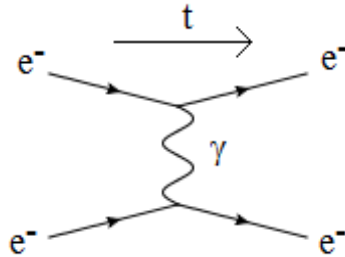


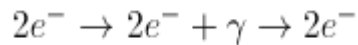
## Lecture 16-17 Notes: 07 / 27 and 07 / 28

### Particle physics Quantum electrodynamics

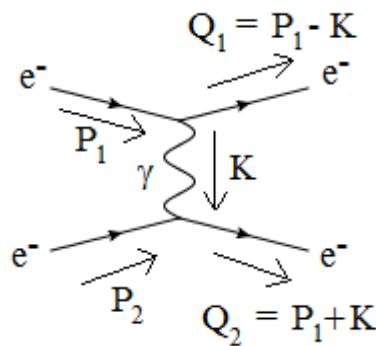
On a microscopic level, electromagnetic interactions between electrons can be viewed as the electrons exchanging photons. One electron emits a photon, and another absorbs it:



A drawing like this is called a **Feynman diagram**. It corresponds to a well-defined mathematical expression that can be used to determine the probability of the electrons interacting in a particular way. Note that this is somewhat similar to a diagram for a chemical reaction that has two electrons in the initial and final state, and two electrons and a photon in the intermediate state:



The photon allows for transfer of energy and momentum between the two electrons. We label the electrons' initial 4-momenta  $P_1$  and  $P_2$ , the final 4-momenta  $Q_1$  and  $Q_2$ , and the 4-momentum of the photon  $K$ :



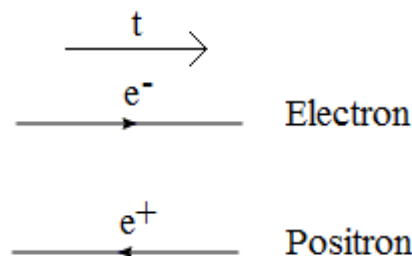
The 4-momentum is conserved at every vertex (point of interaction between a photon and an electron): if an electron emits a photon with 4-momentum  $K$ , its own 4-momentum decreases by  $K$ , while the 4-momentum of the electron that absorbs the photon is increased by  $K$ . This ensures that the 4-momentum for the overall process is conserved:  $Q_1 + Q_2 = P_1 + P_2$ .

The photon appears as an intermediate in the process, but not in the initial or the final state. Intermediate particles like this are known as **virtual particles**. Particles that are actually observed either in the initial or in the final state are known as **real particles**. Real particles obey the energy-momentum relationship  $E^2 = p^2c^2 + m^2c^4$  ( $E = pc$  for the massless photon), but the energy and momentum of virtual particles can deviate from this relationship. However, the process depicted in the diagram is more likely to occur for small deviations than for large ones.

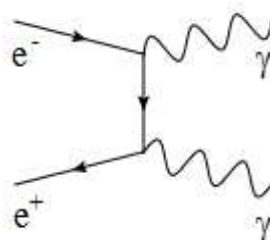
Because the energies and momenta of the electrons change due to the exchange of the photon, the paths of the electrons change direction as a result of this reaction. Calculating exactly how the photons and electrons behave from the Feynman diagrams gives the same results as classical electrodynamics, provided we look at the average behavior of a large number of photons and electrons rather than individual particles.

### Anti-particles

The arrows on the electron lines show the direction of negative charge flow. If the arrow points to the right (forward in time), then negative charge flows in the direction of the particle's motion, just as we expect for a moving electron. It turns out we can turn the arrow backwards. Then, the negative charge follows in a direction opposite to the particle's motion, which corresponds to a positively charged particle. The positively charged counterpart of an electron is known as a **positron**.



The positron has exactly the same properties as an electron (mass, interactions, etc) except that its charge is positive rather than negative. An electron and a positron can **annihilate** to form a pair of photons (it turns out that forming just one photon makes it impossible to satisfy the mass-energy relationship for the incoming particles and the outgoing photon):



Since the electron and the positron both have a rest mass of  $0.511 \text{ MeV} / c^2$ , and the photons have no rest mass, annihilation of an electron and a positron at rest causes  $1.022 \text{ MeV}$  of energy to be liberated in the form of photons. Annihilation into photons is the most efficient way of converting mass to energy: a particle and an antiparticle react to convert 100% of their combined rest mass into energy.

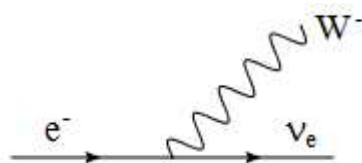
Note that the processes above conserve charge at each vertex. Whatever charge flows into a vertex must flow out, as indicated by the arrows on the electron and positron lines. This ensures the overall conservation of charge: in the case of electron-electron scattering, the net charge is  $-2e$  in both the initial and the final state, while in the case of electron-positron annihilation, the charge is zero in both the initial and the final state.

### Neutrinos and weak interactions

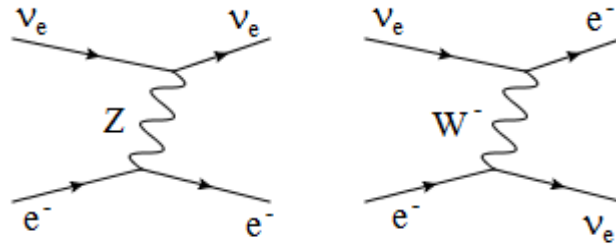
It turns out that for extremely high particle energies or temperatures (such as those found in the early Universe shortly after the Big Bang, or in particle accelerators), there are 4 different photon-like particles. But as the energy is decreased, there is a phase transition that causes three of these particles to acquire masses. The particle that remains massless is the photon; the other three are called the  $W^+$ ,  $W^-$  and  $Z$  bosons.

The photon and the  $Z$  boson have no electric charge, while the  $W$  bosons have charge of either  $+e$  or  $-e$ . The masses of the three massive bosons are very high: the  $W^+$  and  $W^-$  have masses of  $80.4 \text{ GeV}/c^2$ , while the  $Z$  has a mass of  $91.2 \text{ GeV}/c^2$ . For comparison, the mass of a proton is  $0.938 \text{ GeV}/c^2$ , so these particles are almost 100 times as heavy as a proton.

An electron can emit a  $W^-$  boson and turn into a particle called an *electron neutrino*, denoted by the symbol  $\nu_e$ : This is a particle with nearly zero mass ( $m \ll 1 \text{ eV} / c^2$ )



Charge is conserved at the vertex, and the electron's negative charge is carried away by the  $W^-$ . This means that the electron neutrino has zero charge. Therefore, it doesn't participate in electromagnetic interactions, which in the language of quantum electrodynamics means that it doesn't emit or absorb photons. The neutrinos can still interact with other particles (for example, electrons) by exchanging  $W$  or  $Z$  bosons as shown below:

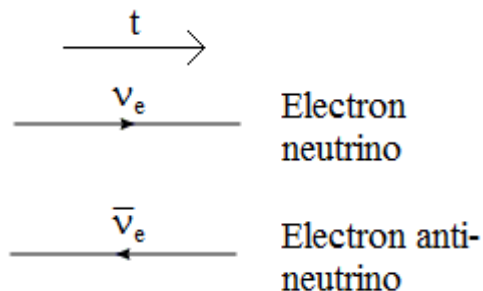


So neutrinos do interact, but only through  $W$  and  $Z$  bosons. The  $W$  and  $Z$  bosons are extremely heavy, and therefore it costs a lot of energy to form an intermediate state containing one of these particles. This acts like having a very high activation energy for a process in chemistry: these reactions are highly improbable, unless the neutrinos have an extremely high energy that can overcome this barrier.

For this reason, low-energy neutrinos (energies much less than 80-90 GeV) very rarely interact with anything. Huge numbers of such neutrinos are produced inside the Sun in nuclear reactions; almost all of these travel through the entire bulk of the Sun without ever reacting with any other particle. Those that reach the Earth simply pass through as if the Earth wasn't there, with very few exceptions. However, there is such a large number of neutrinos coming from the Sun that a few do react, so a large enough detector can in fact detect solar neutrinos. Since these particles can come from the center of the Sun without being blocked by the intervening matter of the outer layers, observing them provides a direct way of studying the Sun's interior.

Interactions involving the  $W$  and  $Z$  bosons are known as **weak interactions**. They are weak because of the high mass of the bosons, which makes producing such bosons in intermediate states highly improbable, at least for low-energy reactions. Of course, if the reacting particles have very high energies, this problem is alleviated as there is enough energy available to make the bosons, and the weak interactions become as strong as electromagnetic ones.

Just like the electrons, neutrinos have an antiparticles called anti-neutrinos. These are denoted by pointing the arrow on the neutrino line backwards, and putting a bar over the neutrino symbol:



Note that the arrows on neutrinos no longer denote the direction of flow of electric charge, since the neutrinos are neutral. However, the direction of the arrows still follows the same rule: if an arrow goes into a vertex, the other arrow must come out. This can be viewed as the conservation of a quantity known as the **electron lepton number**  $L_e$ . Electrons and electron neutrinos  $L_e = +1$ , while positrons and electron anti-neutrinos have electron lepton number  $L_e = -1$ .

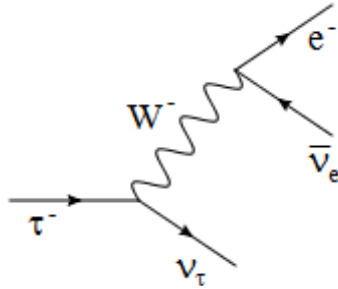
## Generations of particles

We have now encountered electrons and electron neutrinos and their antiparticles, as well as photons and  $W$  and  $Z$  bosons that mediate electromagnetic and weak interactions between these particles. It turns out that for some reason, electrons and electron neutrinos exist in three different versions, known as **generations**, in nature. In addition to the electrons, there are **muons**, which have the same properties as electrons except for a much higher mass, and **tau**, which have an even higher mass. Muons and tau have corresponding muon and tau neutrinos, which just like the electron neutrino have a nearly zero mass. The properties of these particles (collectively known as **leptons**) are summarized below:

Symbol	Name	Mass	Charge
$e^-, e^+$	Electron, positron	$0.511 \text{ MeV} / c^2$	$\pm e$
$\mu^-, \mu^+$	Muon, anti-muon	$105.7 \text{ MeV} / c^2$	$\pm e$
$\tau^-, \tau^+$	Tau, anti-tau	$1777 \text{ MeV} / c^2$	$\pm e$
$\nu_e, \bar{\nu}_e$	Electron (anti)neutrino	$\approx 0$	0
$\nu_\mu, \bar{\nu}_\mu$	Muon (anti)neutrino	$\approx 0$	0
$\nu_\tau, \bar{\nu}_\tau$	Tau (anti)neutrino	$\approx 0$	0

The rule for weak interactions of these particles is that when an electron, muon or a tau interact with a  $W$  boson, they must turn into their corresponding type of neutrino. This can be expressed in terms of conservation of electron, muon and tau lepton numbers; just as there is a conserved number  $L_e$  for electrons and electron neutrinos, so there are corresponding numbers  $L_\mu$  for muons and muon neutrinos, and  $L_\tau$  for tau and tau neutrinos. Each lepton number is separately conserved. Negatively charged particles and neutrinos have a lepton number  $+1$  of the corresponding type; their antiparticles and anti-neutrinos have lepton number  $-1$  of the same type.

Muons and tau are unstable, and can decay into lighter particles via the weak interaction. Since particles must decay into lighter products, muons can decay into electrons while tau can decay into electrons or muons. Below is an example of how a tau can decay to an electron:



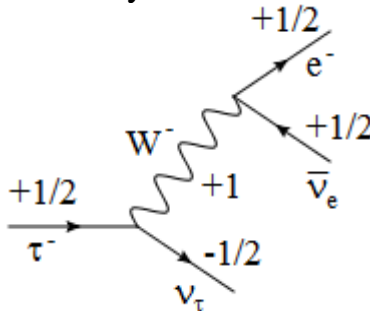
The tau can emit a  $W^-$  boson, which carries away the negative charge and turns the tau into a tau neutrino. The  $W$  boson is too heavy to be present in the final state, so it must turn into other particles. One way is to produce an electron and an electron antineutrino. The tau can thus decay by producing an electron, a tau neutrino and an electron antineutrino. Note that the lepton numbers are conserved: in the initial state, one tau is present, so the tau number is +1 and the electron number is 0. In the final state, one tau neutrino is present, so the tau number is still +1. An electron and an electron antineutrino are also present; the electron has electron number +1, while the antineutrino has electron number -1, so the net electron number is still 0.

### Conservation of spin

We briefly mention conservation of spin. Spin is a quantity that measures the angular momentum of a particle. In classical mechanics, only extended objects that rotate can have angular momentum, but in quantum mechanics, it is possible for an elementary particle to have angular momentum, provided that it is restricted to certain values. Spin simply gives the particle's angular momentum in units of Planck's constant (notice that Planck's constant has the same units as angular momentum.)

Electrons, muons, tau, and all neutrinos can have a spin of  $+1/2$  or  $-1/2$ . Photons can have a spin of  $+1$  or  $-1$ .  $W$  and  $Z$  bosons can have a spin of  $+1$ ,  $0$ , or  $-1$ . Particles with half-integer spin are called *fermions*, while those with integer spins are called *bosons*.

The spin is conserved, but since there is a choice of which value a particle's spin takes, there are often many ways to satisfy conservation of spin by assigning spins to particle lines. Here is an example using tau decay. Note that this is just one way to assign spins:



Note that spin is conserved at every vertex, and the total spin in the initial state and in the final state is  $+1/2$ .

Conservation of spin does place some restrictions on possible reactions. Note that a state with an odd number of fermions (1, 3, etc) will always have a half-integer spin, while a state with an even number of fermions (0, 2, etc) will always have an integer spin, no matter how the spins are assigned. Thus it is impossible to go from a state with an odd number of fermions to one with an even number, and vice-versa. In the tau example, we go from 1 fermion to 3 fermions, so this is allowed. It is impossible to construct a process where the tau turns into 2 fermions and a boson, for example, since such a state could not have a net spin of  $+1/2$  or  $-1/2$ .

### Quarks and the strong force

It turns out that protons and neutrons are not elementary particles, but are instead made up of simpler constituents known as **quarks**. In fact, they are made of different combinations of two types of quarks, known as the **up quark** (u) and the **down quark** (d). The up quark has an electric charge of  $+2/3e$ , while the down quark has an electric charge of  $-1/3e$ . The proton contains two up quarks and a down quark, with a net charge of  $+1$ , while the neutron contains two down quarks and an up quark, with a zero net charge. Quarks have corresponding antiparticles called anti-quarks; the up anti-quark has a charge of  $-2/3e$ , while the down anti-quark has a charge of  $+1/3e$ .

Quarks are bound together by a new type of force known as the **strong force**. The strong force is very similar to the electromagnetic force, and is carried by particles called **gluons**, which are the analogue of photons.

While the description of the forces is similar, there are some important differences between the strong force and the electromagnetic force. First, while there is only one type of photon, there are eight types of gluons. More importantly, while photons don't interact with each other (they are not charged), gluons do. Thus a gluon that is emitted by a quark can proliferate, spawning other gluons which can in turn produce more.

For this reason, while the strong force is not extremely strong if the bound quarks are close together, it becomes stronger and stronger as the quarks are pulled apart. In fact, the potential energy of the strong interaction goes to infinity as the quarks are pulled infinitely far away from each other. Thus it is not possible to completely separate bound quarks from each other, as that would require infinite energy. This property of the strong force is known as **confinement**. It means that quarks are always trapped within bound states.

It turns out that there are three possible ways of forming a bound state of quarks. One is to put three quarks together, another is to put three anti-quarks together, and another is to put together one quark and one anti-quark.

Three-quark systems are called **baryons**, three-antiquark systems are called **anti-baryons**, while a quark-antiquark system is called a **meson**. Baryons have spin of 3/2 or 1/2. Mesons have spin of 0 or 1. Let us see what kind of baryons we can form from up and down quarks:

Quark content	Charge	Mass, MeV	Name
uuu	+2	1232	$\Delta^{++}$
uud	+1	938	Proton
udd	0	940	Neutron
ddd	-1	1232	$\Delta^{-}$

The proton and the neutron are among the possible baryon states. The other particles are called delta particles, with a charge of either +2 or -1. The deltas are heavier than the proton and neutron, and unstable. The reason is this: as you might recall from your chemistry class, you can't put more than two electrons in the same orbital due to the Pauli exclusion principle. The same works with quarks inside baryons: the quarks are in orbitals of sorts, and you can't put more than two quarks of the same type into the same orbital. If there are three quarks of the same type, as for the delta particles, one of these quarks must go into a different orbital with a higher energy. This extra energy contributes to the particle's rest energy, and thus gives it additional mass.

There are additional baryons made from just up and down quarks. These correspond to excited states of protons and neutrons, where one of the quarks is boosted to a higher energy level. These excited states quickly decay back to protons and neutrons by emitting the excess energy in the form of a gamma ray (photon).

The mesons made from up and down quarks and anti-quarks are the charged and neutral **pions** (this term denotes mesons made from only up and down quarks and anti-quarks, not any of the heavier quarks that we will see later):

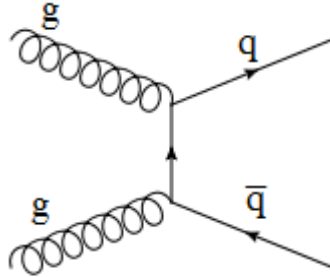
Quark content	Charge	Mass, MeV	Name
$u\bar{d}$	+1	140	$\pi^{+}$
$u\bar{u}$ or $d\bar{d}$	0	135	$\pi^{0}$
$d\bar{u}$	-1	140	$\pi^{-}$

Mesons and baryons are collectively known as **hadrons**.

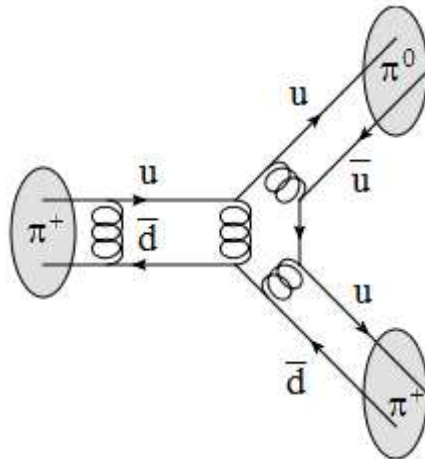


## Disintegration of hadrons

Suppose we try to pull the quark and anti-quark in a meson apart; this can happen, for example, if one of the constituent particles undergoes a high-energy collision with another particle, and ends up being accelerated away from the other constituent. As we discussed before, this will cause the potential energy associated with the strong interaction to increase. This energy is carried by the gluons in the strong field. If they have enough energy, a pair of gluons can form a quark and an anti-quark of the same type. This process is the opposite of annihilation, with gluons instead of photons:



$q$  here stands for any type of quark, but again, the type of quark and the type of anti-quark must be the same. At some point, as the quark and anti-quark in the meson are separated, the gluons trying to hold them together get enough energy that they can produce an additional quark-antiquark pair. This is what can happen when one tries to separate a charged pion,  $\pi^+$ :



The new quark-antiquark pair is produced when there is sufficient energy in the gluon field to produce it. The quark then goes with the original antiquark, and the antiquark goes with the original quark. Thus instead of two free quarks, our attempt to separate them results in the charged pion turning into a charged pion and a neutral pion. A down quark and anti-down quark pair could have formed instead; the final result would still be a charged pion and a neutral pion. Note the conservation of electric charge between the initial and the final state.

The same kind of thing happens when a quark is ejected out of a baryon. A quark-antiquark pair forms; the quark goes to fill the missing spot in the baryon, while the antiquark goes with the ejected quark to form either a neutral or a charged meson.

### Generations of quarks

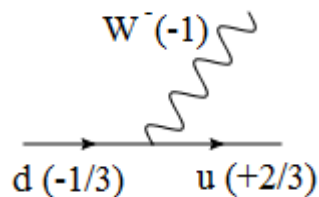
Just as with leptons, quarks come in three generations. In addition to the up and down quarks, there are strange and charm quarks, and top and bottom quarks, as well as their corresponding anti-quarks. These particles are unstable, and heavier than the up and down quarks. Mesons and baryons containing these heavier quarks are also unstable, but they can be produced in accelerators and by cosmic rays interacting with the Earth's atmosphere.

Flavor	Mass, $\text{GeV}/c^2$	Charge, $e$	Flavor	Mass, $\text{GeV}/c^2$	Charge, $e$
d (down)	0.005	-1/3	u (up)	0.002	+2/3
s (strange)	0.101	-1/3	c (charm)	1.27	+2/3
b (bottom)	4.19	-1/3	t (top)	172	+2/3

The top quark is the heaviest known elementary particle. Its mass is  $172.5 \text{ GeV} / c^2$ , almost 200 times the proton mass! Due to its extremely high mass, its lifetime is very short. In fact, it is so short that when produced, the top quark doesn't have time to be bound in a meson or a baryon before decaying. Thus there are no mesons or baryons that contain the top quark.

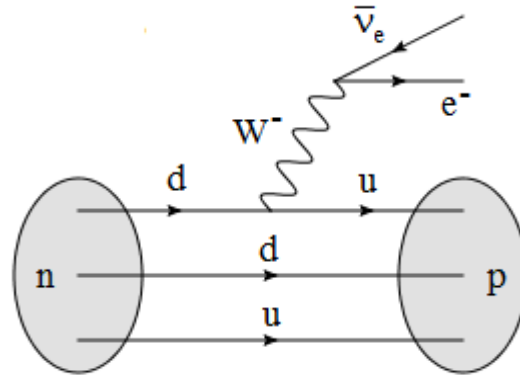
### Weak interactions of quarks

An down quark has a charge of  $-1/3e$ . It can emit a  $W^-$  boson, with a charge of  $-e$ , and transform into an up quark, with a charge of  $+2/3e$ . Note that the total charge of the initial state and that of the final state are the same:



An up quark can similarly produce a  $W^+$  boson and turn into a down quark. This process is responsible for neutron decay. Free neutrons (outside of nuclei) are unstable, with a lifetime of about 15 minutes. They decay into a proton, an electron, and an electron antineutrino (note the conservation of charge, spin and lepton number.)

Neutron decay happens like this:



One of the down quarks in a neutron turns into an up quark by emitting a  $W^-$  boson. The neutron now has two up quarks and a down quark, and has thus turned into a proton. The  $W^-$  then turns into an electron and an electron antineutrino. Since a neutron is slightly heavier than an electron plus a proton (plus a neutrino, which has negligible mass), this reaction is energetically favorable, and neutrons spontaneously decay. However, because the difference in mass is very small, and because a very heavy  $W$  boson is involved as an intermediate state, this reaction is very slow. The neutron lifetime of 15 minutes is extremely long by particle physics standards.

### Nuclear physics

We have seen in the last lecture that when a particle decays, if the total mass of the decay products is less than the mass of the original reactants, the excess mass is released as energy. The amount of energy released can be enormous.

This is what happens in nuclear reactions. Atomic nuclei consist of positively charged **protons** and neutral **neutrons**, bound together by a short-range force known as the **residual strong force** (we will discuss later why it is called that). Protons and neutrons are collectively known as **nucleons**. The number of protons in a nucleus is called the **atomic number  $Z$**  while the total number of nucleons is known as the **mass number  $A$** .

Outside of the nucleus, a proton has a rest energy of  $938.272 \text{ MeV} / c^2$ , while a neutron has a slightly higher mass of  $939.565 \text{ MeV} / c^2$ . The proton is stable (or has an extremely long lifetime) while the neutron decays into a proton, an electron and an electron antineutrino, with a lifetime of about 15 minutes.

Things are slightly different inside the nucleus. The mass of the nucleus is slightly smaller from the total mass of all the protons and neutrons, due to the presence of a negative **binding energy**. Also, neutrons can become stable inside a nucleus; this is because the binding energy can essentially drop their effective mass so low that they cannot decay into a proton and an electron.

For example, deuterium is an isotope of hydrogen. The deuterium nucleus consists of a proton and a neutron, so the rest energy of the individual nucleons in it is 1877.837 MeV. However, the rest energy of the deuterium nucleus is only 1875.622 MeV; the deficit of 2.215 MeV is due to the binding energy.

### **Fusion**

This suggests a way of converting mass to energy: build nuclei out of simpler constituents. This is known as *fusion*. If we were to put a proton and a neutron together to form deuterium, we would liberate 2.215 MeV of energy per every deuterium atom formed. Unfortunately, this is impractical, since it is difficult to get neutrons at a high enough density. There are two difficulties: neutrons are unstable, and because they lack charge, they don't respond to electromagnetic fields and tend to go right through matter, which is primarily solid due to the repulsion of the electrons. Thus they are difficult to get in large numbers, and difficult to confine to a small space where the reaction can occur.

We can get deuterium at a high density, and combining two deuterium nuclei to produce a helium 4 nucleus would liberate even more energy (about 24 MeV). However, deuterium nuclei would have to get very close together before the residual strong force could bind them into a helium nucleus. This is difficult to achieve since both nuclei are positively charged, so they repel at larger distances, where the strong force is not yet significant.

The easiest way to get the nuclei through this electrostatic barrier is to heat the deuterium up. At some point, the atoms bounce around at such a high speed that if two nuclei collide, they can break through the barrier and undergo fusion. The temperature required is on the order of millions of kelvins. Such conditions exist at the center of the Sun and other stars. As expected, fusion takes place there, generating the heat that powers the star.

Artificial controlled fusion is difficult to achieve because of the high temperatures required. The most promising approach is a machine known as a *tokamak*. In this device, a magnetic field holds a plasma of electrons and deuterium nuclei in place while intense electromagnetic pulses heat the plasma up to the required temperature. So far, tokamaks have been able to sustain fusion for a few seconds.

### **Beta decay**

It turns out that for nuclei of a certain mass number, there is an optimal ratio of protons to neutrons. Large deviations from this ratio result in a nucleus that has less binding energy, and therefore more mass, than the nucleus with the optimal ratio. Beta decay can change protons into neutrons and vice versa.

In beta decay, a neutron emits an electron and an electron antineutrino, and turns into a proton. As mentioned before, free neutrons undergo beta decay and turn into protons with a lifetime of about 15 minutes. A nucleus that has an excess of neutrons can similarly undergo beta decay, losing a neutron and gaining a proton in the process. This increases the atomic number  $Z$  by 1 while keeping the mass number  $A$  fixed.

A nucleus that has an excess of protons can instead decay by emitting a positron (which is exactly like an electron, but with a positive charge) and an electron neutrino.

The electron or positron emitted by beta decay is sometimes called a *beta particle*. It's just a normal electron (or positron), typically with a kinetic energy on the order of tens to hundreds of keV. At this energy, the particle can penetrate skin and thin clothing and cause chemical reactions within the human body, producing various toxins and carcinogens. For this reason, exposure to beta radiation is hazardous, and can cause radiation sickness in the short run, and cancer in the long run. Beta radiation, however, is easily stopped with radiation protection, such as a lead-padded radiation suit.

The neutrinos or antineutrinos can go through just about anything, but they are not likely to do any damage to the human body, since they just pass through it as if it wasn't there. Thus they are not a radiation hazard.

In beta decay, the energy released from the reaction is distributed more or less randomly between the electron and the antineutrino. The nucleus gets very little of the energy, since it is so massive that it doesn't move too fast. The energy of the antineutrino can range between zero and the total energy released in the reaction, with the rest of the energy going to the electron.

**Example:** Problem 18 from HW5  
Carbon 14 decays via beta decay.

(a) What isotope does it turn into?

(b) The carbon 14 nucleus has a mass of 14.003241 u, and the nucleus it turns into has a mass of 14.003074 u. Suppose that the electron and the antineutrino are emitted in such a way that the nucleus remains at rest. What are the energies of the electron and the antineutrino? How fast is the electron moving (as a fraction of the speed of light)?

**Solution:**

(a) Beta decay increases the charge of the nucleus by 1 by converting a neutron to a proton, but it leaves the number of nucleons unchanged. Thus the product of this reaction is nitrogen 14 (which is the most common naturally occurring isotope of nitrogen).

(b) First convert the atomic masses to electron-volts:  $1 \text{ u} = 931.46 \text{ MeV} / c^2$ , so the energy decreases by 0.156 MeV. But that's the change in the atomic rest energy; the nitrogen actually has an additional electron, with a rest energy of 0.511 MeV, so the decrease in nuclear rest energy is actually  $0.156 + 0.511 = 0.667 \text{ MeV}$ . By conservation of energy, this must be equal to the total energy of the electron and the antineutrino. Conservation of momentum tells us that since the nucleus is at rest both in the initial state, the total momentum must be zero in the final state as well. Since the nucleus remains at rest, the electron and the antineutrino momenta must add up to zero. Thus we have the following equations for the energy and momentum conservation:

$$\begin{aligned} E_e + E_\nu &= \Delta E = 0.667 \text{ MeV} \\ \vec{p}_e &= -\vec{p}_\nu \end{aligned}$$

We can choose the  $x$ -axis so that the electron moves in the positive  $x$  direction with momentum of magnitude  $p_e$ , while the neutrino moves in the negative  $x$  direction with momentum of magnitude  $p_\nu$ . Then the second equation becomes

$$p_e = p_\nu$$

We have 2 equations and 4 unknowns, so we supplement the equations with energy-momentum relationships for the electron and the antineutrino. The antineutrino is nearly massless, while the electron has mass  $m$ :

$$E_\nu = p_\nu c \quad E_e^2 = p_e^2 c^2 + m^2 c^4$$

Plug this into the equation for the conservation of momentum:

$$\begin{aligned} p_e &= p_\nu \\ p_e^2 c^2 &= p_\nu^2 c^2 \\ E_e^2 - m^2 c^4 &= E_\nu^2 \end{aligned}$$

Plug this into the equation for conservation of energy as follows:

$$\begin{aligned} E_e + E_\nu &= \Delta E \\ E_\nu &= \Delta E - E_e \\ E_\nu^2 &= \Delta E^2 + E_e^2 - 2E_e \Delta E = E_e^2 - m^2 c^4 \\ \Delta E (2E_e - \Delta E) &= m^2 c^4 \\ E_e &= \frac{1}{2} \left( \frac{m^2 c^4}{\Delta E} + \Delta E \right) = \frac{1}{2} \left( \frac{(0.511)^2}{0.667} + 0.667 \right) \text{ MeV} = 0.529 \text{ MeV} \end{aligned}$$

This is the total energy of the electron; the kinetic energy is  $0.529 - 0.511\text{MeV} = 18\text{ keV}$

The energy of the antineutrino is

$$E_{\nu} = \Delta E - E_e = 0.667\text{MeV} - 0.529\text{MeV} = 0.138\text{MeV} = 138\text{keV}$$

This is all kinetic energy, since the neutrino has no mass and thus no rest energy.

The electron's speed can be calculated from the energy as follows

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1 - v^2/c^2}}$$
$$\frac{v}{c} = \sqrt{1 - \frac{m^2 c^4}{E^2}} = 0.259$$

### **Fission and alpha decay**

A nucleus that contains too many nucleons can become so large that the residual strong force is not efficient at holding it together (remember that this force is very short-ranged, and may have trouble holding on to the nucleons on the outer edge of the nucleus). In this case, part of the nucleus can break off due to repulsion between the positively-charged protons. This is known as ***fission***.

A particularly common form of fission is known as ***alpha decay***. Many heavy nuclei decay by emitting a helium nucleus; a helium nucleus produced by alpha decay is known as an ***alpha particle***. Since the alpha particle contains two protons and two neutrons, alpha decay reduces the atomic number by 2 and the mass number by 4.

Alpha particles and other fission products carry enough energy to cause damage to human cells, but are easily stopped by the skin. Thus, radioactive elements that decay only through fission are only dangerous if they enter the body through ingestion or some other means, so that fission products are emitted inside the body and cannot be stopped by the barrier of the skin. The exception is if the fission products are neutrons; these don't have an electric charge, and can penetrate almost any radiation shielding, but still have a high chance of interacting with matter in the human body and thus causing radiation damage.

## Nuclear reactors and atomic bombs

When certain isotopes (notably uranium 235 and plutonium 239, but many others as well) absorb a neutron, they turn into a different, highly unstable isotopes, which almost immediately break up into a number of fission products. These fission products can include neutrons, which in turn can be absorbed by other atoms and cause more fission and even more neutrons. Such a situation, where more and more neutrons are produced and the atoms fission at a higher and higher rate, is known as a ***chain reaction***.

An uncontrolled chain reaction is used in the atomic bomb. It turns out that whether a chain reaction will happen depends on the total mass of the piece of nuclear fuel as well as on its density. For a given mass, a density above a certain threshold will lead to a chain reaction, while for a given density, a mass above a certain threshold will do the same. Thus there are two types of atomic bombs:

The first is the gun type, which was the kind used in Hiroshima. In it, a piece of nuclear fuel is fired at another piece using an explosive charge. Each piece separately does not have enough mass for a chain reaction, but put together, they do.

The other type is an implosion device. The nuclear fuel has a spherical or ellipsoidal shape, and is surrounded by a symmetric array of explosive charges. All the charges go off at the same time, compressing the nuclear fuel. This causes it to achieve a critical density for the chain reaction. Most modern fission bombs are of this type.

A hydrogen bomb is an uncontrolled fusion device that uses a fission bomb to heat up the deuterium (or some other fusion fuel) to a temperature high enough for fusion. Fusion produces much more energy than fission, so hydrogen bombs can generate much more explosive power than fission bombs.

A nuclear reactor is constructed in such a way that enough neutrons are produced to keep the fission reaction going, but not so many that the reaction speeds up. Neutrons can be absorbed by devices known as control rods; when these are inserted into the reactor, neutrons are absorbed and the reaction slows down. When they are removed, the reaction speeds up. Of course, nuclear reactors are typically constructed in such a way that a runaway chain reaction is not possible even with the control rods out, but there are a number of other safety challenges.



## Gamma decay

When a nucleus undergoes one of the other types of nuclear reactions, it is often left in an excited state (this is just like the excited state of atomic orbitals from chemistry). Just as atoms in an excited state can emit light and settle back into the ground state, a nucleus in an excited state can emit a photon and settle back into its ground state. The photon has a much higher energy (and therefore higher frequency and shorter wavelength) than in the case of atomic orbitals, since the nuclear binding energy is much higher than the atomic binding energy.

The photon emitted from gamma decay is known as a *gamma ray* or *gamma particle*. Gamma particles are among the worst radiation hazards, besides perhaps neutrons. They have high penetrating power, and can go through several centimeters or even meters of lead. This makes it impossible to make a radiation suit that can stop gamma rays. However, like neutrons, they have a fairly high chance of interacting within the human body, causing harmful chemical reactions and damaging DNA.