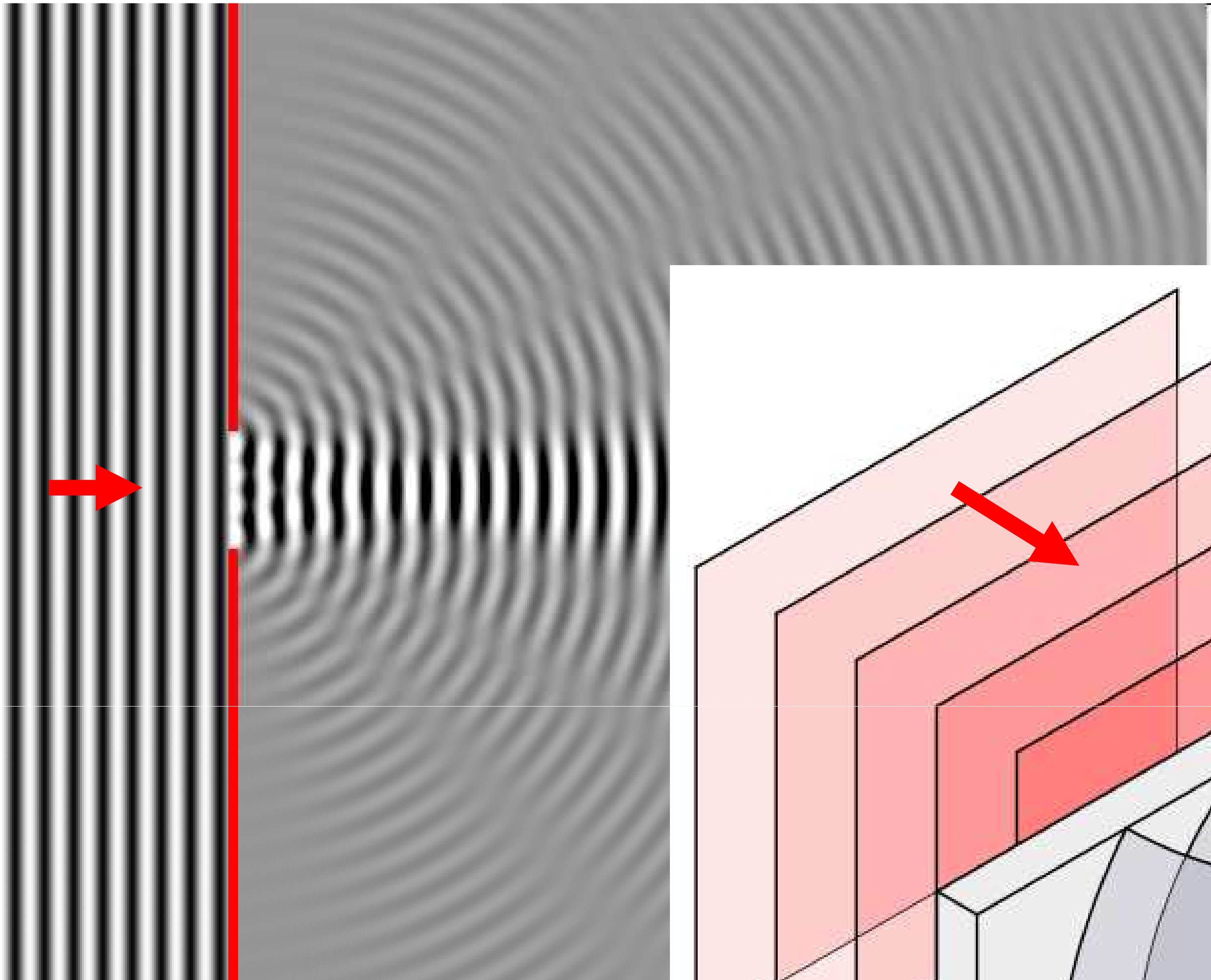
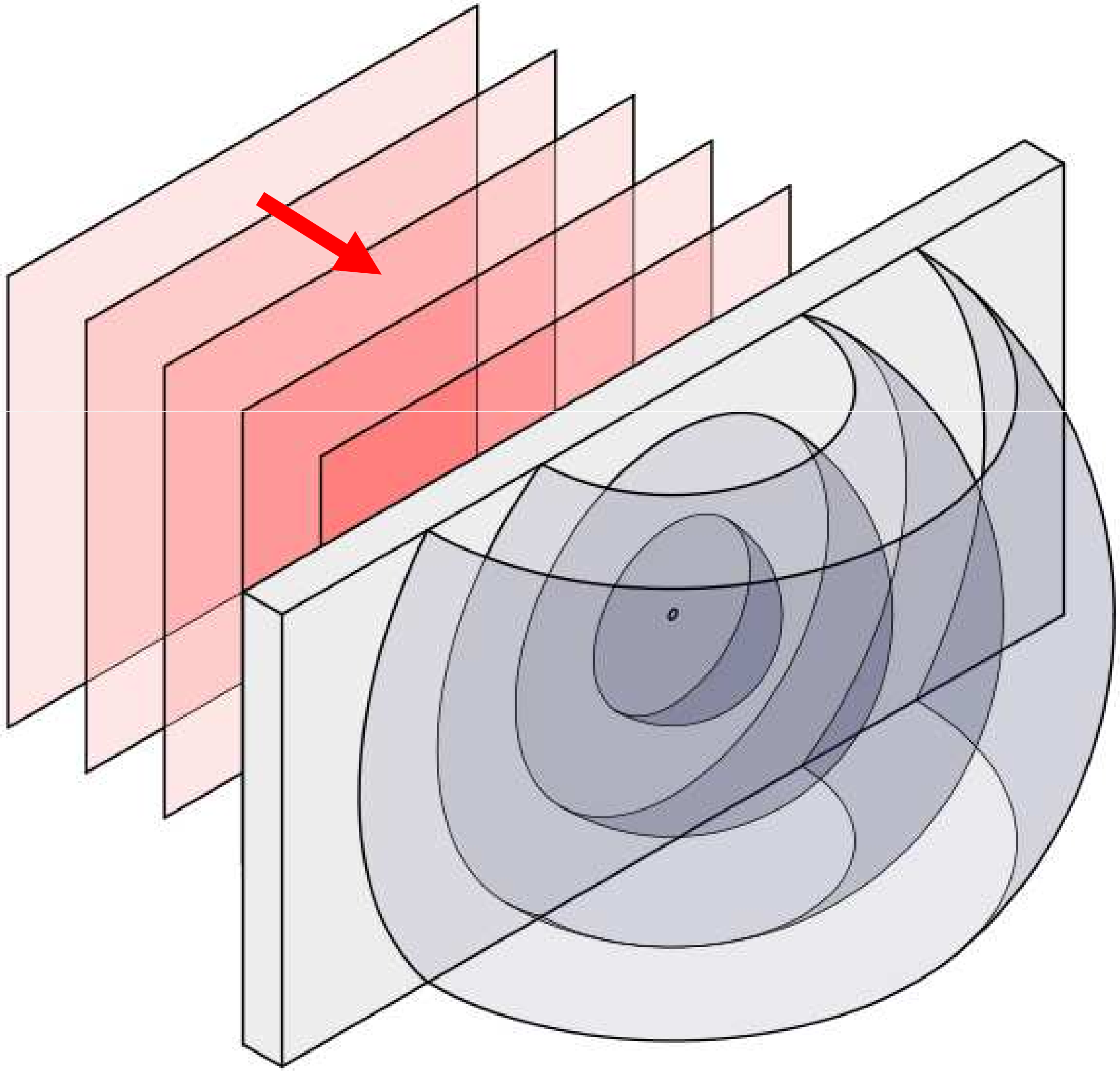
Physics 2C: Diffraction



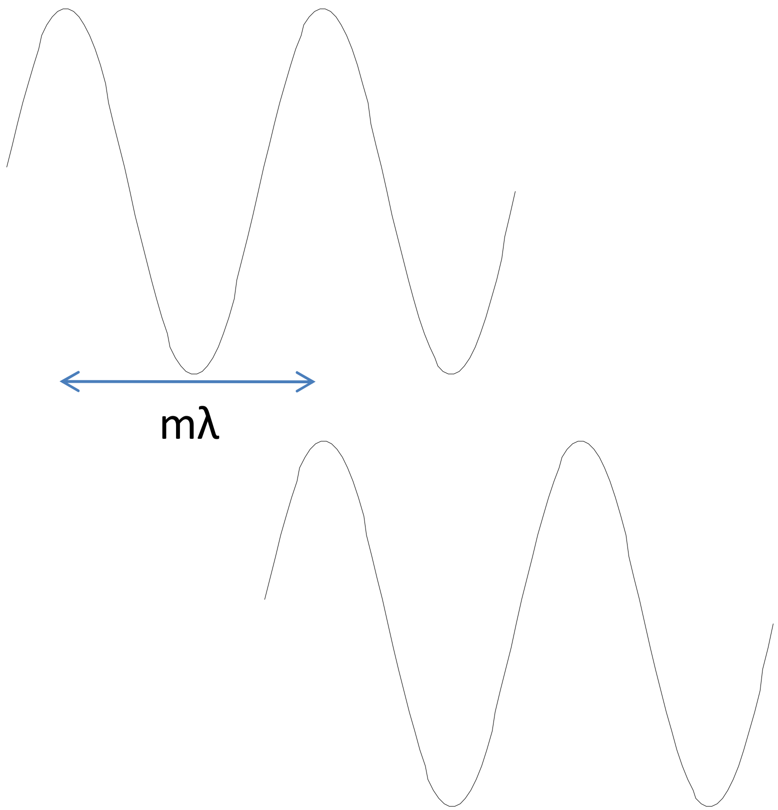


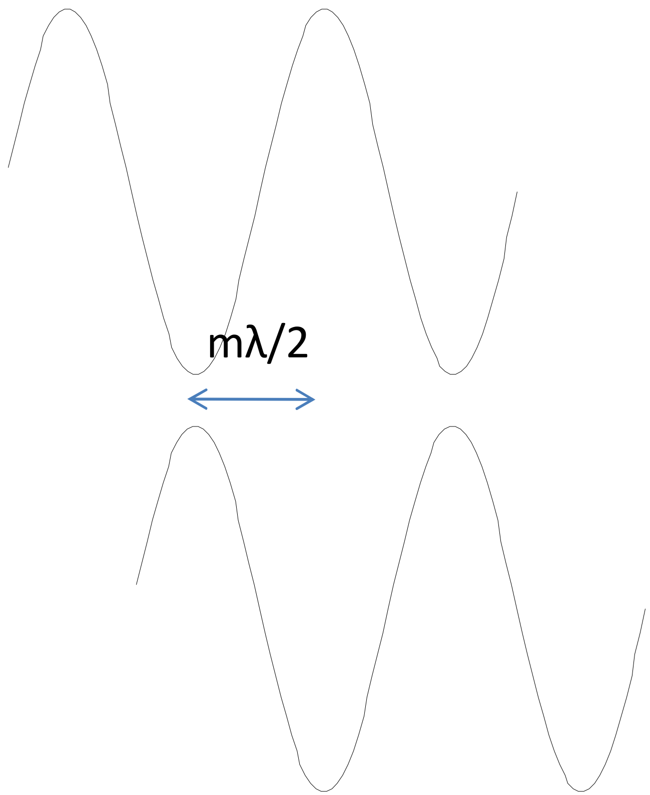
Huygens' principle, double-slit, multi-slit, single-slit

Coherence

- Interference – only if 2 interacting waves maintain a constant phase relationship
- Ordinary sources emit short wave trains w/ random phases (short coherence length)
- Interference from a single light source, split and then recombined
- Lasers – monochromatic (single frequency)

Constructive and Destructive Interference

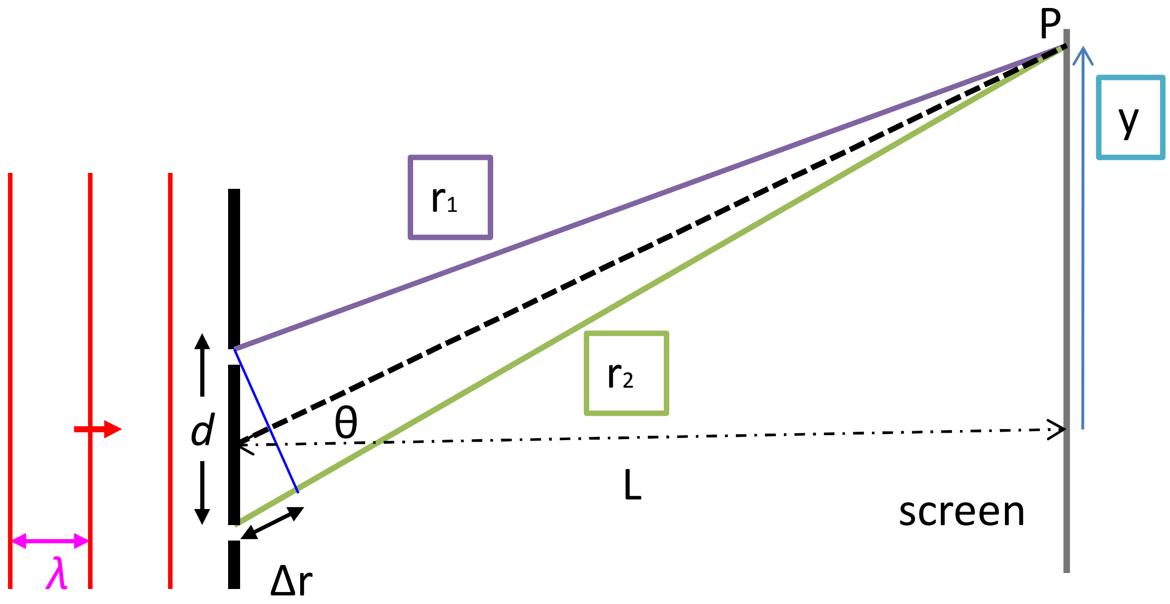




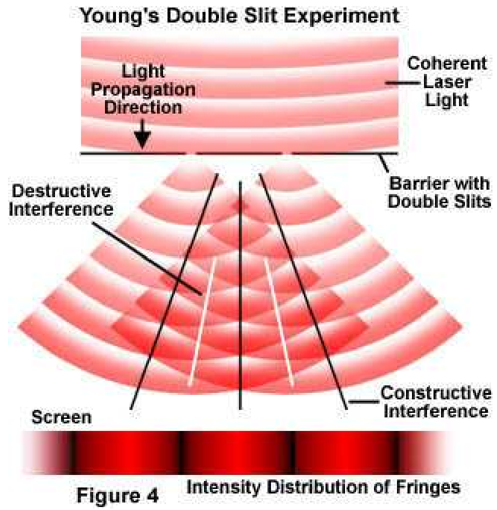
Constructive: path length difference=mλ

Destructive: path length difference=(m+1/2)λ

Double Slit Interference



λ<<d



Interference fringes

Distance between subsequent max(min)?

$$d\sin\theta_1 = m\lambda = d\,\frac{y_1}{L}$$

$$d\sin\theta_2 = (m+1)\lambda = d\,\frac{y_2}{L}$$

$$y_2 - y_1 = \frac{\lambda L}{d}$$

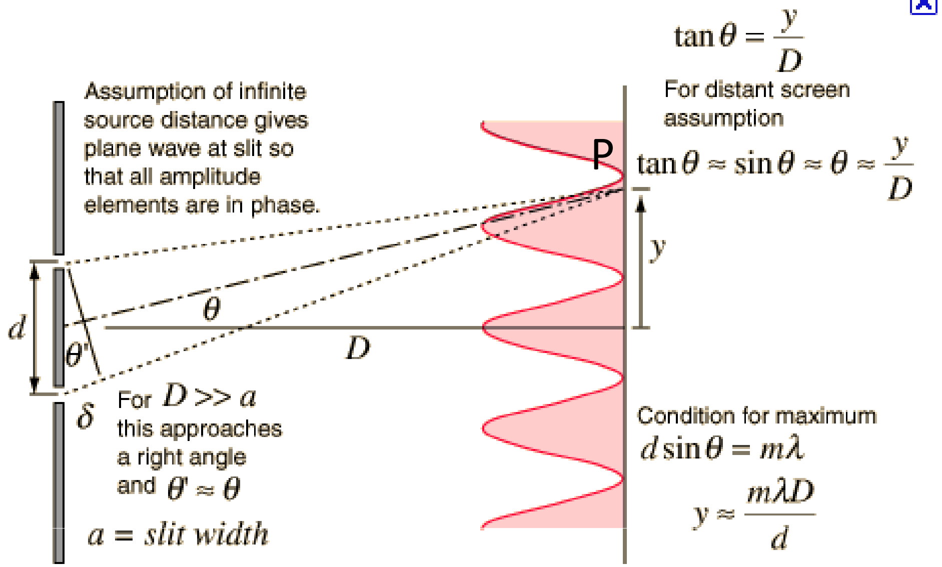
order

Constructive: Δr=mλ

Destructive: Δr=(m+1/2)λ

=d*sin(θ) ~d*tan(θ) =d*y/L

Intensity (double slit)



$$E_1 = E_0 \sin \omega t$$
$$E_2 = E_0 \sin(\omega t + \phi)$$
$$E_P = E_1 + E_2$$

λ<<d

$$A\sin\alpha + A\sin\beta = 2A\sin[(\alpha+\beta)/2]\cos[(\alpha-\beta)/2]$$
$$E_P = 2E_0 \sin(\omega t + \phi/2)\cos(\phi/2)$$
$$\bar{S}_P = 4\bar{S}_0 \cos^2(\phi/2)$$
$$\bar{S}_0 = E_0{}^2 / 2\mu_0 c$$

Path difference (radians)

$$\phi = \Delta r \times \frac{2\pi}{\lambda} = \frac{2\pi d \sin\theta}{\lambda}$$

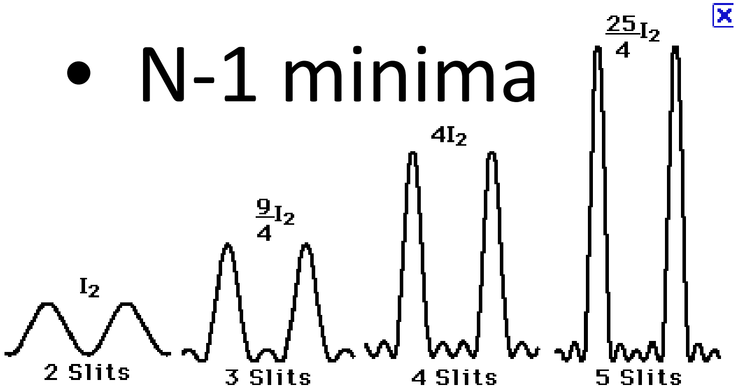
$$\bar{S}_P = 4\bar{S}_0 \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right) \approx 4\bar{S}_0 \cos^2\left(\frac{\pi d y}{\lambda L}\right)$$

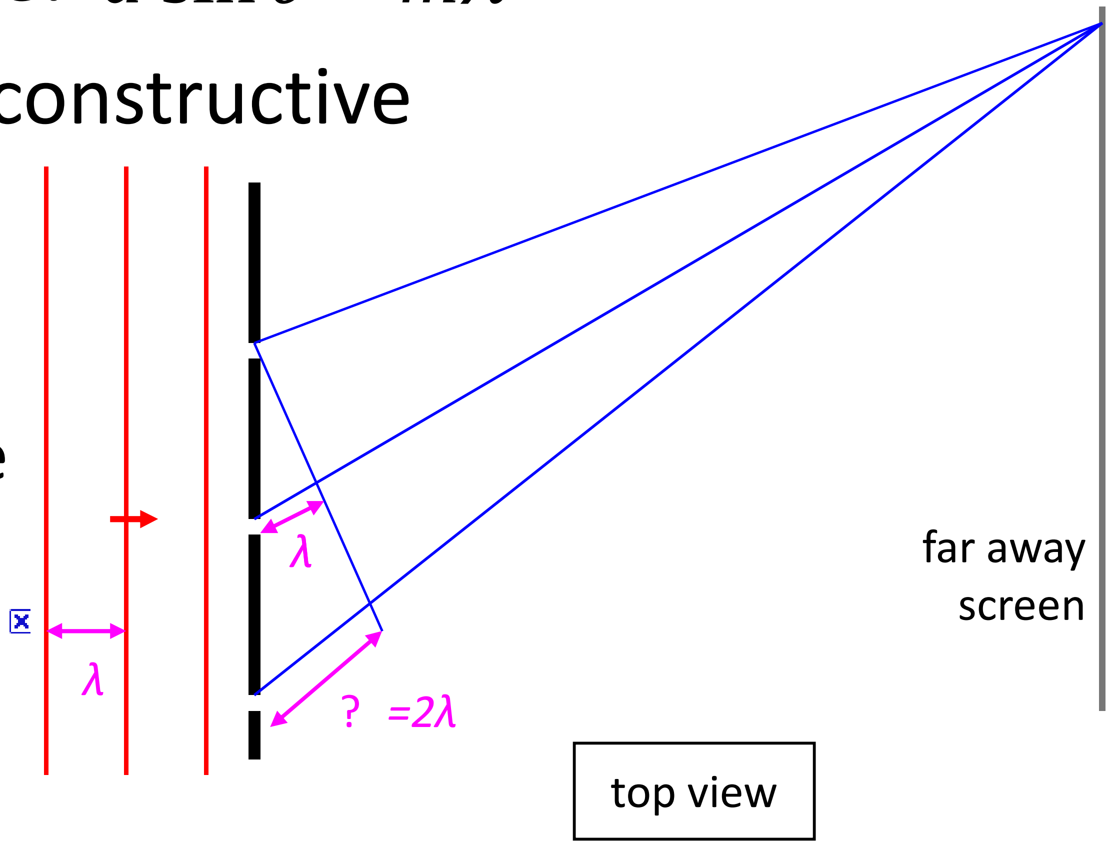
Multiple Slit Interference

- 2 slit constructive: $d\sin\theta = m\lambda$
- Same for >2 slit constructive
- Destructive

  $$d\sin\theta = \frac{m}{N}\lambda$$

  (m NOT a multiple of N)
- N-1 minima



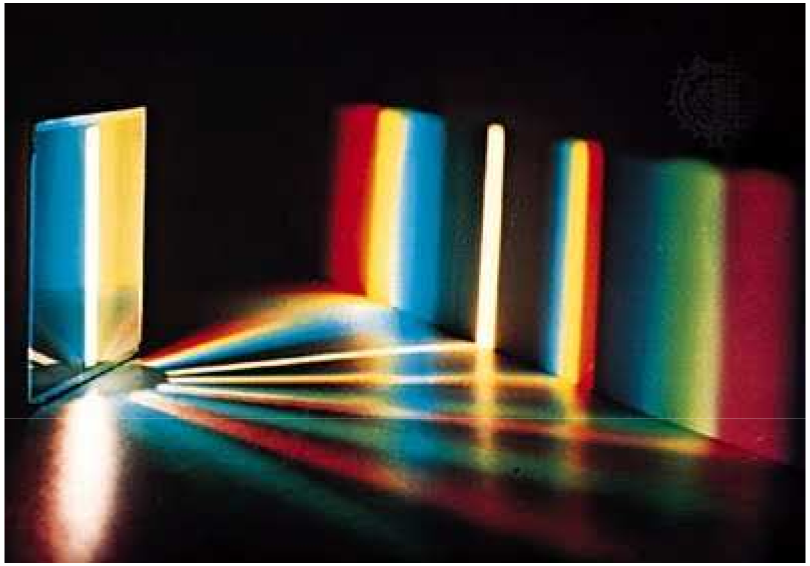


As N increases, primary maxima become brighter and narrower

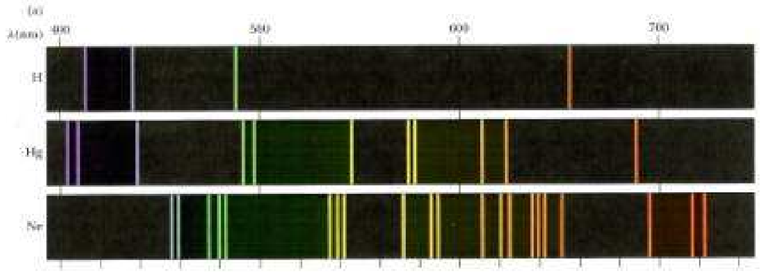
Diffraction Grating

- Very closely, evenly space slits (large N)
- Different wavelengths have different maxima

$$d \sin\theta = m\lambda$$

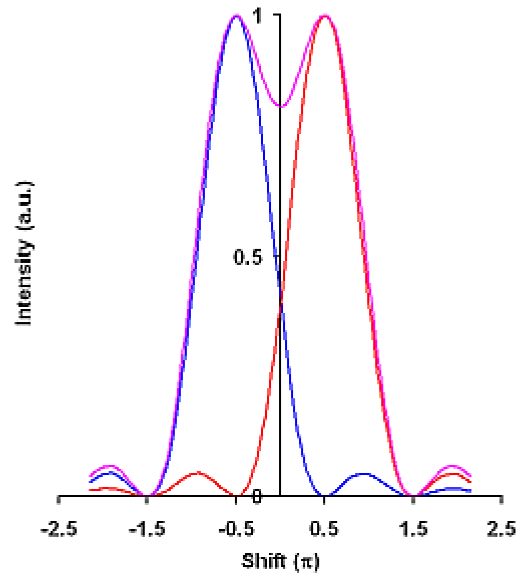


- Grating = disperses light
- m is order of dispersion
- No overlap b/n m=0, 1, 2… but possibility of overlap b/n 2&3, 3&4, etc



Resolving Power

- Light with 2 spectral lines, nearly equal wavelengths: λ, λ'
- Just resolvable if position of 1st max of one @ 1st min of other
- maximum: $d\sin\theta_{\max} = \frac{mN}{N}\lambda'$
- minimum: $d\sin\theta_{\min} = m\frac{N+1}{N}\lambda$



$$\sin\theta_{\min} = \sin\theta_{\max}$$

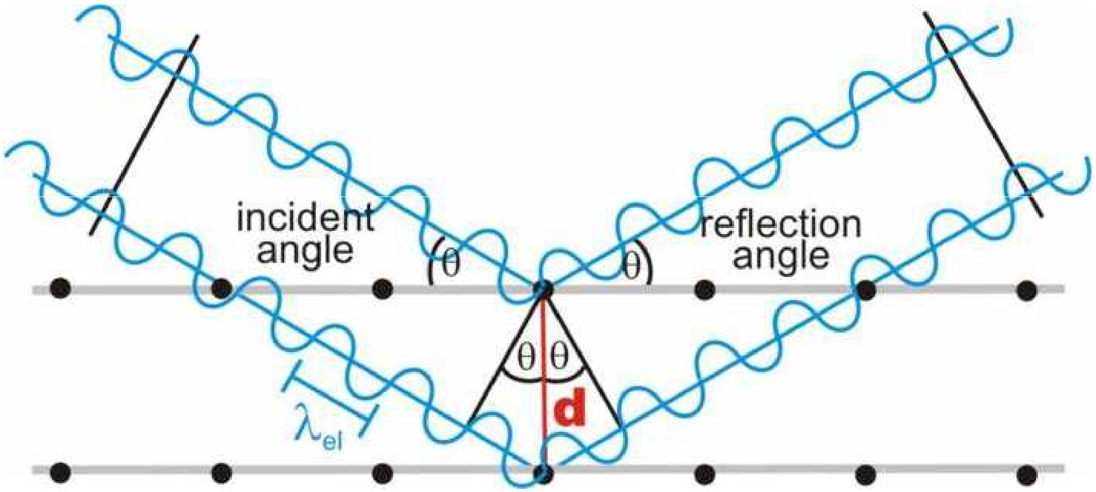
$$m\frac{N+1}{N}\lambda = m\frac{N}{N}\lambda'$$

Resolving power: $\frac{\lambda}{\Delta\lambda} = mN$

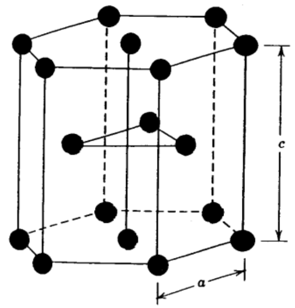
Increases with order, # slits

X-ray (Bragg) Diffraction

- X-ray wavelength (~0.1nm) too short for diffraction gratings (λ<<d)
- Crystal planes in atoms act as diffraction grating

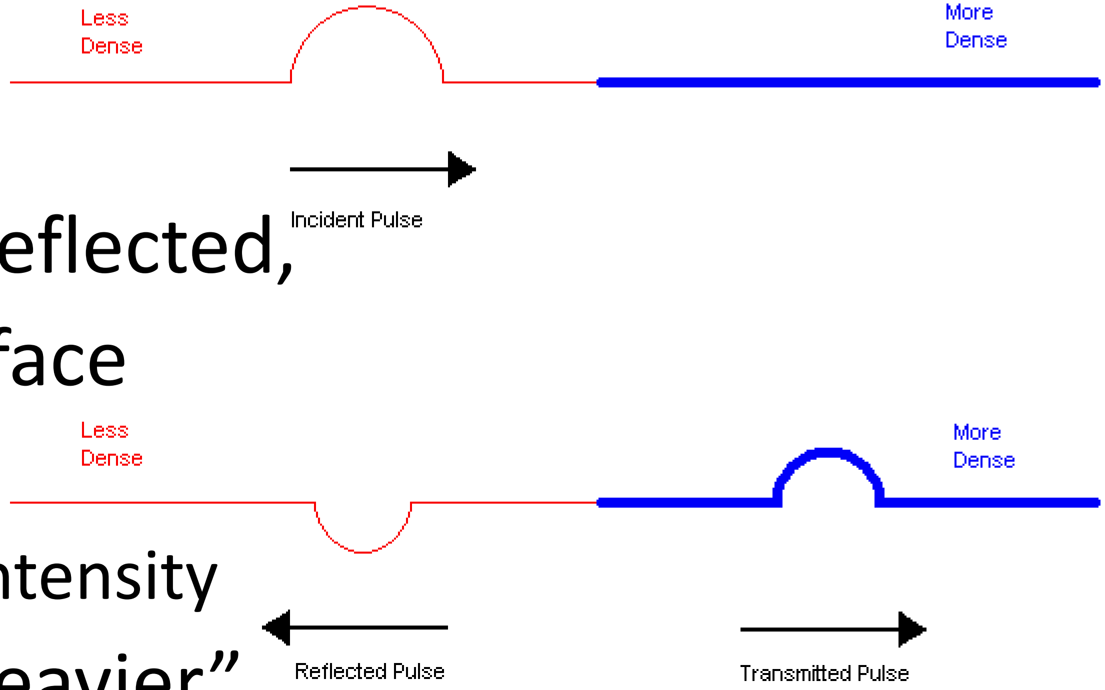


- Bragg condition $2d\sin\theta = m\lambda$

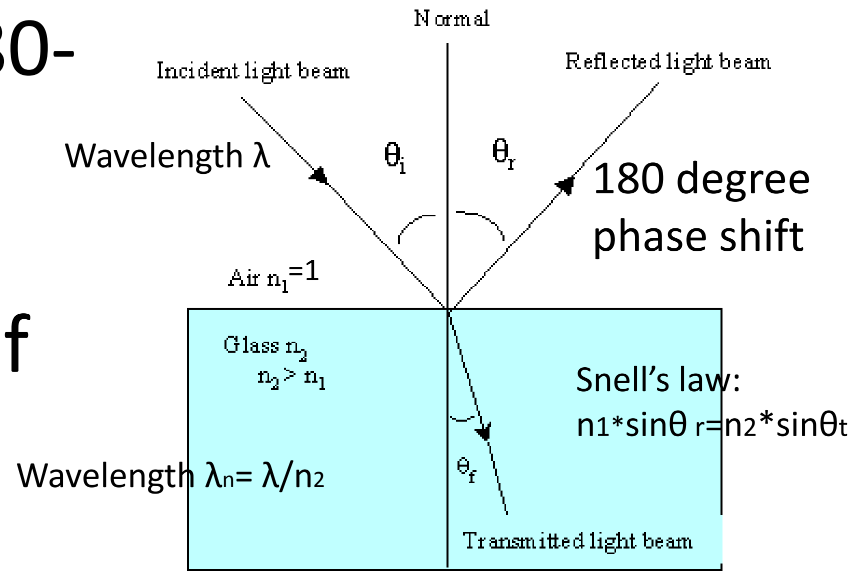


HCP

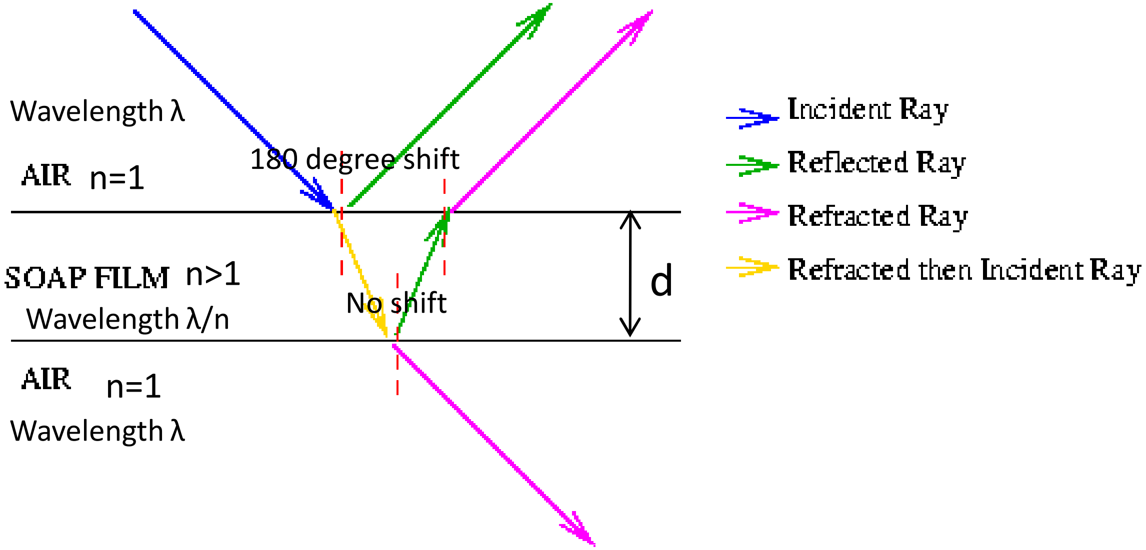
Reflection and Transmission

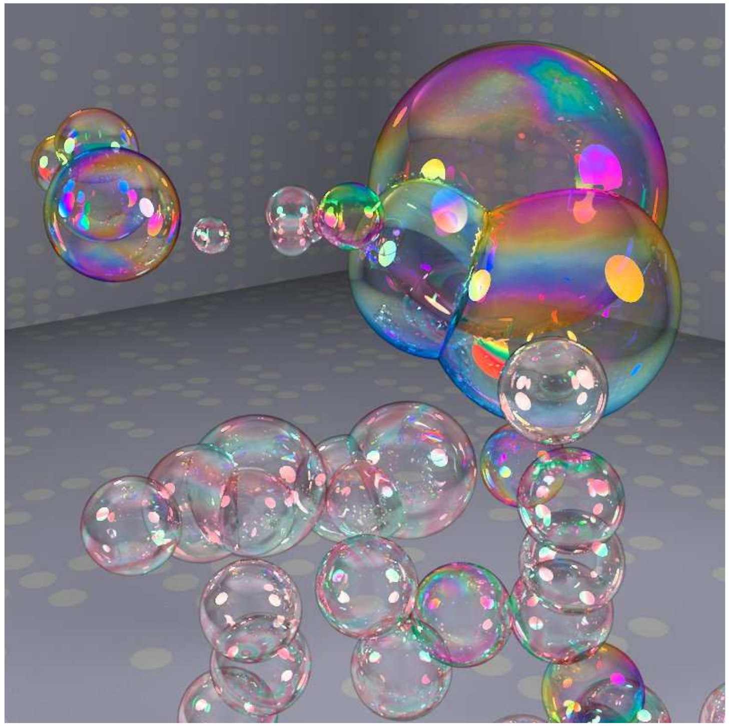


- Wave is partially reflected, transmitted at interface
- Sum amplitude
  - Then square for intensity
- Reflected from "heavier" interface introduces 180-degree phase shift
- For EM, "heaviness" represented by index of refraction



Thin Film Interference

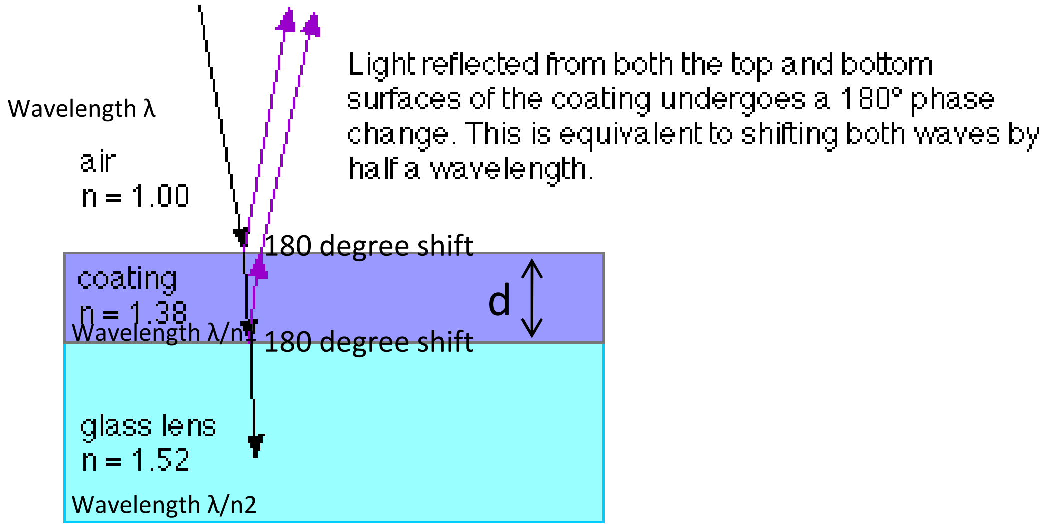


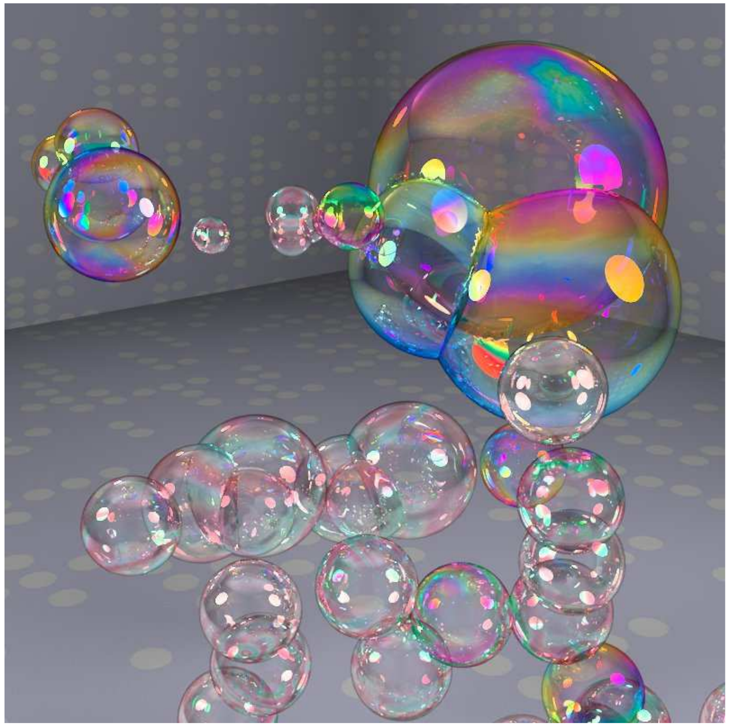


- Other ways than diffraction to see interference
- Light incident on film of thickness d<<λ (b/n air)
- Path difference $\Delta r = 2d \pm 1/2(\lambda/n)$
- Constructive $\Delta r = m(\lambda/n)$,
- Destructive $\Delta r = (m+1/2)(\lambda/n)$

Constructive
$2nd = (m+1/2)\lambda$,
Destructive
$2nd = m\lambda$

Thin Film Interference



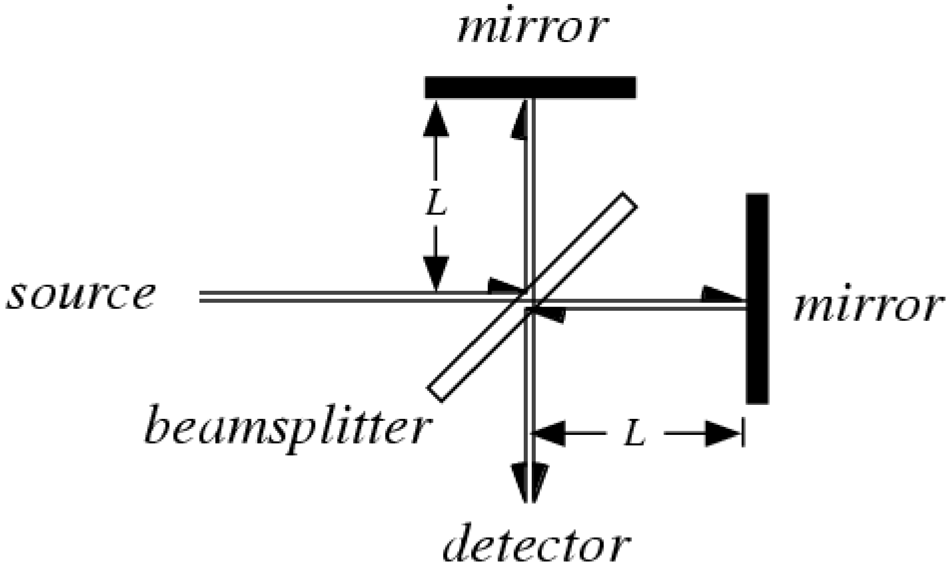


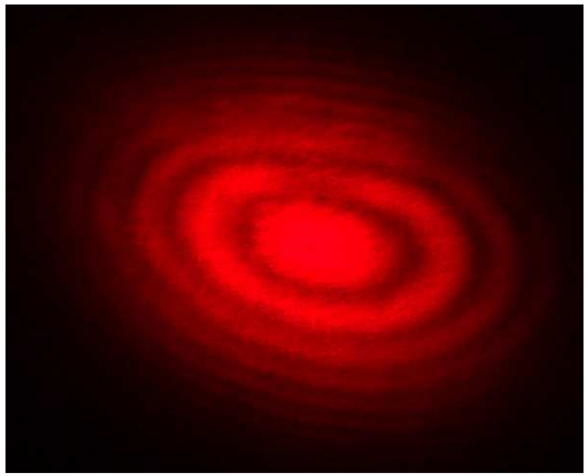
- Thin film of thickness $d \ll \lambda$ $(1, n_1, n_2)$
- Path difference $\Delta r = 2d \pm 1/2(\lambda/n_1) \pm 1/2(\lambda/n_1) = 2d$
- Constructive $\Delta r = m(\lambda/n_1)$,
- Destructive $\Delta r = (m+1/2)(\lambda/n_1)$

Constructive
$2nd = m\lambda$,
Destructive
$2nd = (m+1/2)\lambda$

Michelsen Interferometer

- Interference used for precise measurements of small distances
- Light from monochromatic source split equally by half silvered mirror (beam splitter)
- Each beam reflects off flat mirrors and recombine to form interference fringes

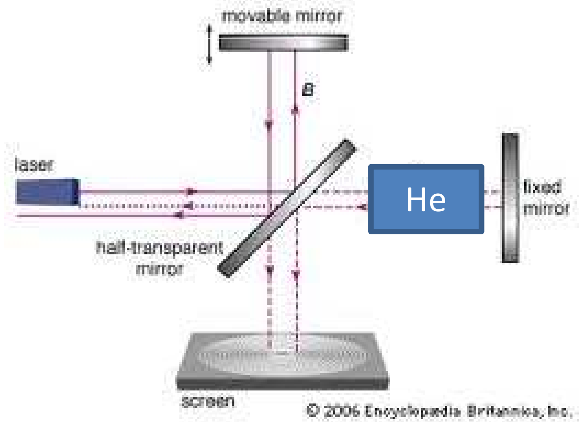




Michelsen Interferometer

- Measurements involve CHANGES in the interference pattern

Example: Light of wavelength λ is being used in a Michelsen interferometer. If one mirror is translated to that the central fringe changes from a light fringe to a dark fringe, by how far was it translated?



Phase b/n min and max = 180degrees
Convert to distance

180 degrees x (λ)/(360 degrees)= **λ/2**

Example: A cylinder of length L is initially evacuated and put into one arm of a Michelsen interferometer operating at a wavelength, λ; there is initially a bright fringe in the center of the pattern. If Helium is let into the cylinder (index of refraction n), how many times does the bright central fringe change to a dark fringe?

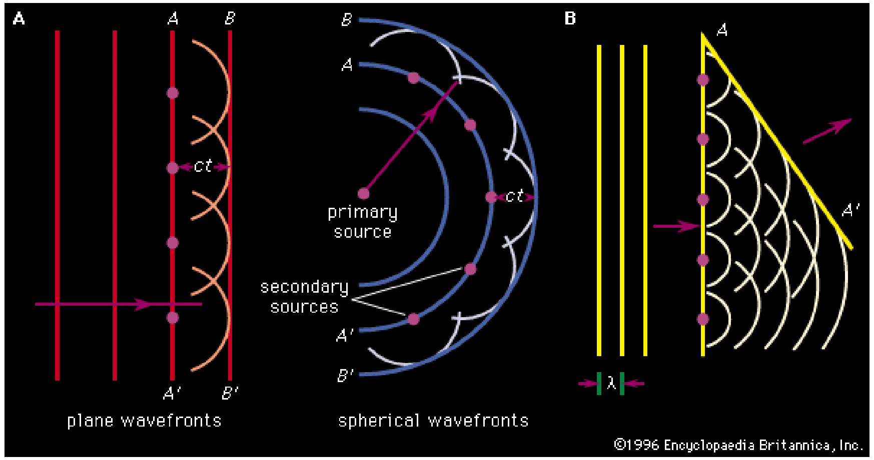
Wavelengths in cylinder with air: L/λ
Cylinder with He: L*n/λ
# of changes from bright to dark = # of additional wavelengths that can fit in cylinder

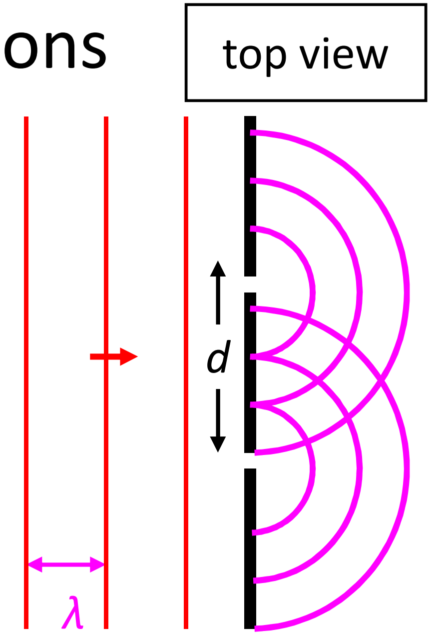
L/ λ*(n-1)

L=2cm, n=1.000036, λ=600nm
#=(2 cm x 10^7 nm/1cm)/(600nm)*0.000036
=I

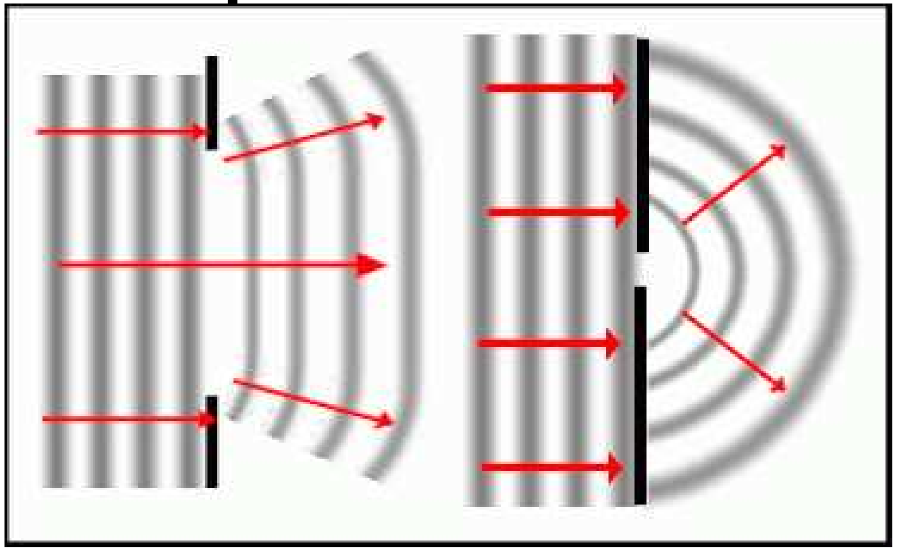
Huygens' Principle

- Diffraction = bending of light (or other waves) as they pass by objects
- Each spot on a wave-front acts like a point source of waves
  - Can be deduced from Maxwell's equations
    - But they weren't known in Huygens' time





Huygen’s Principle

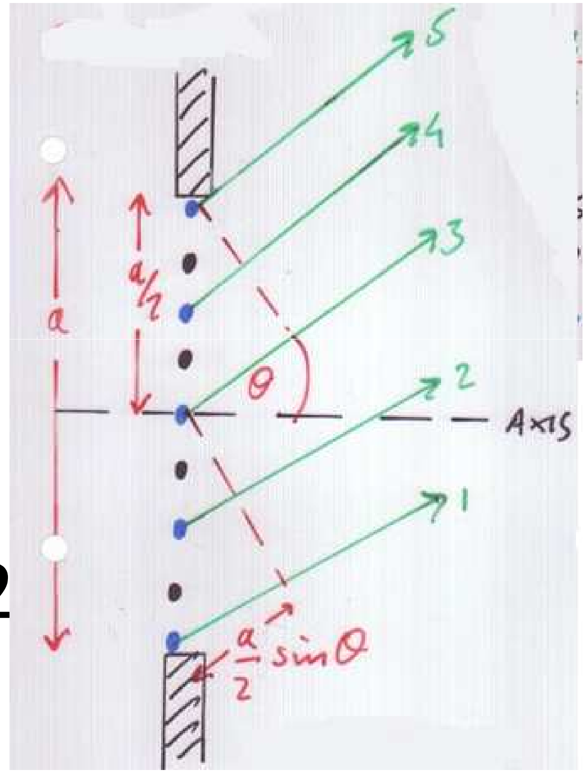


- Plane wave incident on barrier containing hole
- a>>λ, negligible diffraction
- a~λ, diffraction dominates
- There is always diffraction, but can ignore depending on length scale

Diffraction limits our ability to see small objects and focus light

Single slit diffraction

- Single wide slit – grating w/infinitely many slits
- Look in particular θ
- Divide into 3 gratings
- Path difference
- $\Delta r = (a/2)\sin\theta$
- Destructive interference $\Delta r = m\lambda/2$

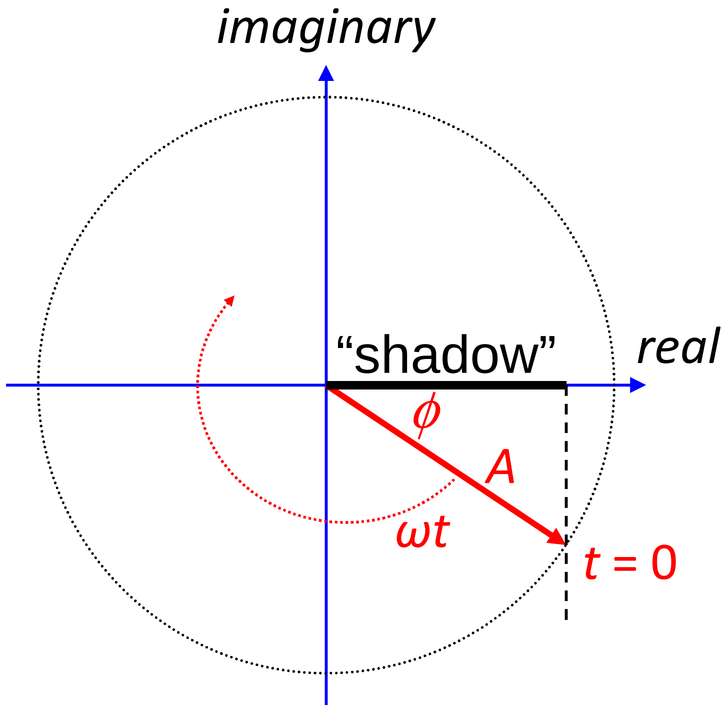


$$\mathbf{a \sin\theta = m\lambda}$$

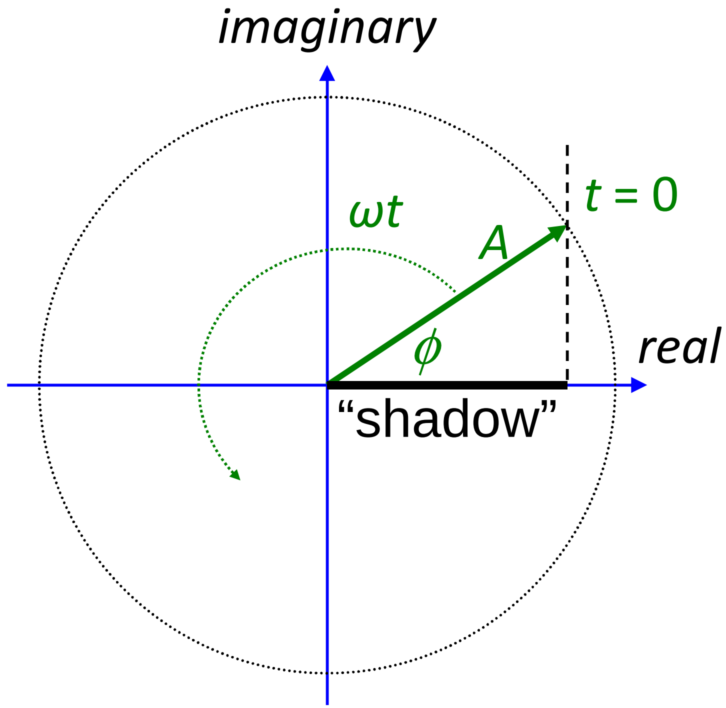
- Secondary maxima ½ b/n minima

Phasors

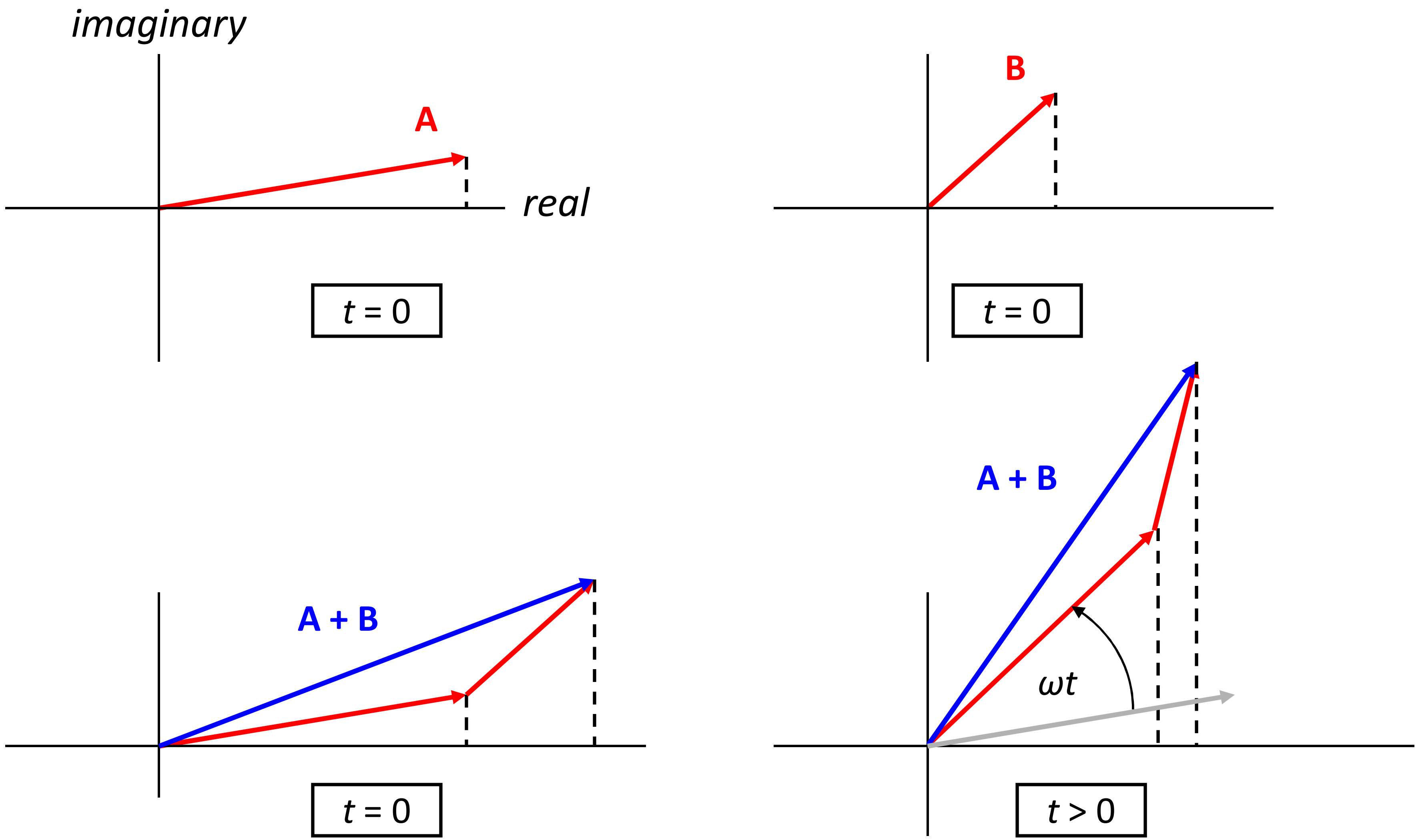
- A complex number which represents a sinusoid: $s(t) = A\cos(\omega t + \phi) = \mathrm{Re}\{Ae^{i(\omega t + \phi)}\}$
- Think of a rotating stick (or "vector")



$\omega t + \phi$

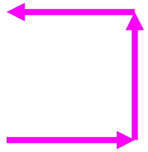


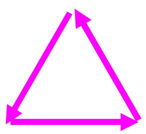
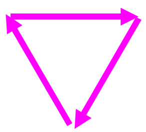
The arithmetic of interfering sinusoids is the arithmetic of complex numbers

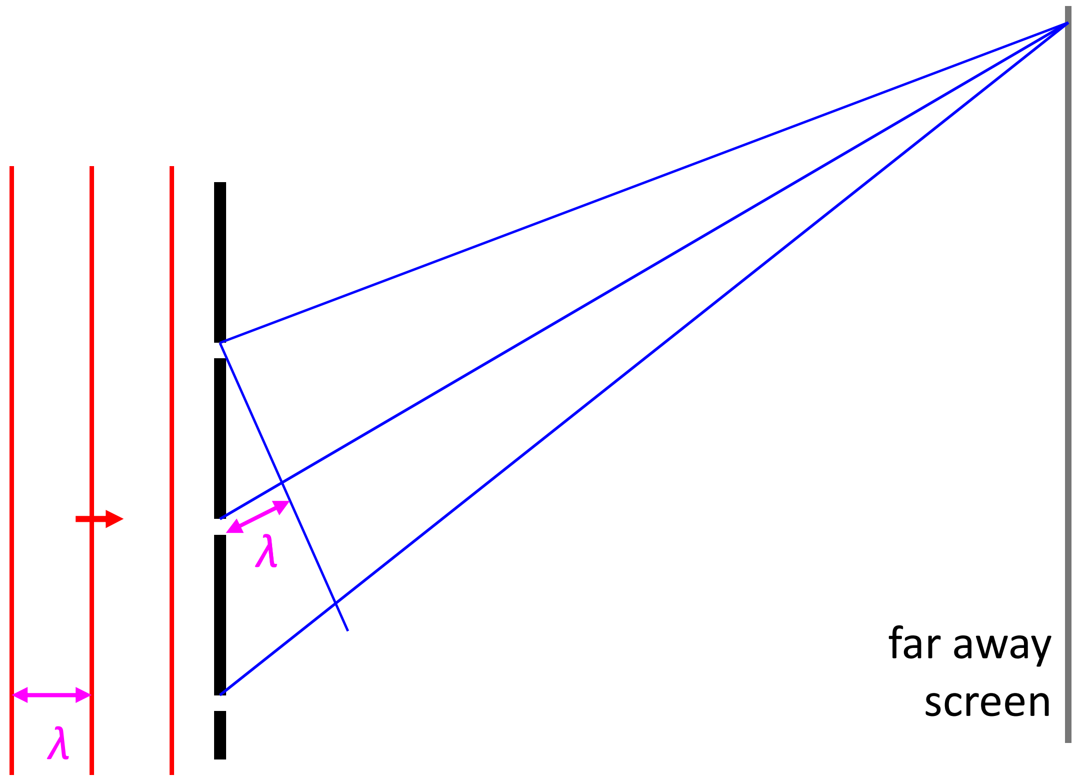


Which diagram shows phasors for the 3 waves at the screen?

- A

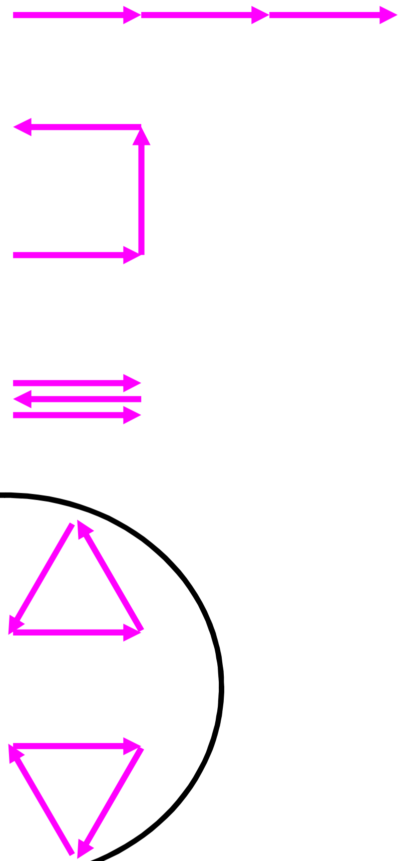
-
- B

- C

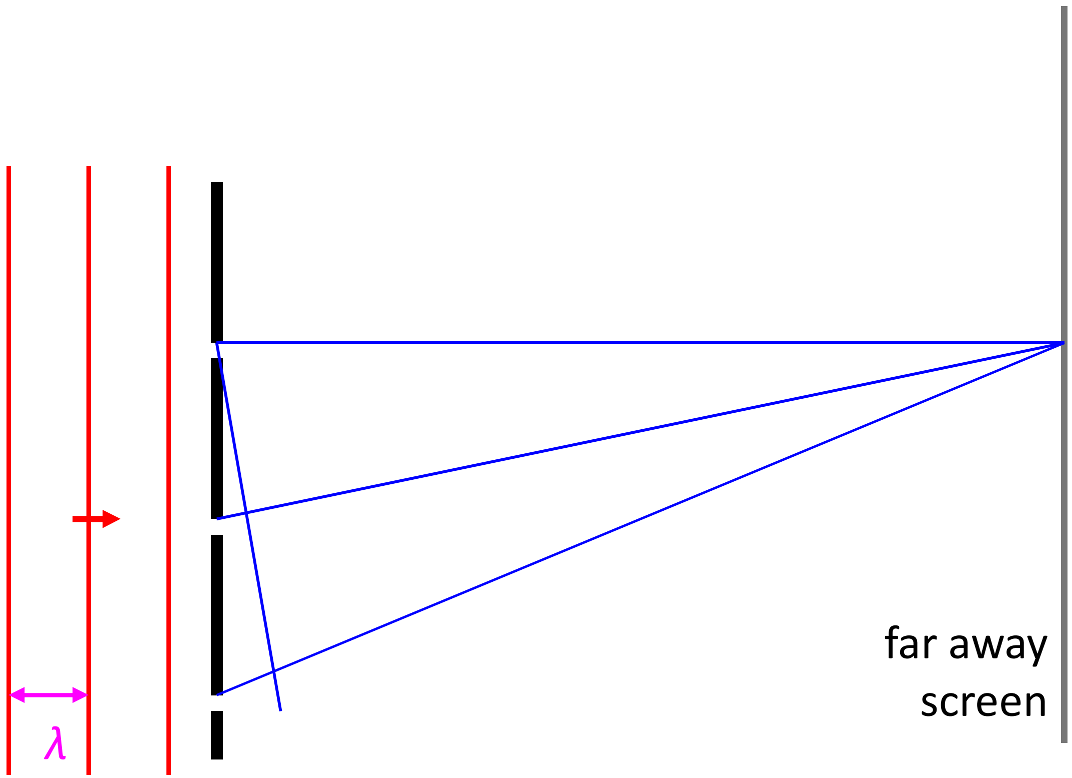
- D

- E




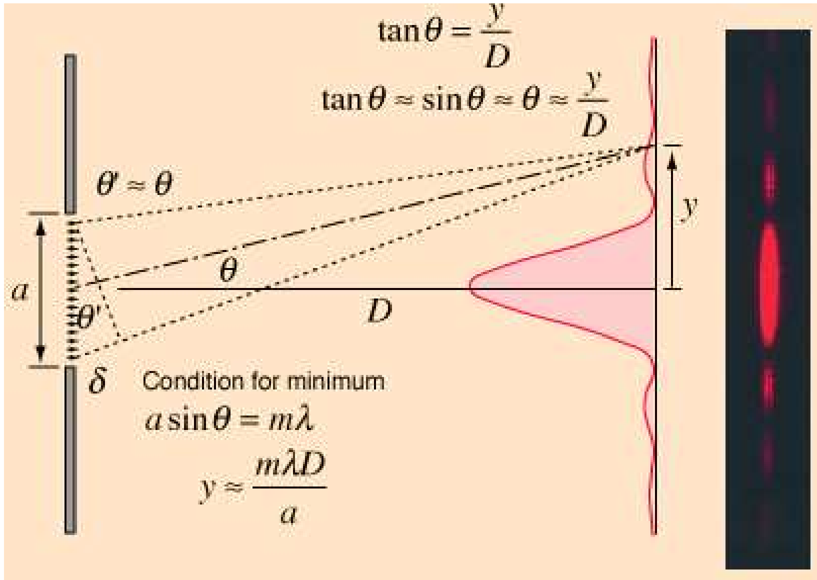
Which phasor diagram shows extinction for the system?

- A
- B
- C
- D
- E





Intensity of Single Slit Diffraction



- Divide into N gratings

$$\Delta\phi = \frac{a}{N-1}\sin\theta \times \frac{2\pi}{\lambda}$$

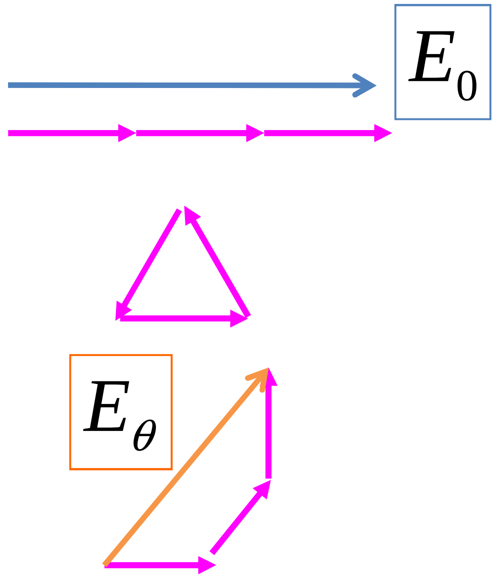
- Total phase of Nth relative to 1st

$$\phi = \frac{2\pi a}{\lambda}\sin\theta$$

- Let N→infinity – arc of radius R
- Pg 993 for pics $\frac{E_\theta}{E_0} = \frac{\sin(\phi/2)}{\phi/2}$

$$\overline{S}_\theta = \overline{S}_0\left[\frac{\sin(\phi/2)}{\phi/2}\right]^2$$

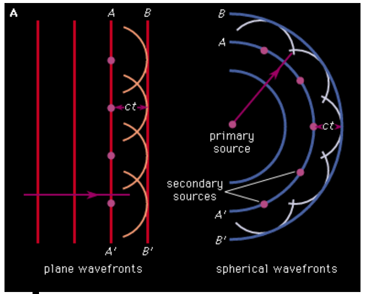
Intensity is 0 for a sin θ=mλ

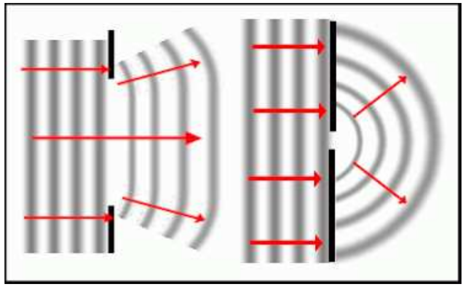


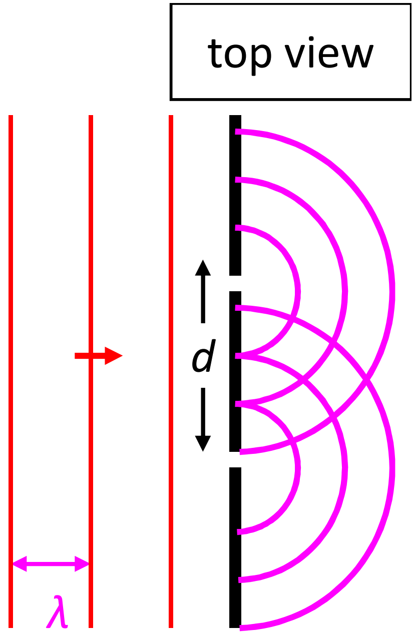
Central peak is wider as a increases relative to **λ**

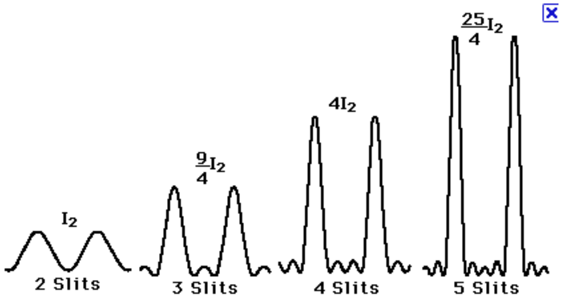
Review

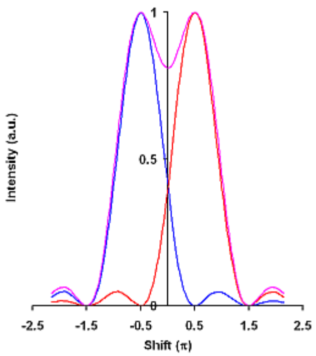
- Coherence
- Huygens' principle
- Diffraction: $d\sin\theta = m\lambda$ (max)
  - Double slit $(m+1/2)\lambda$ (min)
  - Multiple slit $(m/N)\lambda$ (N-1 min)
    - Resolving power $\lambda/\Delta\lambda = mN$
- Single Slit min: $a\sin\theta = m\lambda$
- Interference
  - Thin film
    - $2nd = (m+1/2)\lambda$
    - $2nd = m\lambda$
  - michelsen

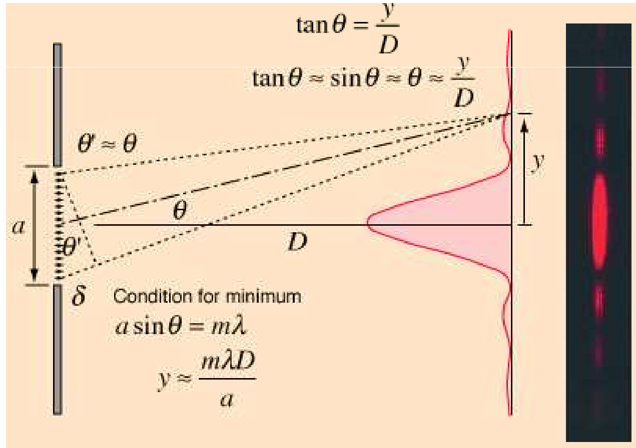


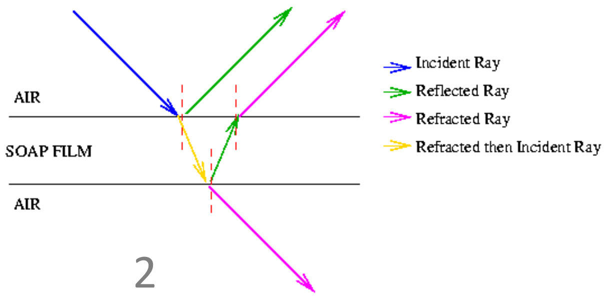


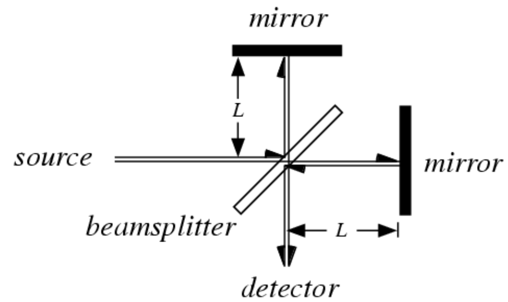






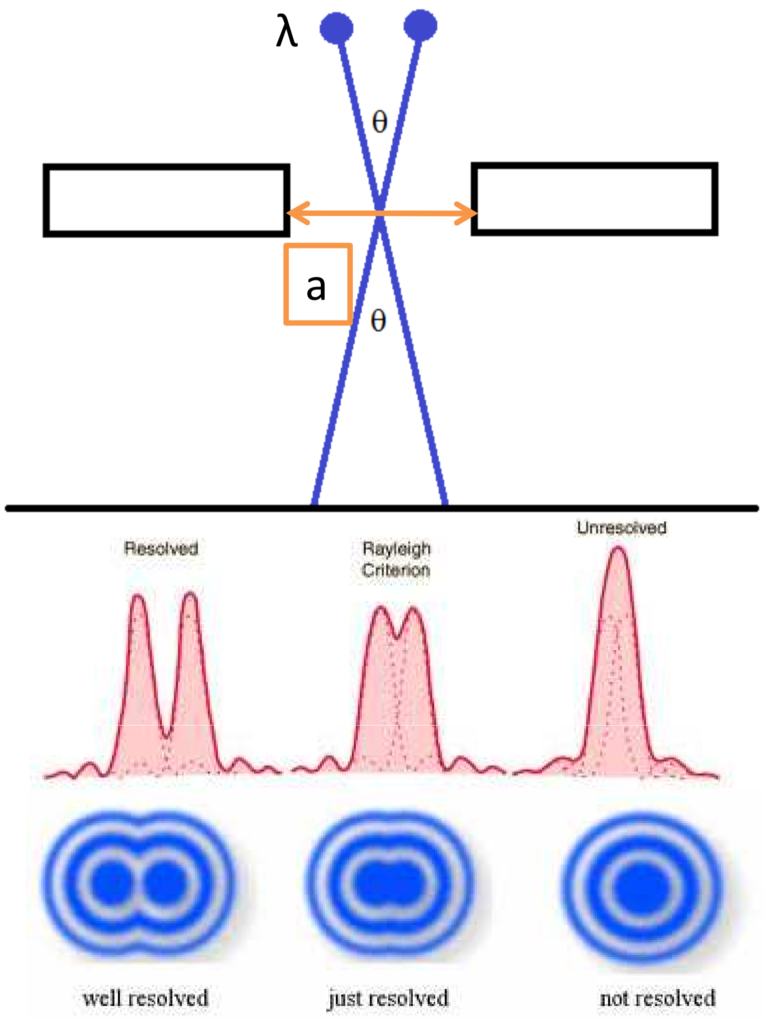






Diffraction Limit (Rayleigh Criterion)

- When the slit width is not negligible (>>λ) – single slit diffraction
- Two (incoherent) sources just resolvable - central peak of one = first minimum of other
- First min (slit): $a\sin\theta=\lambda$
- Small angle: $\sin\theta \sim \theta_{min} = \lambda/a$
- Circular: $\theta_{min}=1.22\ \lambda/a$ (Bessel)



Increasing aperture size allows smaller angular differences to be resolved