Physics 1C **Midterm 1**
Summer Session II, 2011 **Solutions**

1. If $F = -kx$, then $\frac{k}{m}$ is

   (a) A
   (b) ω
   (c) ω^2
   (d) $A\omega$
   (e) $A^2\omega$

   Solution: $F = -kx$ is Hooke's law for a mass and spring system. Angular frequency of this system is:

   $$\omega = \sqrt{\frac{k}{m}}$$

   therefore,

   $$\frac{k}{m} = \omega^2$$

2. A body oscillates with simple harmonic motion along the x-axis. Its displacement varies with time according to the equation $x = 5\sin(\pi t + \frac{\pi}{3})$. The velocity in m/s of the body at $t = 1s$ is

   (a) $+8$
   (b) -8
   (c) -14
   (d) $+14$
   (e) -5

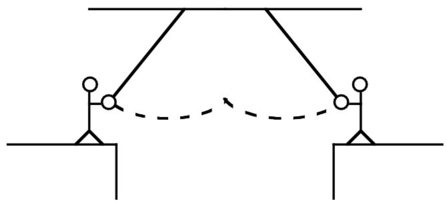
   Solution:

   $$v(t) = \frac{dx(t)}{dt} = \frac{d}{dt}\Big[5\sin(\pi t + \frac{\pi}{3})\Big] = 5\pi\cos(\pi t + \frac{\pi}{3})$$

   at $t = 1s$:

   $$v(1) = 5\pi\cos(\frac{4\pi}{3}) \approx -8$$

3. Two circus clowns (each having a mass of $50kg$) swing on two flying trapezes (negligible mass, length $25m$) shown in the figure. At the peak of the swing, one grabs the other, and the two swing back to one platform. The time for the forward and return motion in s is



(a) 10

(b) 5

(c) 15

(d) 20

(e) 25

Solution: The period of oscillation of a pendulum is independent of mass; therefore, we simply just need to find the value of one period T:

$$T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{25}{10}} \approx 10$$

4. . A simple pendulum on the Earth has a period of one second. What would be its period in s on the moon where the acceleration due to gravity is 1/6 that of Earth?

   (a) 6.00

   (b) 2.45

   (c) 1.00

   (d) 0.408

   (e) 0.167

   Solution: Define the period of oscillation of the Earth as T_E and that of the Moon as T_M.

$$\begin{aligned} T_E &= 2\pi\sqrt{\frac{l}{g_E}} \\ T_M &= 2\pi\sqrt{\frac{l}{g_M}} \end{aligned}$$

   where

$$g_E = 6g_M$$

   therefore,

$$\frac{T_M}{T_E} = \frac{2\pi\sqrt{l/g_M}}{2\pi\sqrt{l/g_E}} = \sqrt{\frac{g_E}{g_M}}$$

T_M is:

$$T_M = T_E\sqrt{\frac{g_E}{g_M}} = 1s \times \sqrt{\frac{1}{6}} \approx 2.45s$$

5. When a damping force is applied to a simple harmonic oscillator which has period T_0 in the absence of damping, the new period T is such that

   (a) $T < T_0$
   (b) $T = T_0$
   (c) $T > T_0$
   (d) $\omega T < \omega_0 T_0$
   (e) $\omega T > \omega_0 T_0$

   Solution: Let's look at spring and mass system as an example. When the SHM is not damped we have a natural angular frequency:

   $$\omega_0 = \sqrt{\frac{k}{m}}$$

   When the SHM is damped we have a different angular frequency:

   $$\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

   from the above equations it is obvious that $\omega < \omega_0$. In other words, the oscillation is slower in presence of damping. Using the relationship between T and ω, $T = 2\pi/\omega$, one can see that $T > T_0$.

6. The speed of light waves in air is $3 \times 10^8 m/s$. The speed of sound waves in air is $333m/s$. How long in s is the time interval between the time a lightning flash is seen and the thunderclap is heard if the lightning flash is 1 kilometer away?

   (a) 3
   (b) 5
   (c) 7
   (d) 10
   (e) 1

   Solution: Both the light and the sound wave travel a distance x which is 1 km or 1,000 m. And each take some time t to get to the observer:

   $$t = \frac{x}{v}$$

Defining, the time it takes the light to get to the observer as t_l and the time it takes the sound to be heard as t_s we have a time difference ΔT:

$$\Delta t = |t_s - t_l| = \left|\frac{x}{v_l} - \frac{x}{v_l}\right| = \left|\frac{10^3 m}{3 \times 10^8 m/s} - \frac{10^3 m}{333 m/s}\right| \approx 3s$$

7. The lowest A on a piano has a frequency of $27.5Hz$. If the tension in the 2.0 meter string is $308N$, and one- half wavelength occupies the wire, what is the mass of the wire in kg?

   (a) 0.025
   (b) 0.051
   (c) 0.072
   (d) 0.081
   (e) 0.037

   Solution:

$$\lambda = 2L = 4 \ m$$

   Also,

$$\begin{aligned} v &= f\lambda \\ v &= \sqrt{T}\mu \end{aligned}$$

   Combining these two equations we have:

$$\frac{T}{\mu} = f\lambda$$

   Now we can solve for μ:

$$\mu = \frac{T}{(f\lambda)^2}$$

   Given this linear mass density, the total mass of the string is:

$$M = \mu L = \frac{(f\lambda)^2}{T} L = \frac{308N}{(27.5Hz \times 4m)^2} \times 2m \approx 0.051Kg$$

8. If $y = 0.02 sin(30x - 400t)$ (SI units), the velocity of the wave in m/s is

   (a) $\frac{3}{40}$
   (b) $\frac{40}{3}$
   (c) $\frac{60\pi}{400}$
   (d) $\frac{400}{60\pi}$

(e) 400

Solution: The argument inside the sin function has the general form of $(kx - \omega t)$; therefore, the wave number ,k, and angular frequency, ω, are both given here. One can easily solve for v:

$$v = f\lambda = \frac{\omega}{2\pi} \times \frac{2\pi}{k} = \frac{\omega}{k} = \frac{40}{3}$$

9. A $500Hz$ tone is sounded at a train station as a train moves toward the station at $20m/s$. What frequency in Hz does the engineer hear if the speed of sound is $335m/s$?

   (a) 530
   (b) 535
   (c) 475
   (d) 495
   (e) 515

   Solution: The train is moving *towards* the train station; therefore, we can use the doppler equation without any changes to the signs of velocities. In this case the observer is stationary ($v_o = 0$) and the source is moving, so we have:

$$f_{engineer} = f_{train}\left(\frac{v}{v - v_s}\right) = 500Hz \times \left(\frac{335m/s}{335m/s - 20m/s}\right) \approx 530Hz$$

10. The longest wavelength that a standing wave can have on a stretched string of length L is

   (a) $2L$
   (b) $3L$
   (c) $5L$
   (d) $7L$
   (e) $9L$

   Solutions: This is the first harmonic on the string.

$$\lambda_n = \frac{2L}{n} \qquad \text{where} \qquad n = 1 \Rightarrow \lambda_1 = 2L$$

11. Two harmonic waves traveling in opposite directions interfere to produce a standing wave described by $y = 3\sin(2x)\cos(5t)$ where x is in m and t is in s. What is the wavelength in m of the interfering waves?

(a) π

(b) $\frac{\pi}{3}$

(c) 2π

(d) 4π

(e) $\frac{\pi}{2}$

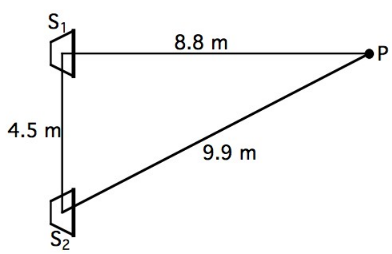
Solution: The general form of a standing wave is:

$$y(x,t) = A\sin(kx)\cos(\omega t) = A\sin(\frac{2\pi}{\lambda})\cos(\omega t)$$

From the above equation, we have:

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{2} = \pi$$

12. Two speakers that are in synchronization are connected to a sine wave source. Waves of $2.2m$ wavelength travel to point P from the speakers. The phase difference, $\Delta\phi_{21}$, between the waves from $S2$ and $S1$ when they arrive at point P is



(a) 0

(b) π

(c) 2.05π

(d) 8π

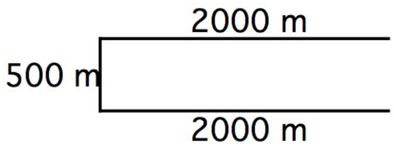
(e) 9π

Solution:

$$\Delta r = \frac{\Delta\phi}{2\pi}\lambda$$

re-write the above equation:

$$\Delta\phi = 2\pi\frac{\Delta r}{\lambda} = 2\pi \times \frac{|9.9m - 8.8m|}{2.2m} = \pi$$



13. An ocean harbor has a rectangular shape, with one shorter side open to the sea. On a day when the speed of waves in the harbor is $20 m/s$, standing waves can be produced along the length of the harbor if their wavelength is

    (a) $1600 m$
    (b) $3200 m$
    (c) $4000 m$
    (d) $5000 m$
    (e) $7000 m$

    Solution: This is like a closed-end pipe; therefore, we have:

$$\begin{aligned} \lambda_n &= \frac{4L}{(2n-1)} \qquad \text{where} \qquad n = 1, 2, 3, \ldots \\ \lambda_1 &= 4L = 8000 m \\ \lambda_2 &= \frac{4L}{3} = \frac{8000 m}{3} \approx 2667 m \\ \lambda_3 &= \frac{4L}{5} = \frac{8000 m}{5} = 1600 m \\ &\vdots \end{aligned}$$

    λ_3 is the only choice that is available.

14. Unpolarized light is passed through three successive Polaroid filters, each with its transmission axis at 45 to the preceding filter. What percentage of light gets through?

    (a) 0%
    (b) 12.5%
    (c) 25%
    (d) 50%
    (e) 33%

    Solution: Let's define I_0, I_1, I_2, and I_3 as the intensity of light as it goes through the first, second, and third polarizer. When the light goes through the first polarizer the intensity drops by a factor of two, in other words:

$$I_1 = I_0$$

The light then goes through the second filter:

$$I_2 = I_1 \cos^2(\theta) = I_1 \cos^2(\pi/4) = I_1 \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{I_1}{2}$$

The angle between the first and second polarizer is also

$$I_3 = I_2 \cos^2(\theta) = \frac{I_2}{2} = \frac{I_1}{4} = \frac{I_0}{8}$$

Therefore, only 12.5% of light comes out of the last polarizer.