirror each have focal length f. The lens is placed a distance $4f$ in front of the mirror. Then an object is placed a distance $2f$ in front of the lens. The image produced by the lens-mirror system will be

(a) $2f$ in front of the mirror and inverted.
(b) $2f$ behind the mirror and upright.
(c) $2f$ in front of the lens and inverted.
(d) $2f$ in front of the lens and upright.
(e) $2f$ behind the mirror and inverted.

Solution: The first image will be formed by the lens:

$$\begin{aligned}\frac{1}{f} &= \frac{1}{p}+\frac{1}{q}\\ \frac{1}{q} &= \frac{1}{f}-\frac{1}{p}\\ q &= \frac{fp}{p-f}=\frac{f\cdot 2f}{2f-f}=2f\end{aligned}$$

Therefore, the first image will be formed in front of the lens at a distance $2f$. This is a real and inverted image. This image can now be treated as the object for the concave mirror. The object is now a distance $2f$ away from the mirror, and the calculation will be identical to the above, and an image will be formed in front of the mirror at a distance $2f$ from the mirror. This is another inverted, real image. This can now be used as another object a distance $2f$ from the lens. The lens will make an image of this real image a distance $2F$ in front of the lens. The final image formation also inverts the object. So the image of the original object is inverted three times, which means the final image is going to be inverted a distance $2f$ in front of the lens.

Another way to do this problem is to find the magnification each time, which is the same in all three different cases:

$$M=-\frac{q}{p}=-\frac{2f}{2f}=-1$$

The final magnification is the the product of all magnificaitons:

$$M=M_1\cdot M_2\cdot M_3=-1$$

This also shows that the final image is inverted.

8. Estimate the distance in cm between the central bright region and the third dark fringe on a screen 5.00 m from two slits 0.500 mm apart, when the slits are illuminated by 500 nm light.

(a) 3.47
(b) 2.15

(c) 1.75

(d) 1.50

(e) 1.25

Solution:

$$y_{dark} = L\tan\theta_{dark} \approx L\sin\theta_{\theta_{dark}} = L\left[(m+\frac{1}{2})\frac{\lambda}{d}\right] \quad ; \quad m = 0, 1, 2, \ldots$$

The third dark region has an m equal to 2:

$$y_{dark} = 5.0m\left(2+\frac{1}{2}\right)\frac{500\times10^{-9}m}{0.5\times10^{-3}m} = 1.25\times10^{-2}m = 1.25cm$$

9. An optical coating ($n = 1.4$) on a glass lens ($n = 1.5$) is designed to minimize reflection of light of 500 nm wavelength. How thick (in nm) should the coating be?

   (a) 84

   (b) 94

   (c) 89

   (d) 99

   (e) 179

   Solution: The light wave will travel a total distance $2t$ and then interferes with itself on the surface between the coating material and the air. However, there is a phase change of 180° when the light wave is reflected off the surface of glass, since the index of refraction of glass is higher than that of the coating material. This 180° phase shift is as if the wave has traveled an extra distance of $\lambda_n/2$:

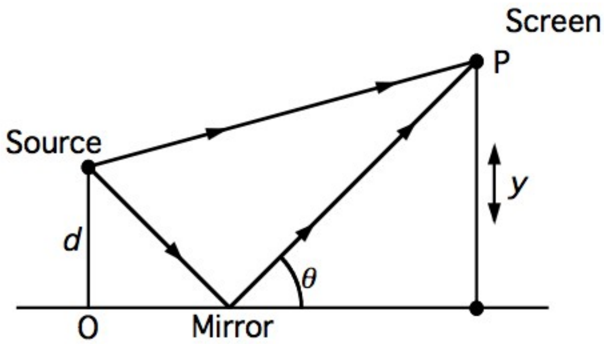
$$\frac{\phi}{2\pi} = \frac{\delta}{\lambda_n} \rightarrow \delta = \frac{\pi}{2\pi}\lambda_n = \frac{\lambda_n}{2}$$

   In order to get destructive interference we have the following equation:

$$\delta = \left(m+\frac{1}{2}\right)\lambda_n = 2t + \frac{\lambda_n}{2} \quad ; \quad m = 0, 1, 2, \ldots$$

   Solving for the thinest possible t, where $m = 1$:

$$\frac{3}{2}\lambda_n = 2t + \frac{\lambda_n}{2} \quad \rightarrow \quad t = \frac{\lambda_n}{2} = \frac{\lambda}{2n} = \frac{500nm}{2\times1.4} \approx 179nm$$



10. An interference pattern is produced at point P on a screen as a result of direct rays and rays reflected off a mirror shown in the figure. If the source is 100 m to the left of the screen, 1 cm above the mirror, and the source is monochromatic ($\lambda = 500nm$), find the distance y in mm to the first dark band above the screen.

    (a) 1.0
    (b) 2.0
    (c) 1.5
    (d) 2.5
    (e) 5.0

    Solution: Because of the reflection of the flat mirror there will be a virtual source behind the mirror a distance d away from the mirror. This source will be 180° out of phase with the real source because the light that reflects off the mirror shifts the phase by this amount. So this is exteremly similar to the double slit experiment except that the phase different between the sources is not zero but 180°. This means that the dark and bright fringes will be switched; therefore, the same equation as a double slit experiment can be used using the bright fringe equations for the dark ones here, and vice versa.

    To find the distance of the first dark fringe, we can use the equation for bright fringes of a double slit experiment:

    $$y_{dark} = L \tan \theta_{bright} \approx L \sin \theta_{dark} = L \left[\frac{m\lambda}{2d} \right]$$

    Where $m = 1$ for the first dark fringe:

    $$y_{dark} = \frac{L\lambda}{2d} = \frac{100m \times 500 \times 10^{-9}m}{0.02m} = 0.25 \times 10^{-4}m = 2.5mm$$

11. A diffraction grating with 4000 lines/cm is illuminated by light from the sun. The solar spectrum is spread out on a white wall across the room. At what angle from the located center line is blue light (400 nm)?

(a) 9.8°
(b) 9.2°
(c) 10.1°
(d) 9.4°
(e) 9.6°

Solution: For a diffraction grating, we have:

$$\sin\theta_{bright} \approx \theta_{bright} = \frac{m\lambda}{d}$$

where $m = 1$ and d is the distance between different slits:

$$d = \frac{1}{4,000}cm = \frac{1}{4\times 10^5}m$$

Hence,

$$\theta_{bright} = \frac{\lambda}{d} = 400\times 10^{-9}\times 4\times 10^5 = 0.16rad \approx 9.18^\circ$$

12. A stopping potential of 3.2 V is needed for radiation whose wavelength is 200 nm. The work function in eV of the material is ($h = 6.626\times 10^{-34}\ J\cdot s$; $c = 3.00\times 10^8\ m/s$; $e = 1.60\times 10^{-19}$ C; $1eV = 1.602\times 10^{-19}J$)

(a) 4
(b) 3
(c) 5
(d) 6
(e) 2

Solution:

$$\begin{aligned} K_{max} &= hf - \phi = e\Delta V_s \\ \phi &= hf - e\Delta V_s = 6.626\times 10^{-34}J\cdot s\times 200\times 10^{-9}m - 1.6\times 10^{-16}C\times 3.2V \\ &= 5.12\times 10^{-16}J \approx 3.2eV \end{aligned}$$

13. The maximum kinetic energy of photoelectrons depends on

(a) the frequency of the light.
(b) the intensity of the light.
(c) the number of photons that reach the surface per second.
(d) the number of quanta.

(e) the speed of light.

Solution: The energy of photoelectrons only depends on the frequency of the light

14. A photon whose energy is 8×10^{-15} J is scattered off an electron at an angle of $90°$. What is the wavelength of the scattered wave in m? ($m_e = 9.11 \times 10^{-31}$ kg; $h = 6.626 \times 10^{-34}$ $J \cdot s$; $c = 3.00 \times 10^8$ m/s; $e = 1.60 \times 10^{-19}$ C)

    (a) 2.73×10^{-11}

    (b) 2.25×10^{-11}

    (c) 2.50×10^{-11}

    (d) 2.40×10^{-11}

    (e) 2.48×10^{-11}

    Solution: We can use the equation for Compton shift with $\theta = 90°$:

$$\begin{aligned} \lambda' - \lambda_0 &= \frac{h}{m_e c}(1 - \cos\theta)\Big|_{\theta=90°} = \frac{h}{m_e c} \\ \lambda' &= \frac{h}{m_e c} + \lambda_0 \end{aligned}$$

We are not given the wavelength of the incoming photon, but we can find this from the total energy of photon:

$$\begin{aligned} c &= f\lambda \\ E &= hf = \frac{hc}{\lambda} \\ \lambda &= \frac{hc}{E} \end{aligned}$$

Now we can rewrite the Compton shift equation:

$$\begin{aligned} \lambda' &= \frac{h}{m_e c} + \frac{hc}{E} = \frac{6.626 \times 10^{-34} J \cdot s}{9.11 \times 10^{-31} kg \times 3.0 \times 10^8 m/s} + \frac{6.626 \times 10^{-34} J \cdot s}{8 \times 10^{-15}} \\ &= 2.73 \times 10^{-11} \end{aligned}$$