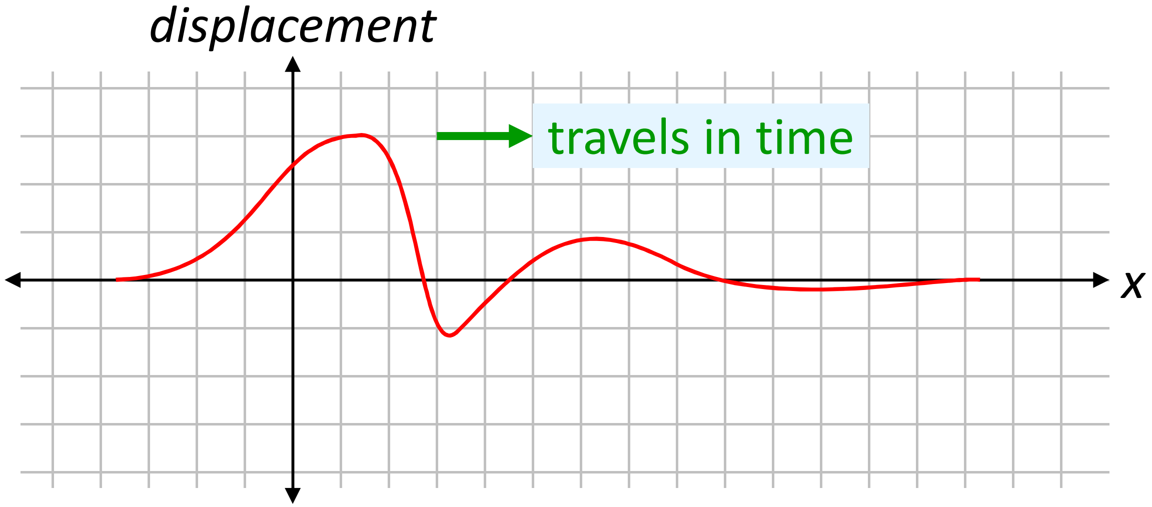
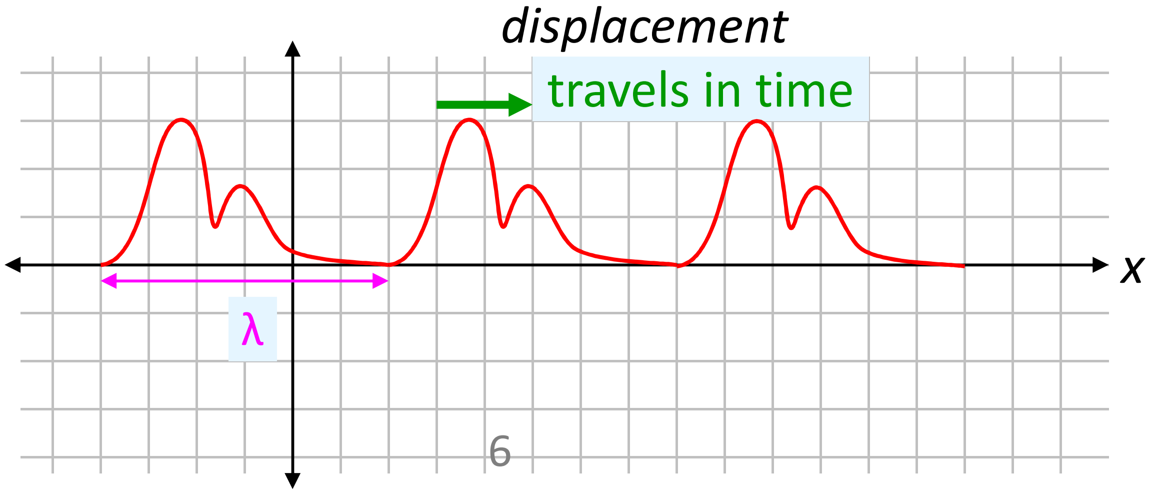
Types of Waveforms

- Pulse: not periodic
  - drops to zero on both sides
  - f(x)



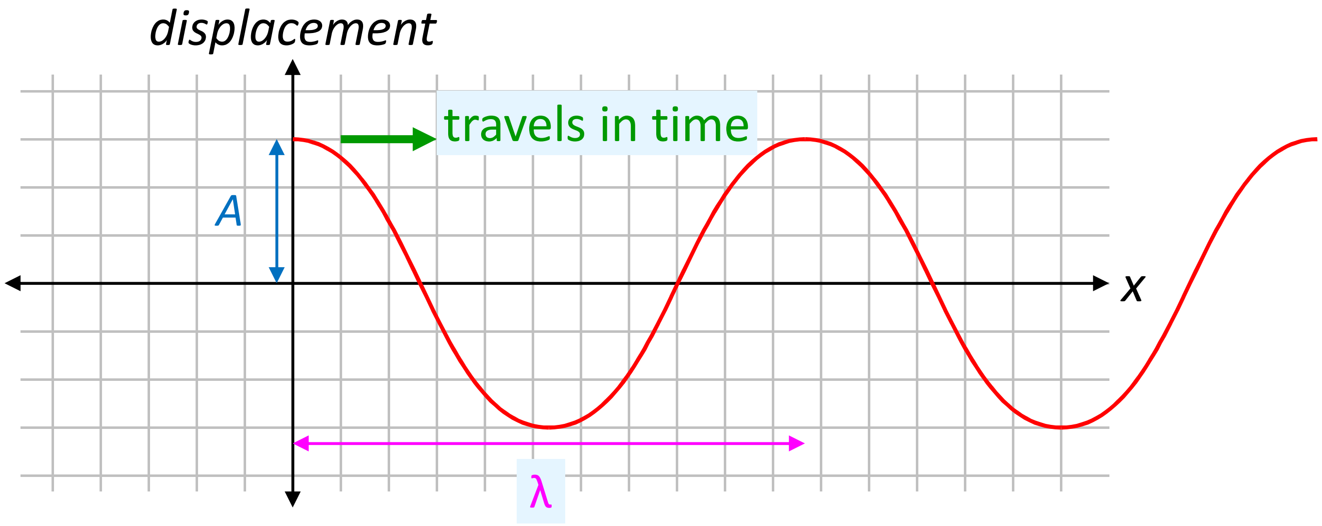
- Continuous (or periodic) wave:



Types of Waveforms

- Harmonic (aka sinusoidal)
  - Special case of periodic

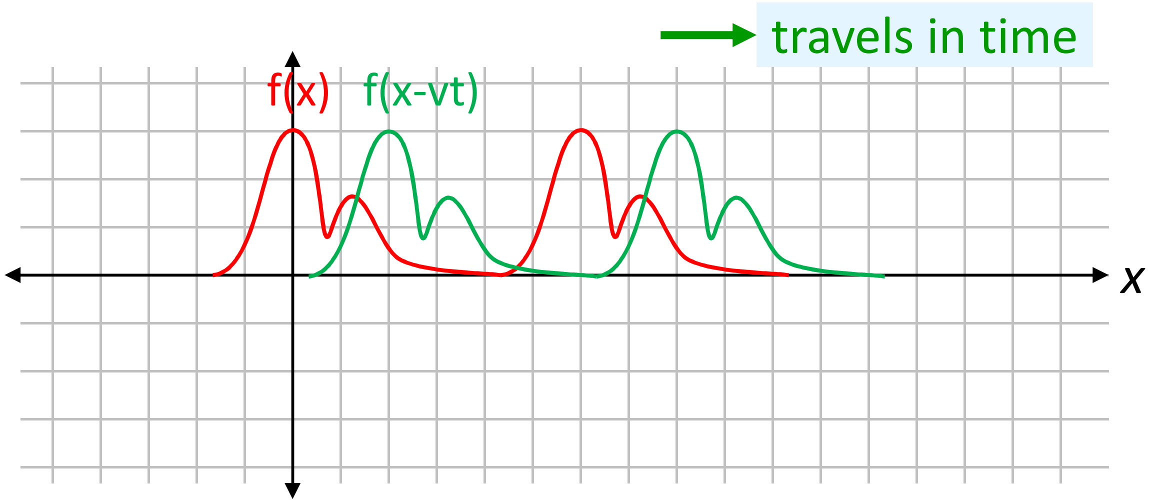
(Longitudinal and
transverse)



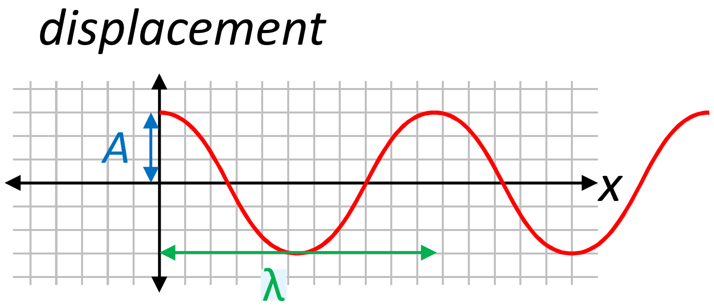
  - Wave speed: $v = \dfrac{\lambda}{T} = \lambda f$

Mathematical Description

- Pulse: $f(x)$
- Moving pulse:
  - "+" direction $f(x-vt)$
  - "-"direction $f(x+vt)$



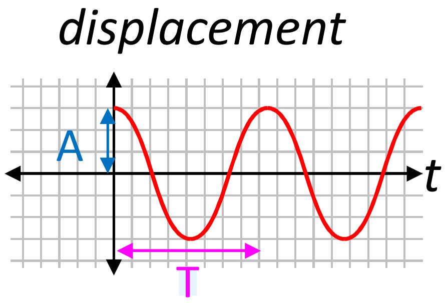
Simple Harmonic Wave



- Position at t=0:
  - $y(x,0) = A\cos\left(\frac{2\pi x}{\lambda}\right) = A\cos(kx)$
- Travelling to the right with speed, v:
  - $y(x,t) = y(x-vt) = A\cos k(x-vt) =$

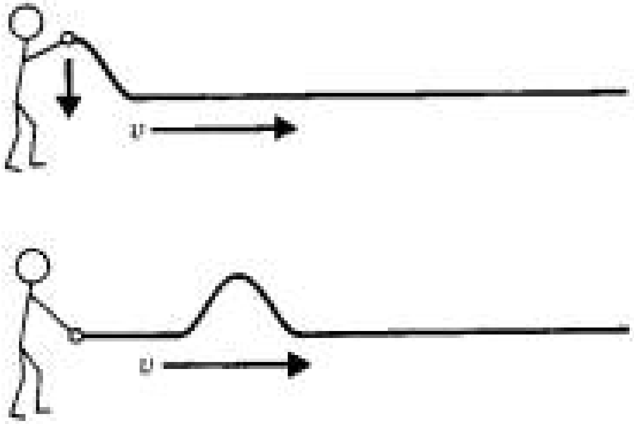
$$A\cos(kx-\omega t)$$

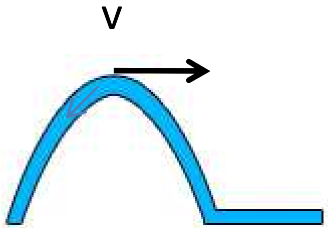
- Angular frequency---> $\omega = 2\pi f = \frac{2\pi}{T}$
- Wave number---------> $k = \frac{2\pi}{\lambda}$
- Velocity------------------> $v = \frac{\omega}{k} = \lambda f$



Waves on a String under tension

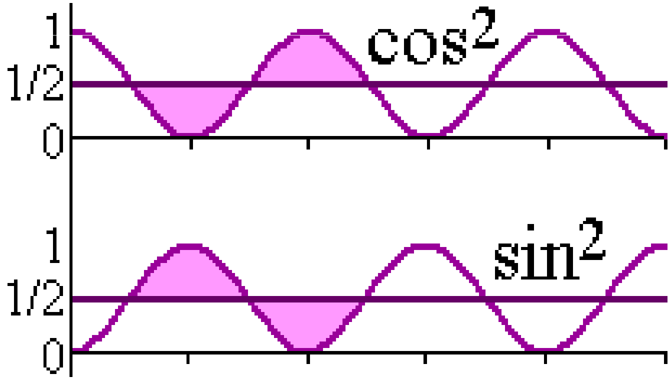
- Small amplitude displacements
- transverse
- Given mass density, μ, tension, F
  - Speed: $v = \sqrt{\frac{F}{\mu}}$
  - Dimensions: [F]/[μ]=[v]

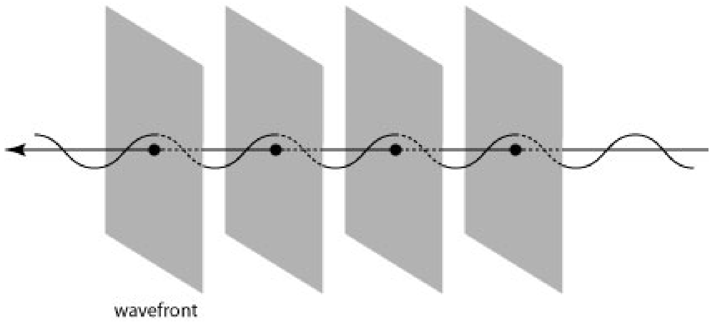




Average Power
(waves propagate energy)

- P=dE/dt= $\vec{F} \cdot \vec{u} \quad \left(W = \vec{F} \cdot \vec{x}\right)$
  - Speed of ***medium*** $u = \frac{\partial y}{\partial t} = \omega A \sin\left(kx - \omega t\right)$
- Power fluctuates throughout the wave (book)
  - $P = F\omega k A^2 \sin^2\left(kx - \omega t\right)$
- Interested in average power
  - $\overline{P} = \frac{1}{2} F\omega k A^2$
- Waves on a string $\overline{P} = \frac{1}{2}\mu\omega^2 A^2 v$

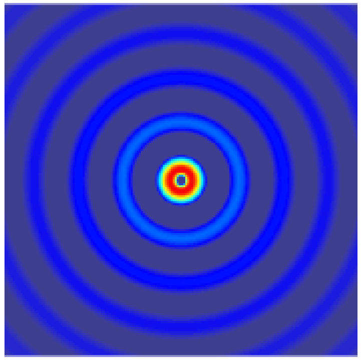




Wave Intensity

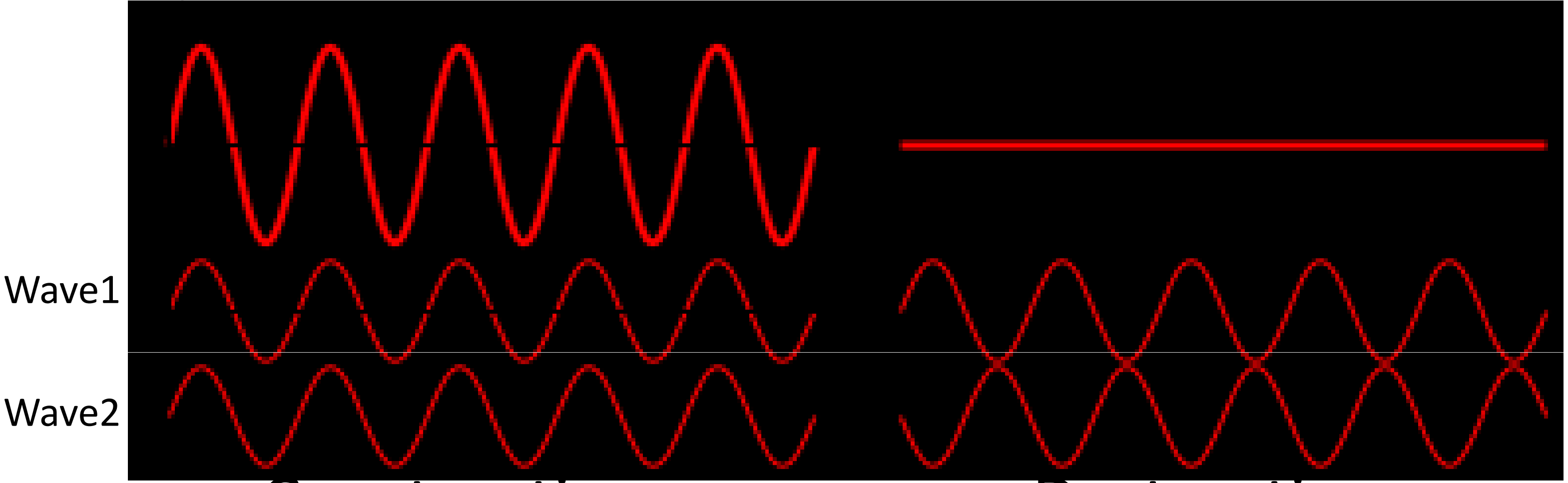
- Intensity= average energy/time/area (W/m^2)
  - Area perpendicular to propagation direction
- Wavefront – surface of constant phase
  - Plane wavefronts (plane wave) – constant intensity
  - Spherical wavefront (spherical wave) – intensity decreases as distance from source increases

$$\bar{I} = \frac{\overline{P}}{A} = \frac{\overline{P}}{4\pi r^2}$$



Superposition

- Wave interference = algebraic sum of displacements



Constructive

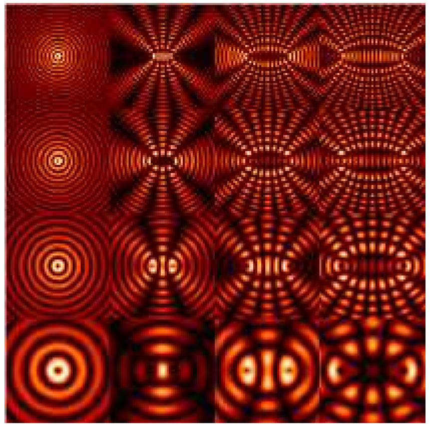
(in phase)

$\Delta\phi = 2\pi n$

Destructive

(180 degree out of phase)

$\Delta\phi = n\pi$

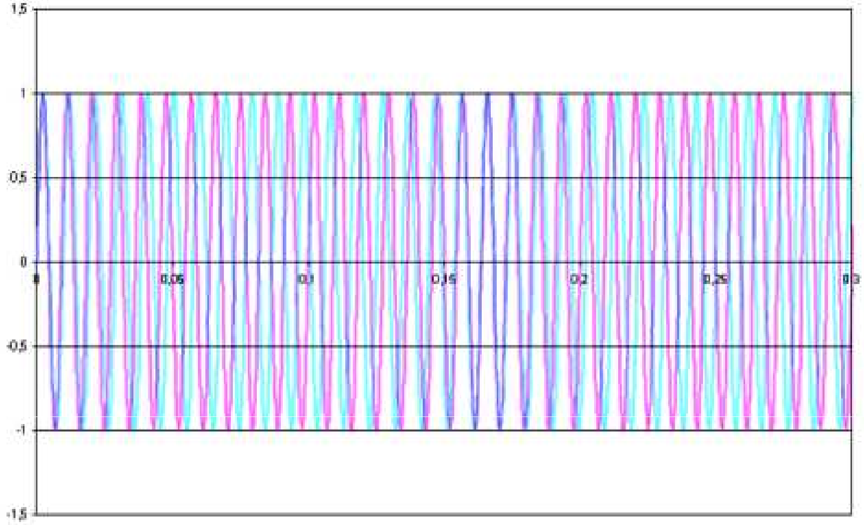


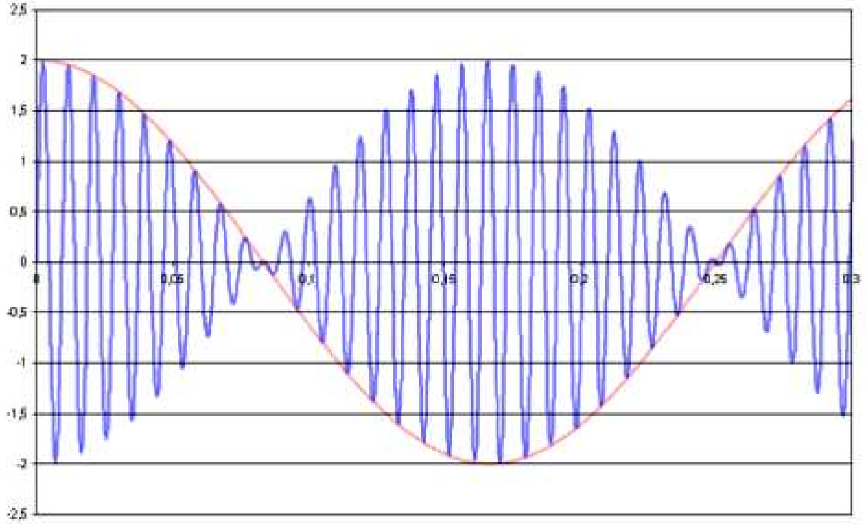
Beats

- Interference of 2 waves w/slightly different frequencies
- At a fixed position, x=0:

$$y(t) = A\cos\omega_1 t + A\cos\omega_2 t$$

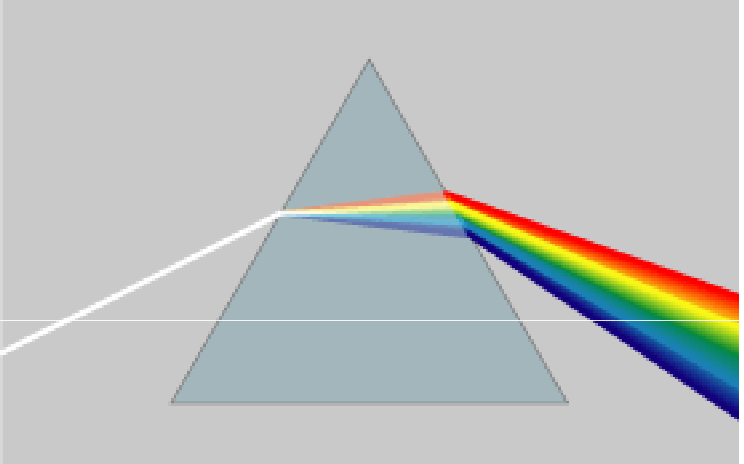
$$= \underbrace{2A\cos\left(\frac{\omega_1 - \omega_2}{2}t\right)}_{\text{“Amplitude”}}\cos\left(\frac{\omega_1 + \omega_2}{2}t\right)$$





Dispersion

- Wave frequency (speed) depends on wavelength $v = \lambda f(\lambda)$



- Waves on the surface

$$v = \sqrt{\frac{\lambda g}{2\pi}}$$

Wave Equation

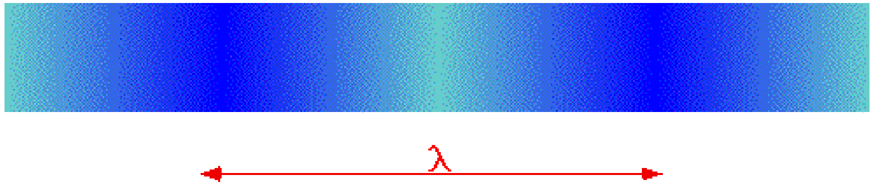
- All equations of the form $f\left(x \pm vt\right)$ satisfy

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$$

- i.e. displacement of waves on a string

$$y\left(x, t\right) = A\cos\left(kx \pm \omega t\right)$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{\mu}{F}\frac{\partial^2 y}{\partial t^2}$$



Sound Waves In Gas

- Longitudinal pressure wave
- Speed of wave $v = \sqrt{\frac{\gamma P}{\rho}}$

pressure

density

- Coefficient, γ, depends on nature of gas
  - – Ideal $\gamma = 5/3$
  - – Diatomic $\gamma = 7/3$

Sound waves in Liquid/Solid

- Speed of sound

  $v = \sqrt{\frac{B}{\rho}}$ (in gas, $v = \sqrt{\frac{\gamma P}{\rho}}$)

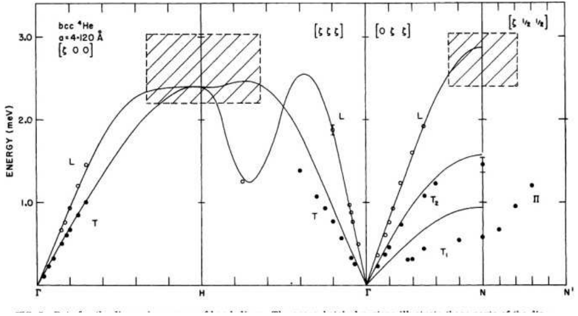
- Bulk modulus (wave speed depends on direction)

  $$B = -\frac{\Delta P}{\Delta V / V}$$

  – Units of pressure

  – Measure of "stiffness" (large B, hard to compress)

- longitudinal & transverse



Sound Intensity

- Recall for a string, $\bar{I} = \dfrac{\overline{P}}{Area}$, $\overline{P} = \dfrac{1}{2}\mu\omega^2 A^2 v$
- In terms of pressure,

$$\frac{Power}{Area} = Force \cdot velocity = (\Delta P)(Area)u$$

- Pressure, displacement vary sinusoidally

$$\Delta P = (\Delta P_0)\cos(kx - \omega t)$$
$$s = -s_0 \sin(kx - \omega t)$$

- See pg 419-420 $\bar{I} = \dfrac{(\Delta P_0)^2}{2\rho v} = \dfrac{1}{2}\rho\omega^2 s_0{}^2 v$

Decibels (dB)

- Human ear can detect a wide range of frequencies → log scale!
- Decibel unit defined as

$$\beta = 10\log\left(I/I_0\right)$$

where $I_0 = 10^{-12} W / m^2$ is threshold of hearing at 1kHz

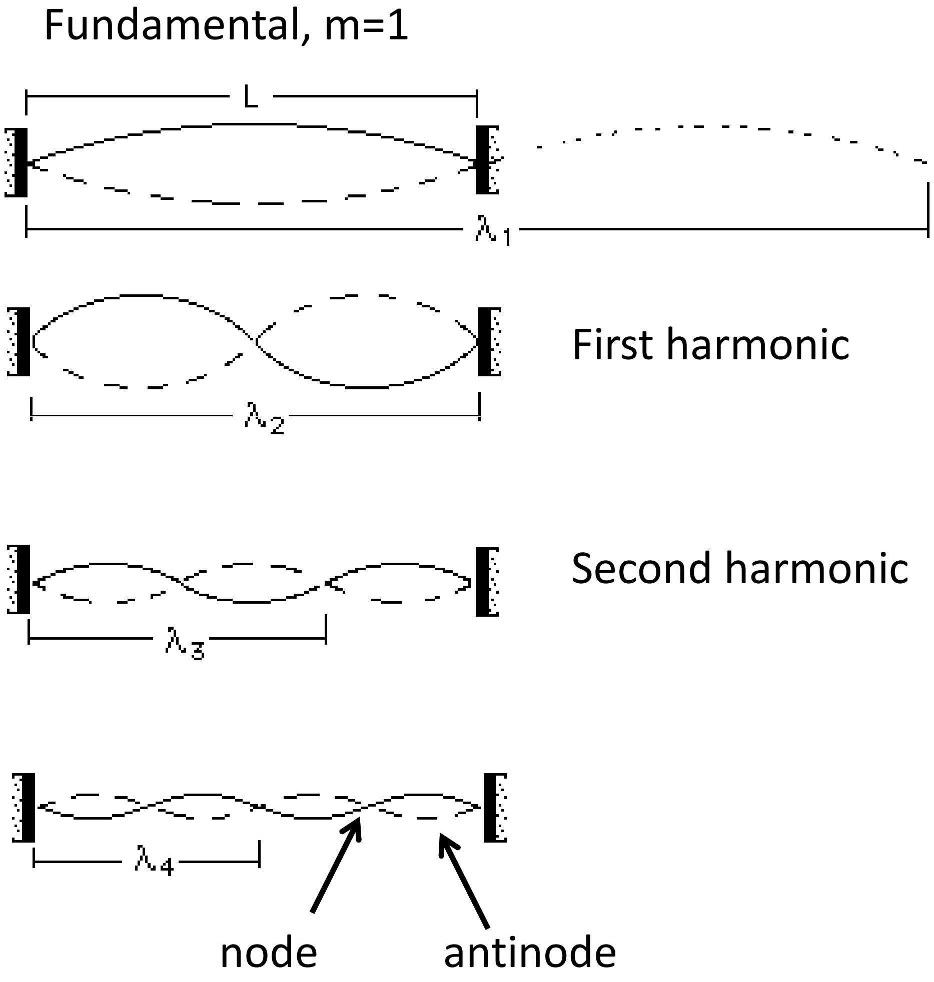
- >40dB, perceived loudness doubles for every 10dB increase

Standing waves

- String (L) clamped tightly at both ends
- No propagation (superposition of waves propagating in opposite directions
- Allowed modes

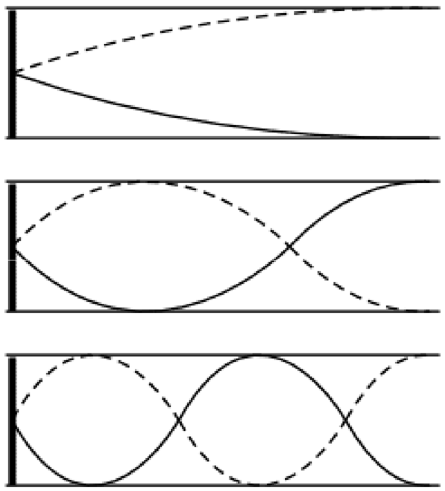
mode number

$$L = \frac{m\lambda_1}{2}$$

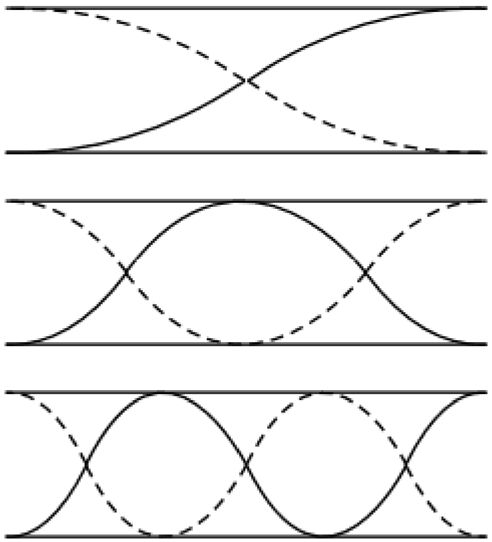


Standing waves

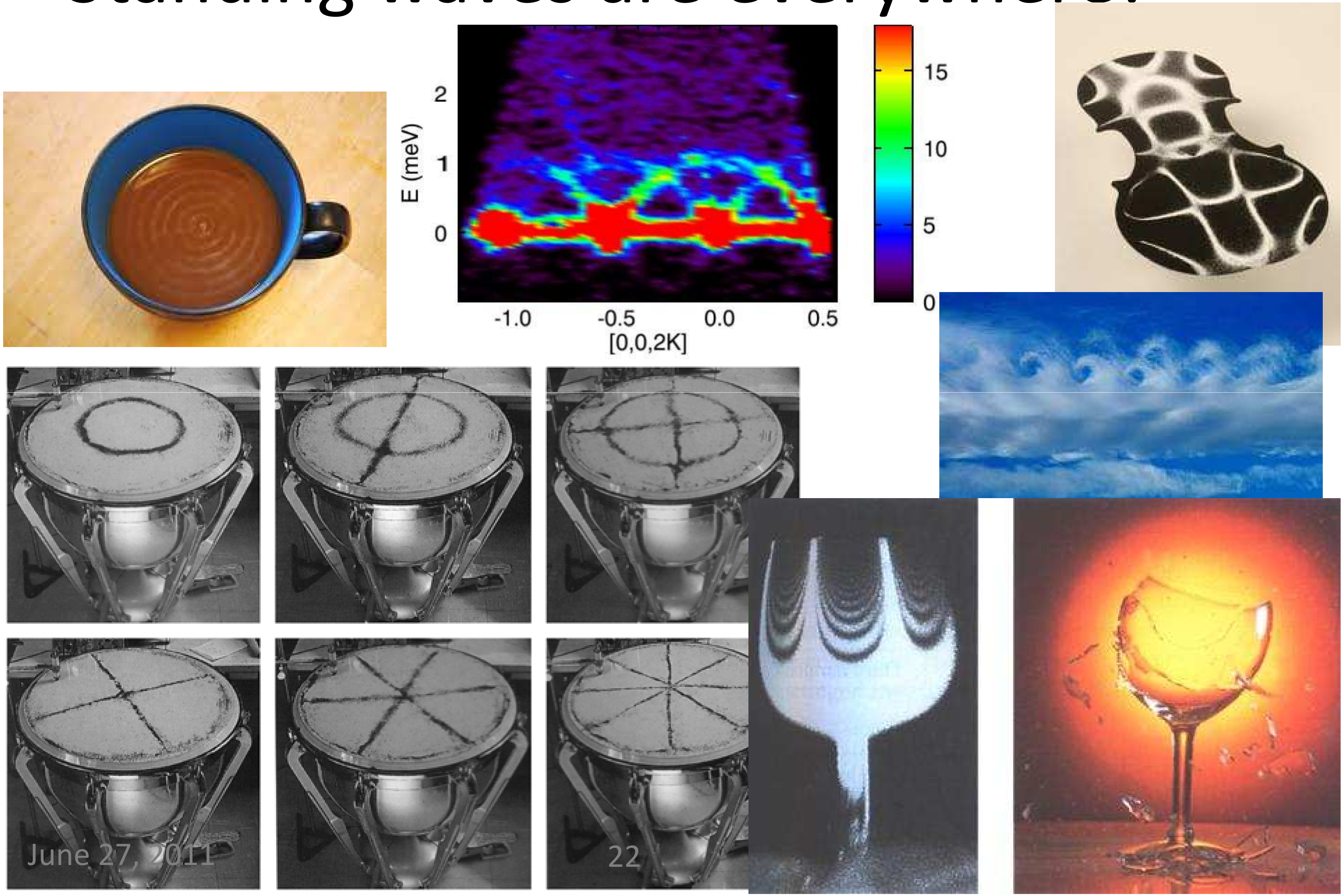
- String clamped at one end



- String open at both ends



Standing waves are everywhere!



*GRADES – will be curved, but only in your favor

Review - waves

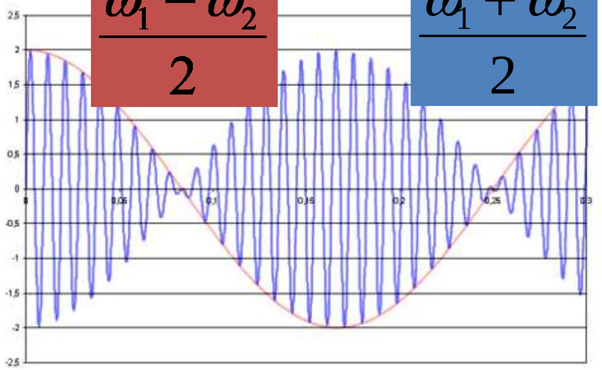
- $v = \lambda f = \frac{\omega}{k}$ (different from velocity of medium)
- Wave equation – $\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} \Longrightarrow f(x \mp vt)$
  - Wave on a string $v = \sqrt{F/\mu}$
  - Sound in gas $v = \sqrt{\gamma P/\rho}$
  - Sound in solid/liquid $v = \sqrt{B/\rho}$

    (bulk modulus $B = -\Delta P/(\Delta V/V)$)
- Longitudinal and/or transverse
- Superposition (i.e. beats)
- Dispersion (wave packets spread out)

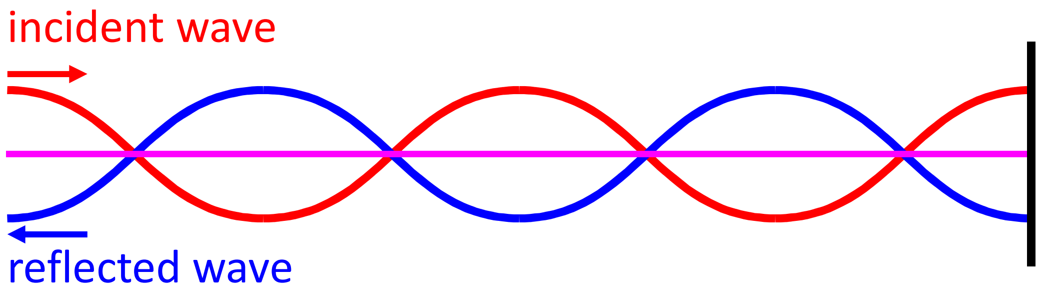
$$A\cos(kx - \omega t)$$

$$k = \frac{2\pi}{\lambda} \qquad \omega = \frac{2\pi}{T}$$

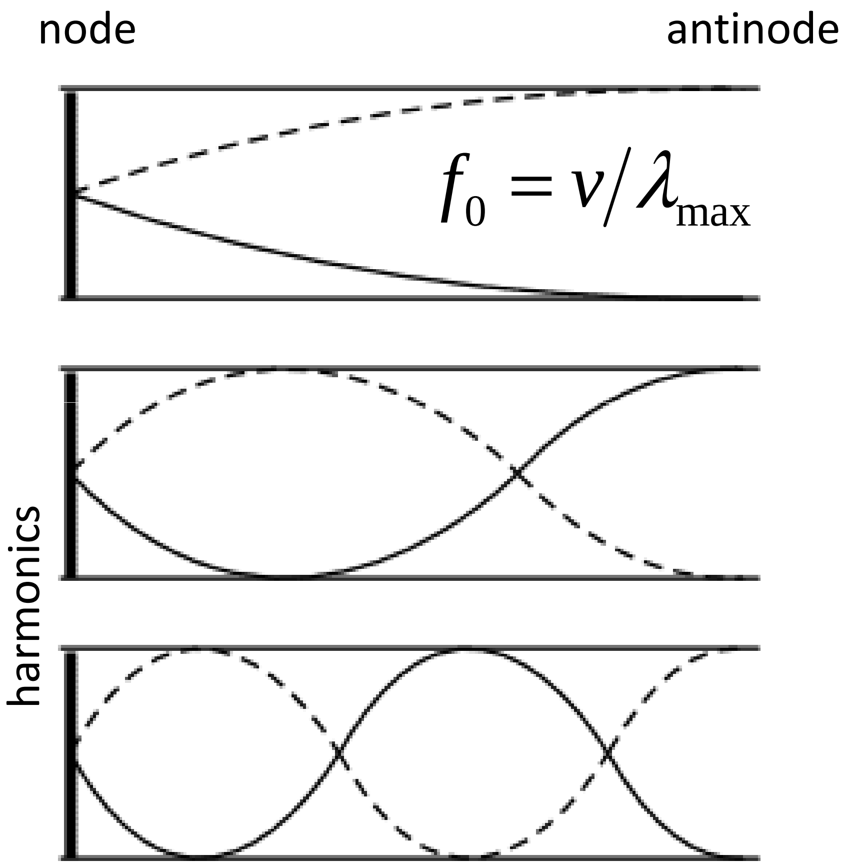
$$\frac{\omega_1 - \omega_2}{2} \qquad \frac{\omega_1 + \omega_2}{2}$$



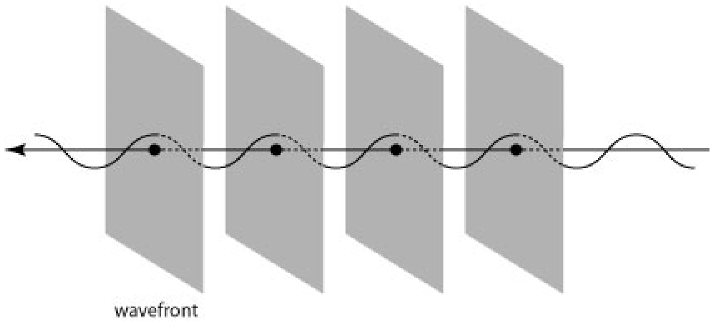
Review – Standing waves



- Propagation through bounded medium
- Superposition of 2 waves travelling in opposite directions
- 3 bounded scenarios
  - – Closed on both $L = \frac{m\lambda}{2}$
  - – Open on both $L = \frac{m\lambda}{4}$
  - – open on one $L = \frac{(m+1/2)\lambda}{2}$



amplitude vs **instantaneous amplitude**



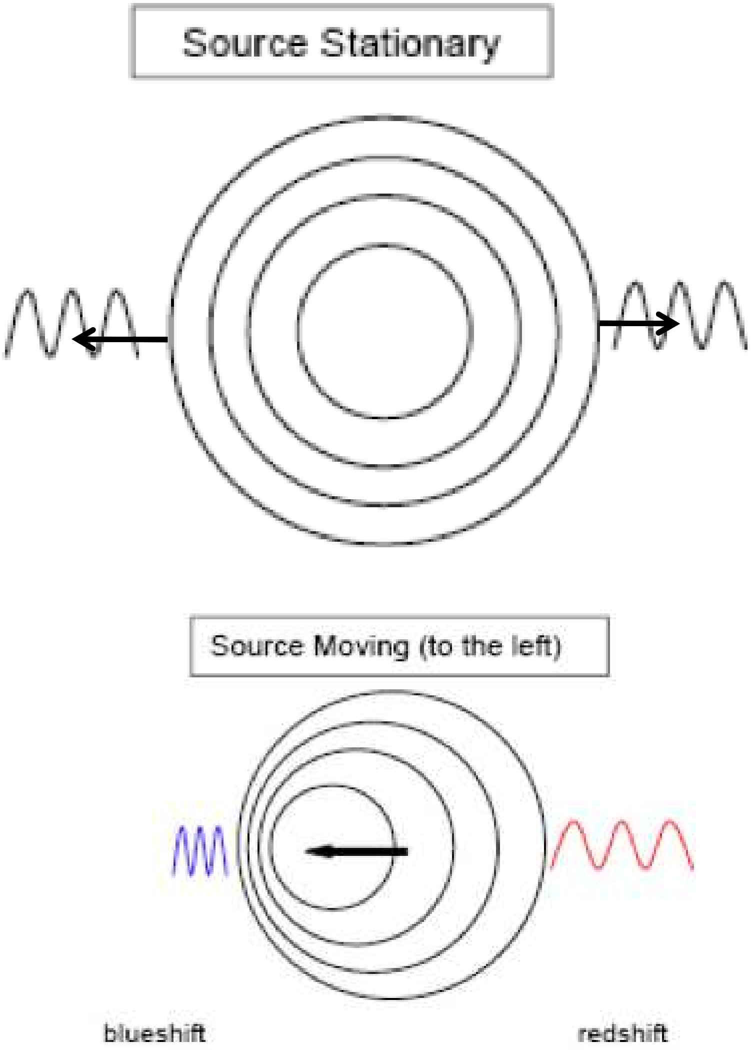
Review - Intensity

- Average power (string)– $\overline{P} = \frac{1}{2}\mu\omega^2 A^2 v$
- "wavefront" – surfaces of constant phase
- Plane/spherical wave – plane/spherical wavefronts
- Intensity – characterizes energy flow in >1D
  - 3D $\bar{I} = \frac{\overline{P}}{A} = \frac{\overline{P}}{4\pi r^2}$ (2D $\bar{I}_{2D} = \frac{\overline{P}}{l} = \frac{\overline{P}}{2\pi r}$)
- Intensity of sound

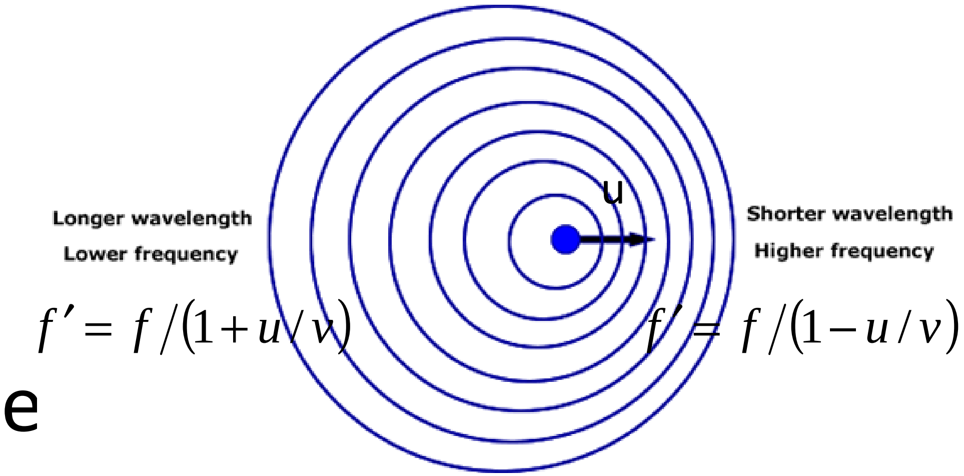
$\bar{I} = \frac{(\Delta P_0)^2}{2\rho v} = \frac{1}{2}\rho\omega^2 {s_0}^2 v$ decibels $\beta = 10\log(I/I_0)$

Doppler effect

- Wave propagates symmetrically in every direction from a point source at rest
- When source/observer moves, the observed **frequency** and/or wavelength is shifted
- Let u be speed of source or observer



Doppler: source moving



$f' = f/(1+u/v)$

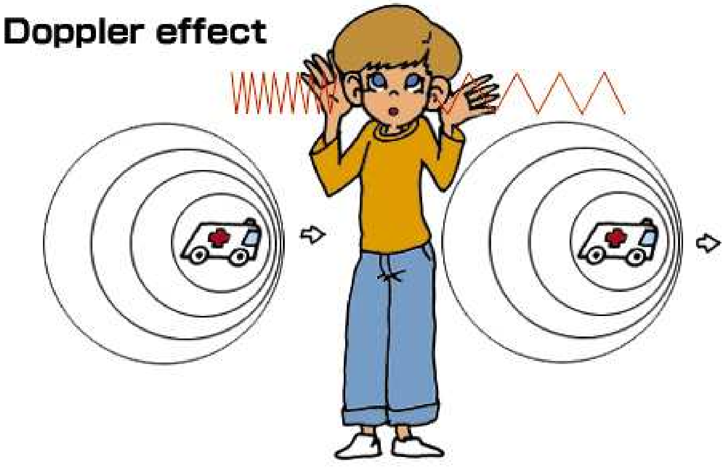
$f' = f/(1-u/v)$

- Source reference frame travel λ in one period, T
- During T, source covers distance of uT
- Observed wavelength:

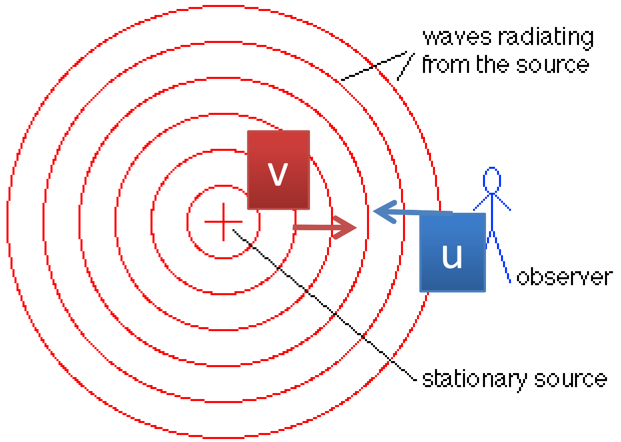
$$\lambda' = \lambda \mp uT$$

$$\lambda' = \lambda(1 \mp u/v)$$

- frequency $f' = f/(1 \mp u/v)$



Doppler Shift: Moving Observer



- Shift in frequency only,
- wavelength does not change
- Speed observed = v+u
- Observed period $T' = \dfrac{\lambda}{v+u}$

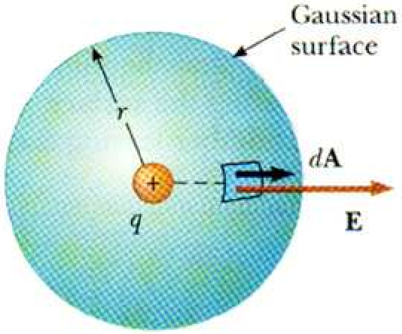
- Observed frequency shift

$$f' = f\left(1 \pm u/v\right)$$

(negative sign means observer moving AWAY)

EM waves

4 laws of EM (up to now)



Gauss

$$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$$

no magnetic monopoles

$$\vec{\nabla}\cdot\vec{B} = 0$$

Faraday

$$\vec{\nabla}\times\vec{E} = -\partial\vec{B}/\partial t$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$$

Ampere

$$\iiint \vec{\nabla}\cdot\vec{F}\,d\tau = \oiint \vec{F}\cdot d\vec{A}$$

$$\iint \vec{\nabla}\times\vec{F}\cdot d\vec{S} = \oint \vec{F}\cdot d\vec{\ell}$$

$$\oint \vec{E}\cdot d\vec{A} = q_{encl}/\varepsilon_0$$

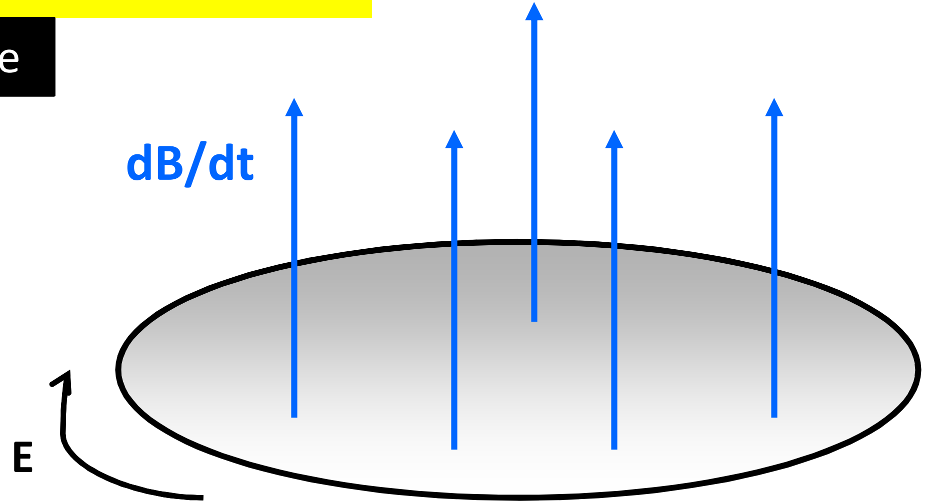
$$\oint \vec{B}\cdot d\vec{A} = 0$$

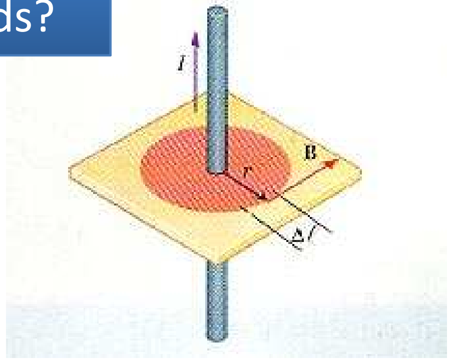
Lenz's law

$$\oint \vec{E}\cdot d\vec{\ell} = -\partial\Phi_B/\partial t$$

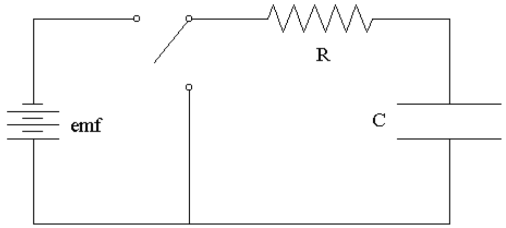
$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{encl}$$

Changing E-fields induce B-fields?





Modify Ampere’s law

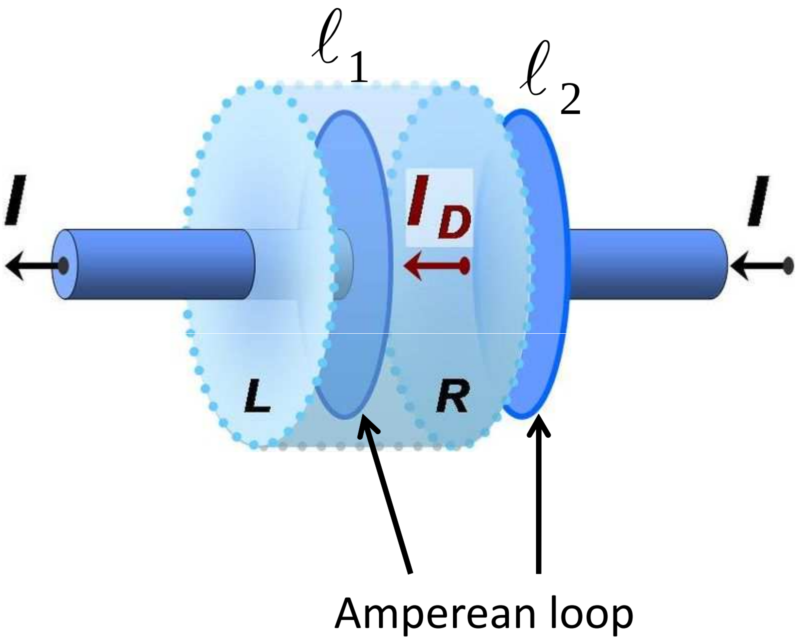


- Ambiguity in ampere’s law

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{encl}$$

$$\oint_{\ell_1} \vec{B}\cdot d\vec{\ell} = \mu_0 I$$

$$\oint_{\ell_2} \vec{B}\cdot d\vec{\ell} = 0!$$



- Displacement current

$$I_D = \varepsilon_0 \partial\Phi_E / \partial t$$

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{encl} + \mu_0\varepsilon_0 \partial\Phi_E / \partial t$$

Maxwell’s Equations - complete

Simplest scenario: in a vacuum

$$\oint \vec{E} \cdot d\vec{A} = q_{encl}/\varepsilon_0$$

Gauss

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{E} \cdot d\vec{\ell} = -\partial\Phi_B/\partial t$$

Faraday

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{encl} + \mu_0\varepsilon_0 \partial\Phi_E/\partial t$$

Ampere (complete)

$$\oint \vec{E} \cdot d\vec{A} = 0$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{E} \cdot d\vec{\ell} = -\partial\Phi_B/\partial t$$

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0\varepsilon_0 \partial\Phi_E/\partial t$$

Electric and Magnetic fields on equal footing → Electromagnetic waves!

EM Waves

- Coupled differential equations

$$\vec{\nabla}\cdot\vec{E}=0$$

$$\vec{\nabla}\cdot\vec{B}=0$$

$$\vec{\nabla}\times\vec{E}=-\partial\vec{B}/\partial t$$

$$\vec{\nabla}\times\vec{B}=\mu_0\varepsilon_0\,\partial\vec{E}/\partial t$$

Take the curl of both sides

$$\boxed{\vec{\nabla}\times\left(\vec{\nabla}\times\vec{E}\right)=\vec{\nabla}\left(\cancelto{0}{\vec{\nabla}\cdot\vec{E}}\right)-\nabla^2\vec{E}}$$

$$\boxed{-\frac{\partial}{\partial t}\left(\vec{\nabla}\times\vec{B}\right)=-\frac{\partial}{\partial t}\left(\mu_0\varepsilon_0\,\partial\vec{E}/\partial t\right)=-\mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}}$$

$$\frac{\partial^2 f}{\partial x^2}=\frac{1}{c^2}\frac{\partial^2 f}{\partial t^2}$$

- In one dimension

$$\boxed{\frac{\partial^2\vec{E}}{\partial x^2}=\mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}}$$

$$\boxed{c=\frac{1}{\sqrt{\mu_0\varepsilon_0}}=3.0\times10^8\,m/s}$$

EM plane Wave

- E,B harmonic waves, oscillate in perpendicular directions
- We use *x* for the propagation direction

$$\vec{E}(x,t) = E_p \cos(kx - \omega t)\hat{j}$$

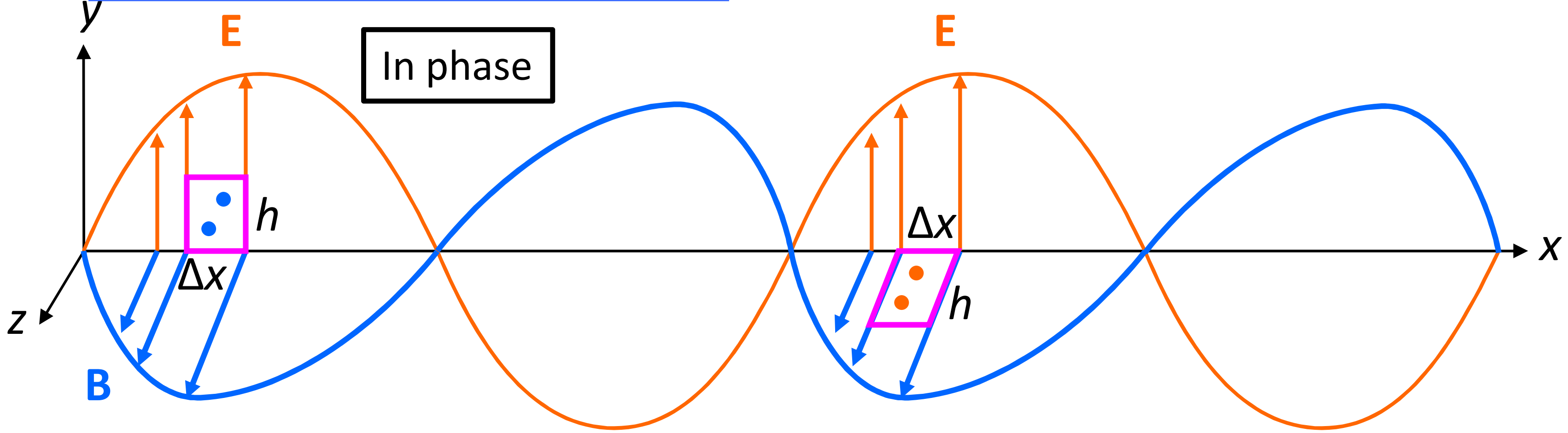
$$\vec{B}(x,t) = B_p \cos(kx - \omega t)\hat{k}$$

Amplitudes related

$$E_p = cB_p$$

speed of wave

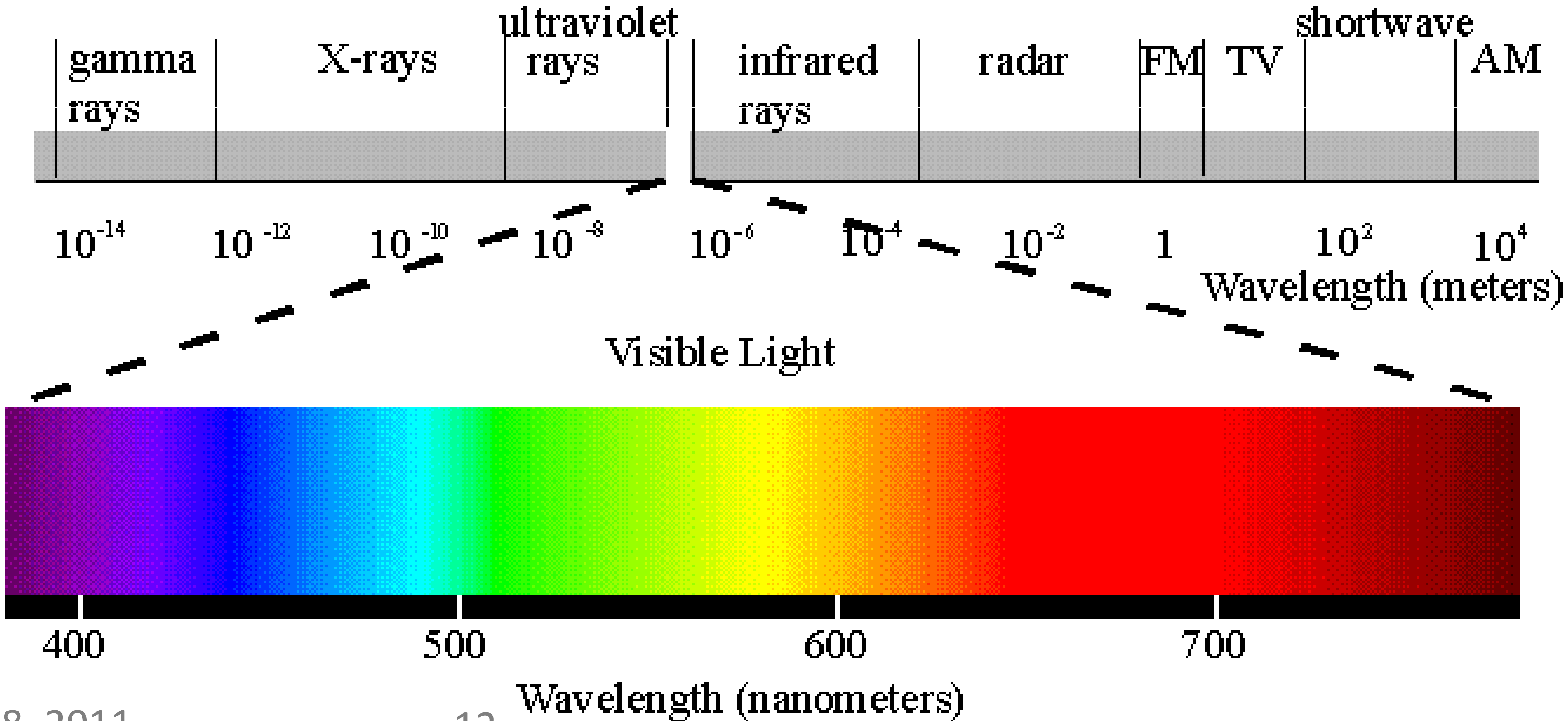
$$c = \omega/k$$



In phase

Electromagnetic Spectrum

- Continuum of allowed wavelengths (frequencies) as long as $\boxed{c = \lambda f}$



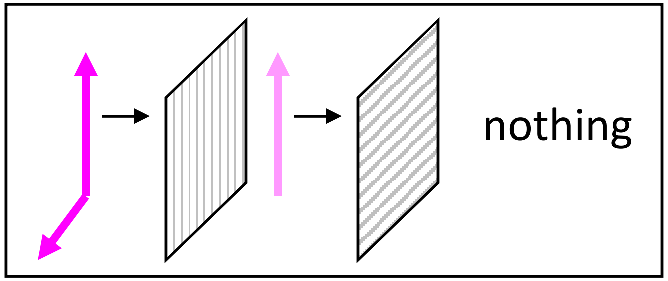
Polarization

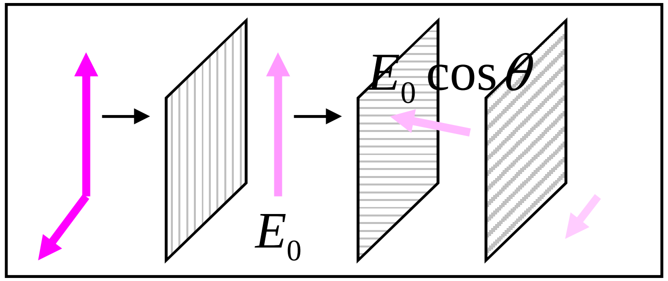
- Specifies direction of **E** field

$$\vec{E}(x,t) = E_p \cos(kx - \omega t)\hat{\imath}$$

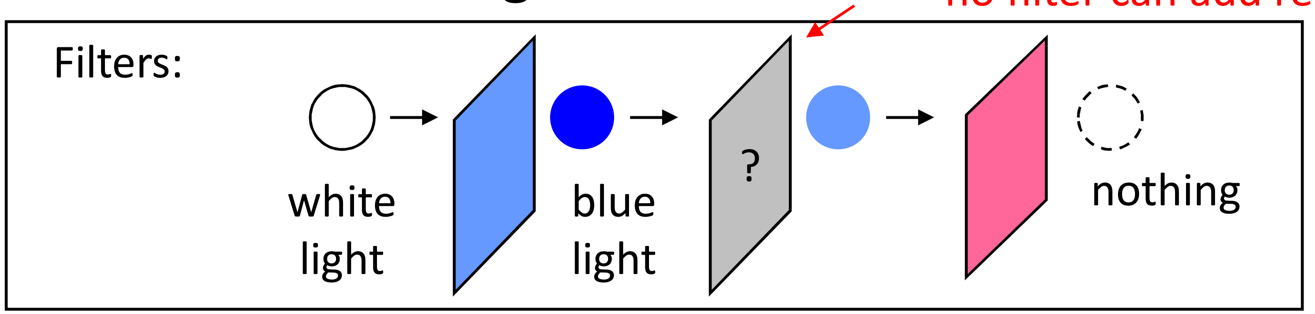
  - Polarization is perpendicular to **B**, **v**
- Unpolarized: superposition of waves with a random orientation of polarizations
  - i.e. visible light from sun, light bulb
- Unpolarized becomes polarized…
  - Reflection from surfaces (partial)
  - POLARIZER (crystal with preferred direction of transmission, called transmission axis)

Polarizers



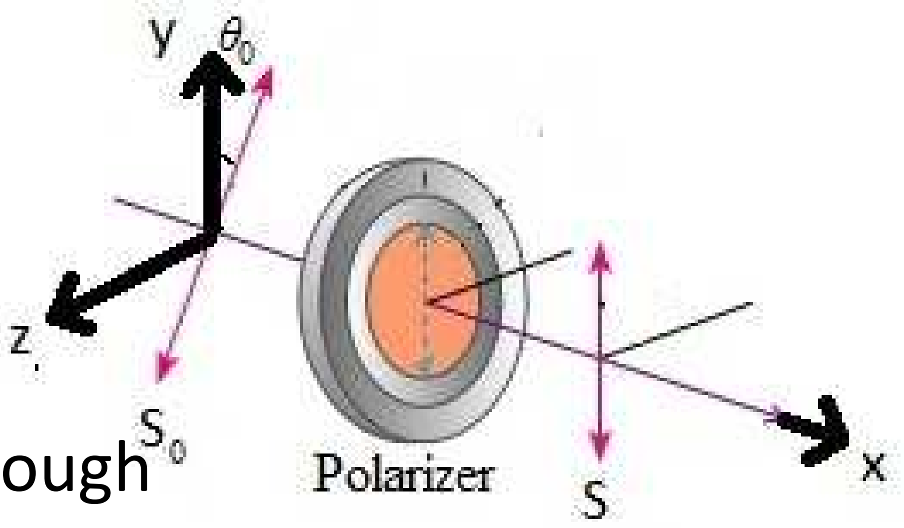


- Polarizers *reorient a component* of the incident wave
  - The output light is *completely* polarized along the polarizer axis
  - Polarizers *change* the light
  - Polarizers are *not* filters
    - Filters can only decrease (or leave unchanged) the light
    - Filters cannot *add* light



no filter can add red light

Polarizers – Law of Malus



- Only component of E along transmission axis will pass through unattenuated

$$E_f = E_0 \cos\theta_0$$

- Intensity (S) varies as amplitude squared

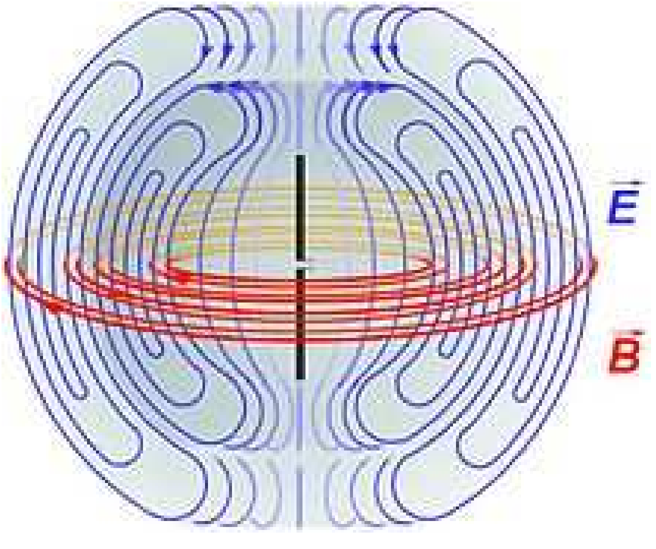
$$S = S_0 \cos^2 \theta_0$$

- For unpolarized light, average over ever direction to get emerging intensity

$$S_{unpolarized} = \frac{1}{2} S_0$$

Producing EM Waves

- Changing B→ E; changing E→B
  - Accelerated electrical charges
- If motion of charges is periodic, EM with that frequency
- Systems are “good” transmitter/receivers if size of system d~λ
- Directional antenna (TV bunny)
- Approximate plane wave

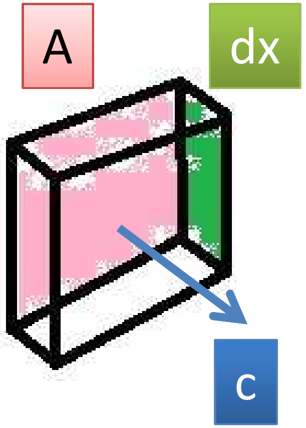


Energy in EM Waves

- Energy density (energy/volume)

$$u = \frac{1}{2}\left(\varepsilon_0 \left|\vec{E}\right|^2 + \left|\vec{B}\right|^2 \middle/ \mu_0\right)$$

- Want intensity (energy/time/area)



-

$$dU = udVol = uAdx$$

$$\frac{dU}{dt} = uA\frac{dx}{dx/c}$$

$$S = \frac{1}{A}\frac{dU}{dt} = \frac{c}{2}\left(\varepsilon_0 \left|\vec{E}\right|^2 + \left|\vec{B}\right|^2 \middle/ \mu_0\right) \qquad \left(E_p = cB_p\right)$$

- Can rewrite

$$\begin{tikzcd} & S = \varepsilon_0 c\left|\vec{E}\right|^2 \\ S = \dfrac{\left|\vec{E}\right|\left|\vec{B}\right|}{\mu_0} \arrow[ur] \arrow[dr] & \\ & S = \dfrac{c\left|\vec{B}\right|^2}{\mu_0} \end{tikzcd}$$

Poynting Vector

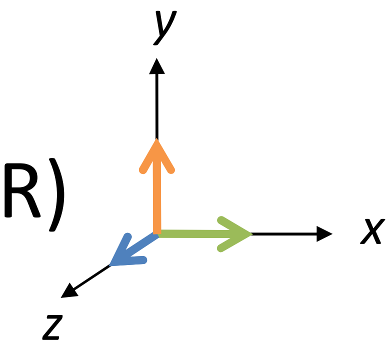
$$\boxed{\vec{E}(x,t) = E_p \cos(kx - \omega t)\hat{j}}$$

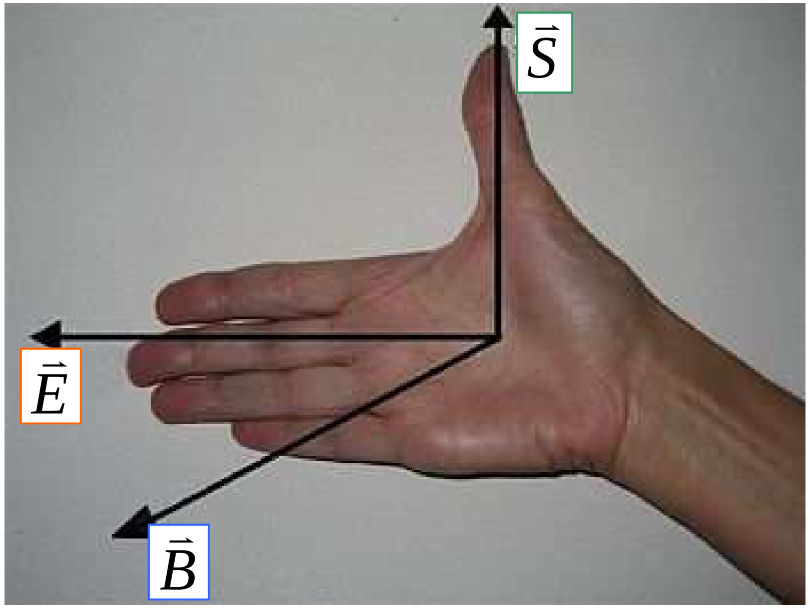
$$\boxed{\vec{B}(x,t) = B_p \cos(kx - \omega t)\hat{k}}$$

- Energy/area/time

$$\boxed{\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}}$$

- Points in direction of propagation (RHR)
- Wavevector $\boxed{\vec{k}}$





Average intensity (magnitude)

- Poynting vector magnitude in terms of E

$$\boxed{\left|\vec{S}\right| = \frac{\left|\vec{E}\right|\left|\vec{B}\right|}{\mu_0} = \frac{E_p B_p}{\mu_0}\cos^2\left(kx - \omega t\right)}$$

- Average value cos^2=1/2

$$\begin{tikzcd}
 & \bar{S} = \frac{\varepsilon_0 c E_p{}^2}{2} \\
\boxed{\bar{S} = \frac{E_p B_p}{2\mu_0}} \arrow[ur] \arrow[dr] & \\
 & \bar{S} = \frac{c\vec{B}_p{}^2}{2\mu_0}
\end{tikzcd}$$

2 lasers emit light of the same color, but the E field in laser 1 is 2x that of laser 2. compare their:

B-fields
intensities
wavelengths

Waves from point source

- Spherical waves

$$S = \frac{Power}{4\pi r^2}$$

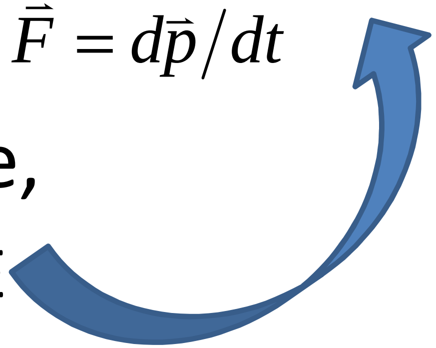
- Energy spreads in space over time, amplitude does not change
- Field strength falls off as 1/r
  - EM waves dominate in all but the immediate vicinity of accelerating charges

Wave Momentum

- EM wave energy and momentum related by

$$p = \frac{U}{c}$$

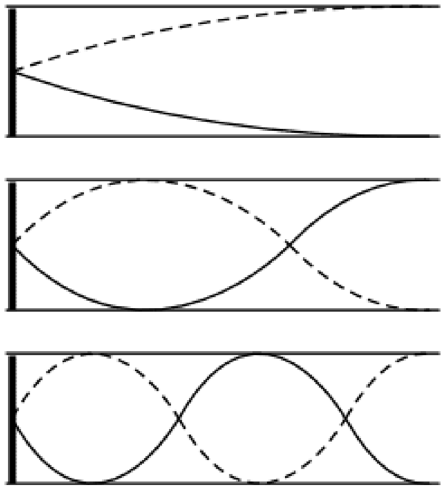
  - If average energy/t/area= $\bar{S}$
- Average momentum/t/area=> $\frac{\bar{S}}{c}$
- Newton's second law $\vec{F} = d\vec{p}/dt$
- Force/Area = Pressure,



  - RADIATION PRESSURE
- EM wave delivers 2x pressure to object on reflection

Ch 17 Review (Sound & more Waves)

- Speed of (longitudinal) wave is
  - Gases $v = \sqrt{\frac{\gamma P}{\rho}}$ (in air, 340 m/s)
  - Liq/Sol $v = \sqrt{\frac{B}{\rho}}$ $B = -\frac{\Delta P}{\Delta V / V}$
- Intensity
  - $\bar{I} = \frac{(\Delta P_0)^2}{2\rho v} = \frac{1}{2}\rho\omega^2 s_0{}^2 v$
- Decibels
  - $\beta = 10\log(I/I_0)$
- Standing waves
- Doppler effect
  - Moving source $f' = f/(1 \pm u/v)$
  - Moving Observer $f' = f(1 \pm u/v)$



Examples

- Ch 17-22

A 1-dB increase in sound level is about the minimum change the human ear can perceive. By what factor is the sound intensity increased for this dB change?

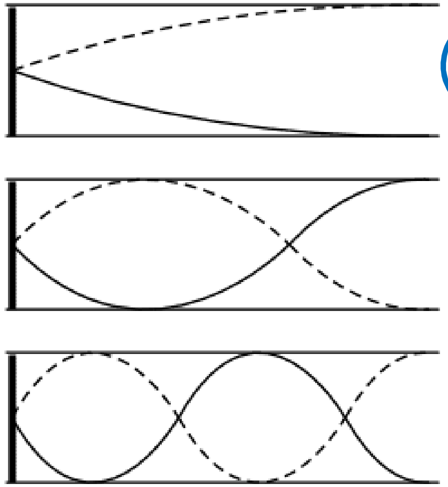
$$\Delta\beta = \beta_2 - \beta_1 = 10\log(I_2/I_1)$$

$$I_2/I_1 = 10^{\Delta\beta/10}$$

$$I_2/I_1 = 10^{1/10} = 1.26dB$$

- Ch 17 46

Estimate the fundamental frequency of the human vocal tract by assuming it to be a cylinder 15 cm long that is closed on one end.



$$L = \lambda_{\max}/4$$

$$L = 3\lambda/4$$

$$L = 5\lambda/4$$

$$v = \lambda_{\max} f_0 = 340m/s$$

$$f_0 = v/4L = (340m/s)\Big/4\left(15cm\times\frac{1m}{100cm}\right) = 570Hz$$

Ch 34 Review (EM Waves)

- Maxwell's Complete Equations

$$\oint \vec{E}\cdot d\vec{A} = q_{encl}/\varepsilon_0$$

$$\oint \vec{B}\cdot d\vec{A} = 0$$

$$\oint \vec{E}\cdot d\vec{\ell} = -\partial\Phi_B/\partial t$$

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{encl} + \mu_0(\varepsilon_0\,\partial\Phi_E/\partial t)$$

Displacement current

$$\oint \vec{E}\cdot d\vec{A} = 0$$

$$\oint \vec{B}\cdot d\vec{A} = 0$$

$$\oint \vec{E}\cdot d\vec{\ell} = -\partial\Phi_B/\partial t$$

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0\varepsilon_0\,\partial\Phi_E/\partial t$$

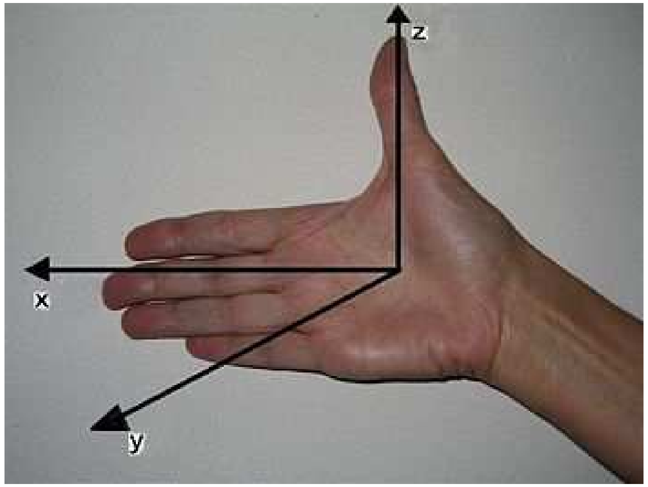
$$\boxed{\frac{\partial^2\vec{E}}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}}$$

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = \frac{\omega}{k}$$

$$\boxed{\vec{E}(x,t) = E_0\cos(kx-\omega t)\hat{j}} \quad \boxed{\vec{B}(x,t) = B_0\cos(kx-\omega t)\hat{k}} \qquad E_0 = cB_0$$

polarization

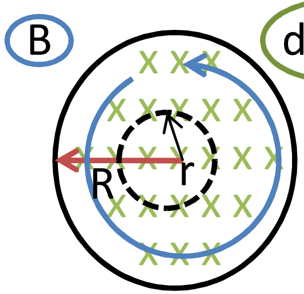
- Intensity:
  - Poynting vector $\boxed{\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}}$
- $\bar{S}_0 \to \bar{S}_0\cos^2\theta \qquad (\bar{S}_0 \to \bar{S}_0/2)$
- EM momentum $p = \frac{U}{c}$
  - Rad Press $\frac{\bar{S}}{c}$



Examples

- Ch 34-6

An E-field points into the page and occupies a circular region of radius 1m. There is B-field forming CCW loops. The B-field strength 50cm from center is 2.0-uT. (a) What is the rate of change of E-field? (b) Is the E-field increasing or decreasing?



dE/dt=?

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0\varepsilon_0 \partial\Phi_E/\partial t$$

$$B(2\pi r) = \mu_0\varepsilon_0(\partial E/\partial t)\pi r^2$$

$$B(@\, r) = \mu_0\varepsilon_0\, r/2(\partial E/\partial t)$$

$$(\partial E/\partial t) = 2Bc^2/r$$

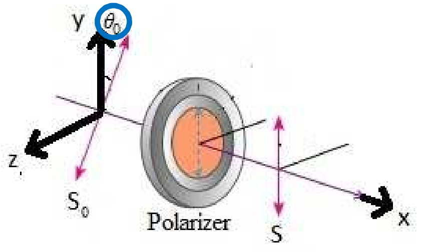
CCW B → dE/dt out of page by RHR!
So strength of E - decreasing

- Ch 34-64

Find the angle between 2 polarizers if upolarized light incident on the pair emerges with 10% of its incident intensity.

After 1st polarizer…

$$S_1 = \frac{1}{2}S_0$$



After 2nd polarizer

$$S_2 = S_1\cos^2\theta_0$$
$$= (S_0/2)\cos^2\theta_0$$

$$\frac{S_2}{S_0} = 0.10 = \frac{\cos^2\theta_0}{2}$$