Lecture 16 Notes: 07 / 27

Nuclear physics

We have seen in the last lecture that when a particle decays, if the total mass of the decay products is less than the mass of the original reactants, the excess mass is released as energy. Depending on the mass difference, the amount of energy released can be enormous.

This is what happens in nuclear reactions. Atomic nuclei consist of positively charged ***protons*** and neutral ***neutrons***, bound together by a short-range force known as the ***residual strong force*** (we will discuss later why it is called that). Protons and neutrons are collectively known as ***nucleons***. The number of protons in a nucleus is called the ***atomic number*** Z while the total number of nucleons is known as the ***mass number*** A.

Outside of the nucleus, a proton has a rest energy of 938.272 MeV / c^2, while a neutron has a slightly higher mass of 939.565 MeV / c^2. The proton is stable (or has an extremely long lifetime) while the neutron decays into a proton, an electron and another particle called a neutrino, with a lifetime of about 15 minutes. However, the mass of the nucleus is slightly smaller from the total mass of all the protons and neutrons, due to the presence of a negative ***binding energy***. Also, neutrons can become stable inside a nucleus; this is because the binding energy can essentially drop their effective mass so low that they cannot decay into a proton and an electron.

For example, deuterium is an isotope of hydrogen. The deuterium nucleus consists of a proton and a neutron, so the rest energy of the individual nucleons in it is 1877.837 MeV. However, the rest energy of the deuterium nucleus is only 1875.622 MeV; the deficit of 2.215MeV is due to the binding energy.

Fusion

This suggests a way of converting mass to energy: build nuclei out of simpler constituents. This is known as ***fusion***. If we were to put a proton and a neutron together to form deuterium, we would liberate 2.215 MeV of energy per every deuterium atom formed. Unfortunately, this is impractical, since it is difficult to get neutrons at a high enough density. There are two difficulties: neutrons are unstable, and because they lack charge, they don't respond to electromagnetic fields and tend to go right through matter, which is primarily solid due to the repulsion of the electrons. Thus they are difficult to get in large numbers, and difficult to confine to a small space where the reaction can occur.

We can get deuterium at a high density, and combining two deuterium nuclei to produce a helium 4 nucleus would liberate even more energy (about 24 MeV). However, deuterium nuclei would have to get very close together before the residual strong force could bind them into a helium nucleus. This is difficult to achieve since both nuclei are positively charged, so they repel at larger distances, where the strong force is not yet significant.

The easiest way to get the nuclei through this electrostatic barrier is to heat the deuterium up. At some point, the atoms bounce around at such a high speed that if two nuclei collide, they can break through the barrier and undergo fusion. The temperature required is on the order of millions of kelvins. Such conditions exist at the center of the Sun and other stars. As expected, fusion takes place there, generating the heat that powers the star.

Artificial controlled fusion is difficult to achieve because of the high temperatures required. The most promising approach is a machine known as a ***tokamak***. In this device, a magnetic field holds a plasma of electrons and deuterium nuclei in place while intense electromagnetic pulses heat the plasma up to the required temperature. So far, tokamaks have been able to sustain fusion for a few seconds.

Beta decay

It turns out that for nuclei of a certain mass number, there is an optimal ratio of protons to neutrons. Large deviations from this ratio result in a nucleus that has less binding energy, and therefore more mass, than the nucleus with the optimal ratio. There exists a reaction, known as ***beta decay*** that can change protons into neutrons and vice versa.

In beta decay, a neutron emits an electron and another particle called an electron antineutrino, and turns into a proton. As mentioned before, free neutrons undergo beta decay and turn into protons with a lifetime of about 15 minutes. A nucleus that has an excess of neutrons can similarly undergo beta decay, losing a neutron and gaining a proton in the process. This increases the atomic number Z by 1 while keeping the mass number A fixed.

A nucleus that has an excess of protons can instead decay by emitting a positron (which is exactly like an electron, but with a positive charge) and an electron neutrino.

The electron or positron emitted by beta decay is sometimes called a ***beta particle***. It's just a normal electron (or positron), typically with a kinetic energy on the order of tens to hundreds of keV. At this energy, the particle can penetrate skin and thin clothing and cause chemical reactions within the human body, producing various toxins and carcinogens. For this reason, exposure to beta radiation is hazardous, and can cause radiation sickness in the short run, and cancer in the long run. Beta radiation, however, is easily stopped with radiation protection, such as a lead-padded radiation suit.

The neutrino or antineutrino is a particle similar to the electron, but with almost zero mass and zero charge. Because the neutrino has no charge, it does not interact via electromagnetic interactions, so it can go through matter almost unhindered. Neutrons don't have charge either, but at least they interact via the strong force, and will interact with a nucleus if they hit one directly. A neutrino can even go through a nucleus.

Neutrinos can go through just about anything, but they are not likely to do any damage to the human body, since they just pass through it as if it wasn't there. Thus they are not a radiation hazard.

Example: Problem 18 from HW5
Carbon 14 decays via beta decay.

(a) What isotope does it turn into?

(b) The carbon 14 nucleus has a mass of 14.003241 u, and the nucleus it turns into has a mass of 14.003074 u. Suppose that the electron and the antineutrino are emitted in such a way that the nucleus remains at rest. What are the energies of the electron and the antineutrino? How fast is the electron moving (as a fraction of the speed of light)?

Solution:
(a) Beta decay increases the charge of the nucleus by 1 by converting a neutron to a proton, but it leaves the number of nucleons unchanged. Thus the product of this reaction is nitrogen 14 (which is the most common naturally occurring isotope of nitrogen).

(b) First convert the atomic masses to electron-volts: 1 u = 931.46 MeV / c^2, so the energy decreases by 0.156 MeV. But that's the change in the atomic rest energy; the nitrogen actually has an additional electron, with a rest energy of 0.511 MeV, so the decrease in nuclear rest energy is actually 0.156 + 0.511 = 0.667 MeV. By conservation of energy, this must be equal to the total energy of the electron and the antineutrino. Conservation of momentum tells us that since the nucleus is at rest both in the initial state, the total momentum must be zero in the final state as well. Since the nucleus remains at rest, the electron and the antineutrino momenta must add up to zero. Thus we have the following equations for the energy and momentum conservation:

$$E_e + E_\nu = \Delta E = 0.667 MeV$$
$$\vec{p}_e = -\vec{p}_\nu$$

We can choose the *x*-axis so that the electron moves in the positive *x* direction with momentum of magnitude p_e, while the neutrino moves in the negative *x* direction with momentum of magnitude p_ν. Then the second equation becomes

$$p_e = p_\nu$$

We have 2 equations and 4 unknowns, so we supplement the equations with energy-momentum relationships for the electron and the antineutrino. The antineutrino is nearly massless, while the electron has mass *m*:

$$E_\nu = p_\nu c \qquad E_e^2 = p_e^2 c^2 + m^2 c^4$$

Plug this into the equation for the conservation of momentum:

$$p_e = p_\nu$$
$$p_e^2 c^2 = p_\nu^2 c^2$$
$$E_e^2 - m^2 c^4 = E_\nu^2$$

Plug this into the equation for conservation of energy as follows:

$$E_e + E_\nu = \Delta E$$
$$E_\nu = \Delta E - E_e$$
$$E_\nu^2 = \Delta E^2 + E_e^2 - 2E_e \Delta E = E_e^2 - m^2 c^4$$
$$\Delta E \left(2E_e - \Delta E\right) = m^2 c^4$$
$$E_e = \frac{1}{2}\left(\frac{m^2 c^4}{\Delta E} + \Delta E\right) = \frac{1}{2}\left(\frac{(0.511)^2}{0.667} + 0.667\right) MeV = 0.529 MeV$$

This is the total energy of the electron; the kinetic energy is 0.529 - 0.511MeV = 18 keV

The energy of the antineutrino is

$$E_\nu = \Delta E - E_e = 0.667 MeV - 0.529 MeV = 0.138 MeV = 138 keV$$

This is all kinetic energy, since the neutrino has no mass and thus no rest energy.

The electron's speed can be calculated from the energy as follows

$$E = \gamma m c^2 = \frac{mc^2}{\sqrt{1 - v^2/c^2}}$$
$$\frac{v}{c} = \sqrt{1 - \frac{m^2 c^4}{E^2}} = 0.259$$

Fission and alpha decay

A nucleus that contains too many nucleons can become so large that the residual strong force is not efficient at holding it together (remember that this force is very short-ranged, and may have trouble holding on to the nucleons on the outer edge of the nucleus). In this case, part of the nucleus can break off due to repulsion between the positively-charged protons. This is known as ***fission***.

A particularly common form of fission is known as ***alpha decay***. Many heavy nuclei decay by emitting a helium nucleus; a helium nucleus produced by alpha decay is known as an ***alpha particle***. Since the alpha particle contains two protons and two neutrons, alpha decay reduces the atomic number by 2 and the mass number by 4.

Alpha particles and other fission products carry enough energy to cause damage to human cells, but are easily stopped by the skin. Thus, radioactive elements that decay only through fission are only dangerous if they enter the body through ingestion or some other means, so that fission products are emitted inside the body and cannot be stopped by the barrier of the skin. The exception is if the fission products are neutrons; these don't have an electric charge, and can penetrate almost any radiation shielding, but still have a high chance of interacting with matter in the human body and thus causing radiation damage.

Nuclear reactors and atomic bombs

When certain isotopes (notably uranium 235 and plutonium 239, but many others as well) absorb a neutron, they turn into a different, highly unstable isotopes, which almost immediately break up into a number of fission products. These fission products can include neutrons, which in turn can be absorbed by other atoms and cause more fission and even more neutrons. Such a situation, where more and more neutrons are produced and the atoms fission at a higher and higher rate, is known as a ***chain reaction***.

An uncontrolled chain reaction is used in the atomic bomb. It turns out that whether a chain reaction will happen depends on the total mass of the piece of nuclear fuel as well as on its density. For a given mass, a density above a certain threshold will lead to a chain reaction, while for a given density, a mass above a certain threshold will do the same. Thus there are two types of atomic bombs:

The first is the gun type, which was the kind used in Hiroshima. In it, a piece of nuclear fuel is fired at another piece using an explosive charge. Each piece separately does not have enough mass for a chain reaction, but put together, they do.

The other type is an implosion device. The nuclear fuel has a spherical or ellipsoidal shape, and is surrounded by a symmetric array of explosive charges. All the charges go off at the same time, compressing the nuclear fuel. This causes it to achieve a critical density for the chain reaction. Most modern fission bombs are of this type.

A hydrogen bomb is an uncontrolled fusion device that uses a fission bomb to heat up the deuterium (or some other fusion fuel) to a temperature high enough for fusion. Fusion produces much more energy than fission, so hydrogen bombs can generate much more explosive power than fission bombs.

A nuclear reactor is constructed in such a way that enough neutrons are produced to keep the fission reaction going, but not so many that the reaction speeds up. Neutrons can be absorbed by devices known as control rods; when these are inserted into the reactor, neutrons are absorbed and the reaction slows down. When they are removed, the reaction speeds up. Of course, nuclear reactors are typically constructed in such a way that a runaway chain reaction is not possible even with the control rods out, but there are a number of other safety challenges.

Gamma decay

When a nucleus undergoes one of the other types of nuclear reactions, it is often left in an excited state (this is just like the excited state of atomic orbitals from chemistry). Just as atoms in an excited state can emit light and settle back into the ground state, a nucleus in an excited state can emit a photon and settle back into its ground state. The photon has a much higher energy (and therefore higher frequency and shorter wavelength) than in the case of atomic orbitals, since the nuclear binding energy is much higher than the atomic binding energy.

The photon emitted from gamma decay is known as a ***gamma ray*** or ***gamma particle***. Gamma particles are among the worst radiation hazards, besides perhaps neutrons. They have high penetrating power, and can go through several centimeters or even meters of lead. This makes it impossible to make a radiation suit that can stop gamma rays. However, like neutrons, they have a fairly high chance of interacting within the human body, causing harmful chemical reactions and damaging DNA.

The structure of nucleons

It turns out that protons and neutrons are not elementary particles, but are instead composed of more fundamental constituents known as ***quarks***. The quarks come in different types, known as ***flavors***. The most common types of quarks are the called the ***up quark*** or ***u quark*** and the ***down quark*** or ***d quark***. Quark charges are fractions of the electric charge. The *u* quark has a charge of +2/3*e*, while the *d* quark has a charge of -1/3*e*. The proton consists of two *u* quarks and a *d* quark, with a total charge of +2/3+2/3-1/3 = +1, while a neutron consists of a *u* quark and two *d* quarks, and thus has a total charge of +2/3-1/3-1/3 = 0.

Quarks have counterparts known as the ***anti-quarks***. The anti-quarks have the same mass as the quarks, but an opposite charge. Thus the up anti-quark has a charge of -2/3, while the down anti-quark has a charge of +1/3.

Quarks interact via the ***strong nuclear force***. The strong nuclear force is very similar to electromagnetism at a short range, but when quarks bound by the strong force too far away from each other, the force gets stronger and stronger, approaching infinity for infinite separation. For this reason, it is not possible to produce a free quark. As particles that carry the electromagnetic force are called photons, the particles that carry the strong force are known as ***gluons***.

Bound states of quarks can consist of either three quarks (or three anti-quarks) or a quark or an anti-quark. The three-quark states are known as ***baryons*** (or ***anti-baryons***); the quark-antiquark states are known as ***mesons*** (or ***pions*** if they consist of only up and down quarks and antiquarks). All particles made of quarks and antiquarks are known as ***hadrons***.

Let's see what kind of bound states we can construct from up and down quarks. The possible baryons are:

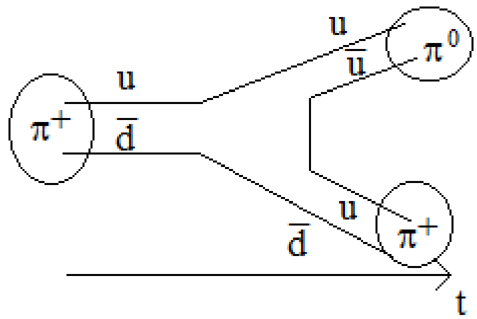
Quark content	Charge	Mass, MeV	Name
uuu	+2	1232	Delta^{++}
uud	+1	938	Proton
udd	0	940	Neutron
ddd	-1	1232	Delta^{-}

The possible mesons (pions) are

Quark content	Charge	Mass, MeV	Name
$u\bar{d}$	+1	140	π^+
$u\bar{u}$ or $d\bar{d}$	0	135	π^0
$d\bar{u}$	-1	140	π^-

In reality, there are several more types of quarks, known as strange, charm, bottom and top quarks. These all have higher masses, and produce higher-mass, unstable mesons and baryons. Because of the six types of quarks, the large variety of possible combinations and the additional complication of excited states, there is a very large variety of mesons and baryons.

What happens if we try to pull a quark-antiquark pair apart? This can happen if a pion undergoes a collision, and a quark and an antiquark get accelerated from each other as a result. The binding potential energy will increase as the particles move apart from each other, and at some point, it can become high enough to produce the rest energies of another quark-antiquark pair. In this case, a quark and an antiquark of the same type can be produced from nothing. The process looks something like this:



Time is on the horizontal axis. At some point, the quarks comprising the positively charged pion begin to come apart. When the separation becomes great enough, a quark-antiquark pair appears; the quark goes with the separated antiquark, while the antiquark goes with the separated quark. This particular process produces a positively charged pion and a neutral pion.

Thus protons, neutrons and other hadrons are composed of quarks held together by the strong force. What holds the protons and neutrons in the nuclei together? Just as the electromagnetic force that holds electrons inside atoms produces residual effects that then bind atoms together into molecules, so the strong force between quarks produces a residual force that binds protons and neutrons into nuclei. This residual force is very complicated and short-range, just as the interatomic forces are.

Leptons and weak interactions

The electron and the electron neutrino are known as ***leptons***. Just like quarks, they have antiparticles, the positron and the electron antineutrino.

There are other kinds of leptons as well; the muon and the tau are basically heavier, unstable versions of the electron, and they have corresponding neutrinos as well, known as the muon and the tau neutrino. Electrons, muons and tau all have charge of -1*e*; their antiparticles have the same mass but an opposite charge. Neutrinos are uncharged.

Leptons do not participate in the strong interactions that hold quarks together; thus they are not bound together in hadrons, and can exist as free particles. Electrons, muons and tau have charge, and therefore participate in electromagnetic interactions. Neutrinos do not.

All leptons, even neutrinos, as well as quarks, participate in the ***weak interactions***. These involve the creation of a very heavy intermediate particle, known as a ***weak boson***. There is a neutral Z boson and charged W^+ and W^- bosons, with charges of +1*e* or -1*e*, respectively.

We will finish up with the weak interactions on Thursday.